# MATH 5071 - Problem Set 10

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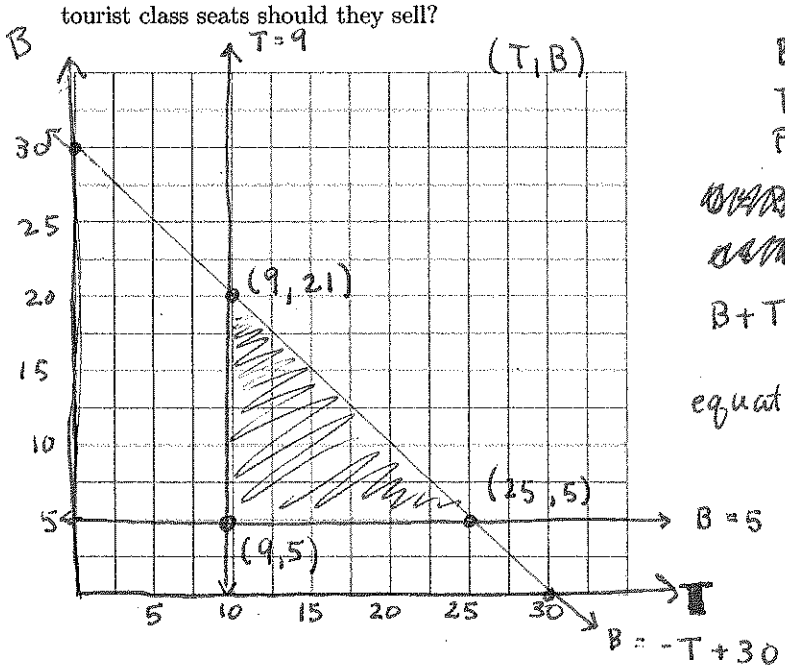
**1)** For the following Linear Programming problems, specify your variables, make a list of the constraints (inequalities), sketch the feasible region, find the vertices, and minimize/maximize the problem.

**i)** Fly-High Airlines sells business class and tourist class seats for its charter flights. To charter a plane, at least 5 business class tickets must be sold and at least 9 tourist class tickets must be sold. The plane does not hold more than 30 passengers. Fly-High makes \$40 profit for each business class ticket sold and \$45 profit for each tourist class ticket sold. In order for Fly-High Airlines to maximize its profits, how many tourist class seats should they sell?




B = business class tickets
T = tourist class tickets
P = profit (\$)

$$\left.\begin{array}{l} B \geq 5 \\ T \geq 9 \\ B+T \leq 30 \end{array}\right\} \qquad \left\{\begin{array}{l} B \geq 5 \\ T \geq 9 \\ B \leq -T+30 \end{array}\right.$$

equation to maximize: $P = 40B + 45T$

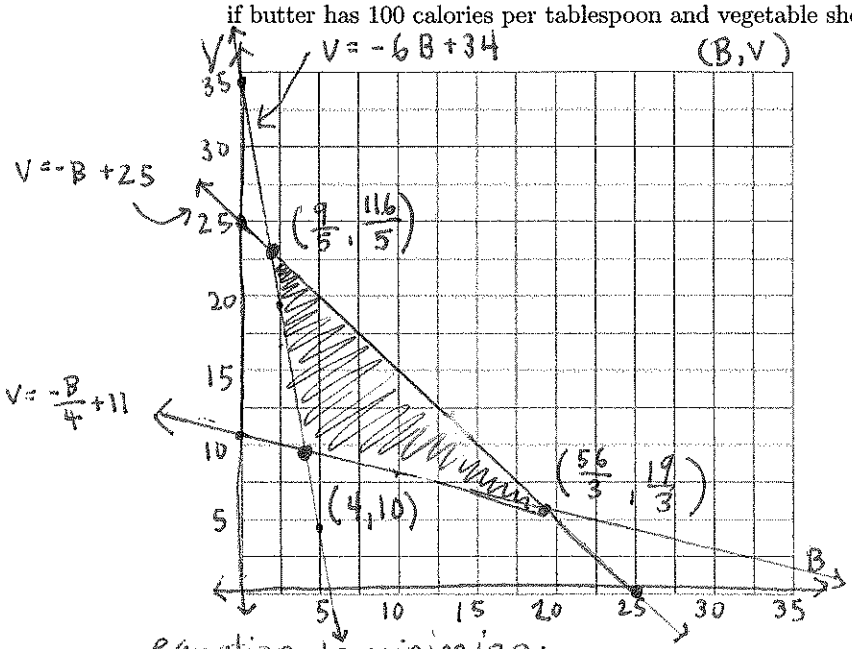
$(9,5)$: $P = 40(5) + 45(9) = \$605$

$(9,21)$: $P = 40(21) + 45(9) = \$1245$

$(25,5)$: $P = 40(5) + 45(25) = \$1325$

they should sell 25 tourist class seats and 5 business class seats in order to maximize their profits.

**ii)** Marcus is creating a low-fat pie crust recipe for his pie shop. Butter has six grams of saturated fat and one gram of polyunsaturated fat per tablespoon. Vegetable shortening has one gram of saturated fat and four grams of polyunsaturated fat per tablespoon. In the recipe, the butter and vegetable shortening will not be more than 25 tablespoons. The butter and vegetable shortening combine for at least 34 grams of saturated fat and at least 44 grams of polyunsaturated fat. Minimize the number of calories in the recipe if butter has 100 calories per tablespoon and vegetable shortening has 115 calories per tablespoon.




| | Sat. | Unsat. |
|---|---|---|
| Butter | 6 g/tbs. | 1 g/tbs. |
| Veg. | 1 g/tbs. | 4 g/tbs. |

B = tbs. of butter
V = tbs. of veg. shortening
C = calories

$B \geq 0$
$V \geq 0$
$B + V \leq 25$
sat: $6B + V \geq 34$
unsat: $B + 4V \geq 44$

$$\begin{cases} B \geq 0 \\ V \geq 0 \\ V \leq -B + 25 \\ V \geq -6B + 34 \\ V \geq -\frac{1B}{4} + 11 \end{cases}$$

equation to minimize:

$C = 100B + 115V$

Find intersections:

$-B + \cancel{25} = -6\cancel{B} + 34$
$+6B - \cancel{25} \quad +\cancel{6B} - 25$
$5B = 9 \quad B = \frac{9}{5}$
$V = -\frac{9}{5} + \frac{125}{5} \quad V = \frac{116}{5}$
$(\frac{9}{5}, \frac{116}{5})$

$-\frac{B}{4} + \cancel{11} = -6\cancel{B} + 34$
$+6B \quad \cancel{-11} \quad +\cancel{6B} \quad -11$
$\frac{24B}{4} - \frac{B}{4} = 23$
$\frac{\cancel{4}}{\cancel{23}} \cdot \frac{\cancel{23}B}{\cancel{4}} = \cancel{23} \cdot \frac{4}{\cancel{23}}$
$B = 4$
$V = -\frac{4}{4} + 11 = 10$
$(4, 10)$

$-\frac{B}{4} + \cancel{11} = -\cancel{B} + 25$
$+B \quad \cancel{-11} \quad +\cancel{B} \quad -11$
$\frac{4B}{4} - \frac{B}{4} = 14$
$\frac{\cancel{4}}{\cancel{3}} \cdot \frac{\cancel{3}B}{\cancel{4}} = 14 \cdot \frac{4}{3} = \frac{56}{3} = B$
$V = -\frac{56}{3} + \frac{75}{3} = \frac{19}{3}$
$(\frac{56}{3}, \frac{19}{3})$

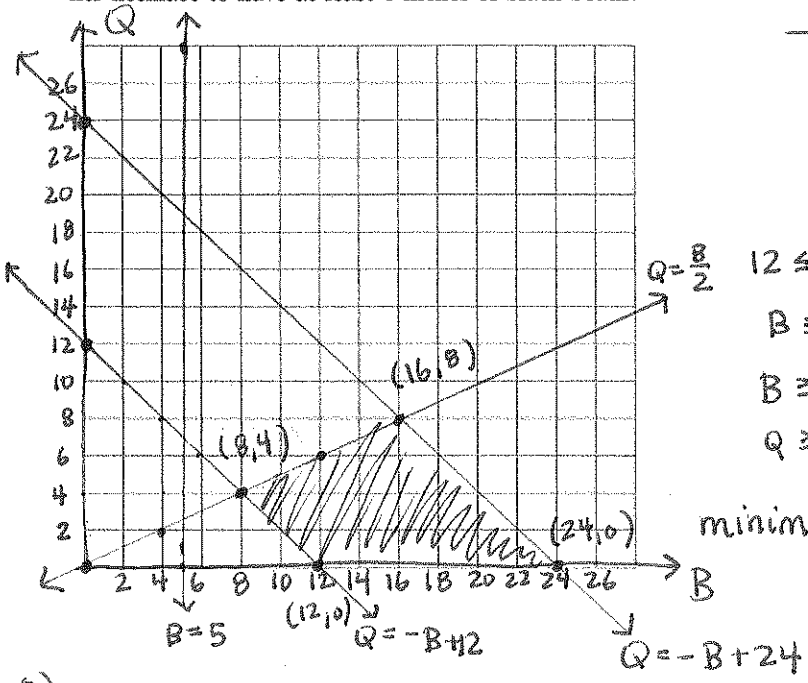
$(\frac{9}{5}, \frac{116}{5})$
$C = 100(\frac{9}{5}) + 115(\frac{116}{5})$
$= 180 + 2668 = 2848$ calories

$C = 100(4) + 115(10)$
$C = 400 + 1150$
$= 1550$ calories
minimum

$C = 100(\frac{56}{3}) + 115(\frac{19}{3})$
$= 2595$ calories

Minimum calories when using 4 tablespoons of butter & 10 tablespoons of veg. shortening.

**iii)** The Bead Store sells material for customers to make their own jewelry. Customer can select beads from various bins. Grace wants to design her own Halloween necklace from orange and black beads. She wants to make a necklace that is at least 12 inches long, but no more than 24 inches long. Grace also wants her necklace to contain black beads that are at least twice the length of orange beads. Finally, she wants her necklace to have at least 5 inches of black beads. Minimize the cost if an inch of black beads costs \$.25 and an inch of orange costs \$.40.




B = inches of black beads

Q = inches of orange beads

C = cost of the necklace (\$)

$12 \le B + Q \le 24$

$B \ge 2Q$

$B \ge 5$

$Q \ge 0$

$$\begin{cases} Q \ge -B + 12 \\ Q \le -B + 24 \\ B \ge 2Q \quad Q \le \frac{B}{2} \\ B \ge 5 \\ Q \ge 0 \end{cases}$$

minimize: $C = .25B + .4Q$

(B, Q)

(12, 0): $C = .25(12) + .4(0) = \$3$ ← minimum

(8, 4): $C = .25(8) + .4(4) = \$3.60$
$2 + 1.6$

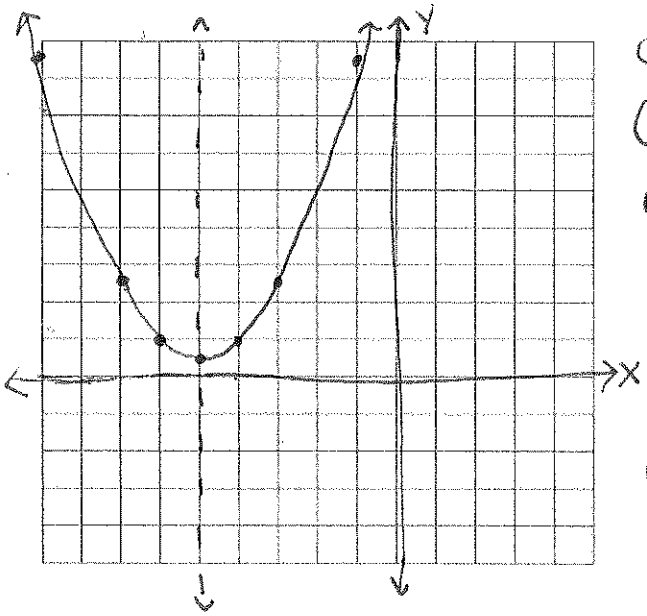
(16, 8): $C = .25(16) + .4(8) = \$7.20$
$4 + 3.20$

(24, 0): $C = .25(24) + .4(0) = \$6$

lowest cost necklace made with 12 inches of black beads and no orange beads

**2)** For the following polynomial functions, find: a. the coordinates of the vertex, b. the equation for the line of symmetry, c. the x-intercepts, d. the y-intercepts, e. whether the vertex is a max. or min., f. the domain, g. the range, and then h. graph the function.

**i)** $y = \frac{1}{2}x^2 + 5x + 13$ $\quad a = \frac{1}{2} \quad b = 5 \quad c = 13$




(a) $y = \frac{1}{2}(-5)^2 + 5(-5) + 13 = \frac{25}{2} - 25 + 13$
$= \frac{25}{2} - 12 = \frac{1}{2}$ $\boxed{(-5, \frac{1}{2})}$

(b) $x = \frac{-b}{2a} = \frac{-5}{2(\frac{1}{2})} = -5$ $\boxed{x = -5}$

(c) $0 = \frac{1}{2}x^2 + 5x + 13$ $\quad x = \frac{-5 \pm \sqrt{5^2 - 4 \cdot \frac{1}{2} \cdot 13}}{2 \cdot \frac{1}{2}}$

$x = -5 \pm \sqrt{25 - 26} = -5 \pm \sqrt{-1} = -5 \pm i$

complex = $\boxed{\text{no x-intercepts}}$

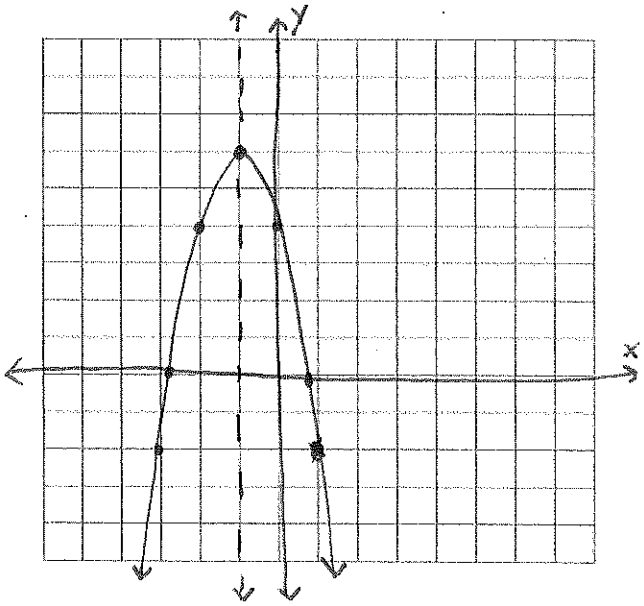
(d) $y = \frac{1}{2}(0)^2 + 5(0) + 13$ $\boxed{(0, 13)}$

(e) $\boxed{\text{min}}$ because $a > 0$

(f) $\boxed{\mathbb{R}}$

(g) $\boxed{y \geq \frac{1}{2} \text{ or } [\frac{1}{2}, \infty)}$

**ii)** $y = -2(x+1)^2 + 6$




(a) $\boxed{(-1, 6)}$

(b) $\boxed{x = -1}$

(c) $0 = -2(x+1)^2 + 6$ $\quad -6 = -2(x+1)^2$ $\quad 3 = (x+1)^2$

$\sqrt{3} = \sqrt{(x+1)^2}$ $\quad x + 1 = \pm\sqrt{3}$ $\quad \boxed{\begin{array}{l} x = (-1 + \sqrt{3}, 0) \\ x = (-1 - \sqrt{3}, 0) \end{array}}$

$-2.73$

(d) $y = -2(0+1)^2 + 6$

$y = -2 + 6$ $\quad y = 4$ $\boxed{(0, 4)}$

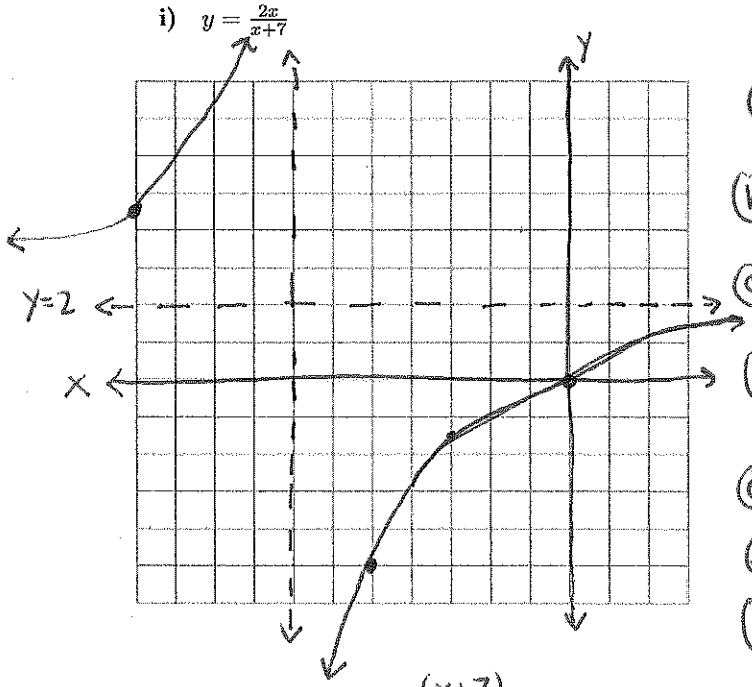
(e) $\boxed{\text{max}}$ because $a < 0$

(f) $\boxed{\mathbb{R}}$

(g) $\boxed{y \leq 6 \text{ or } (-\infty, 6]}$

**3)** For the following rational functions, find: a. the equations for any vertical asymptotes, b. the equations for any horizontal asymptotes, c. the x-intercepts, d. the y-intercepts, e. the coordinates for any holes., f. the domain, g. the range, and then h. graph the function.

i) $y = \frac{2x}{x+7}$




(a) $x+7=0$ $\boxed{x=-7}$ V.A.

(b) $y=\frac{2}{1}$ $\boxed{y=2}$ H.A.

(c) $0=\frac{2x}{x+7}$ $2x=0$ $x=0$ $\boxed{(0,0)}$

(d) $y=\frac{2(0)}{0+7}=0$ $\boxed{(0,0)}$

(e) no holes

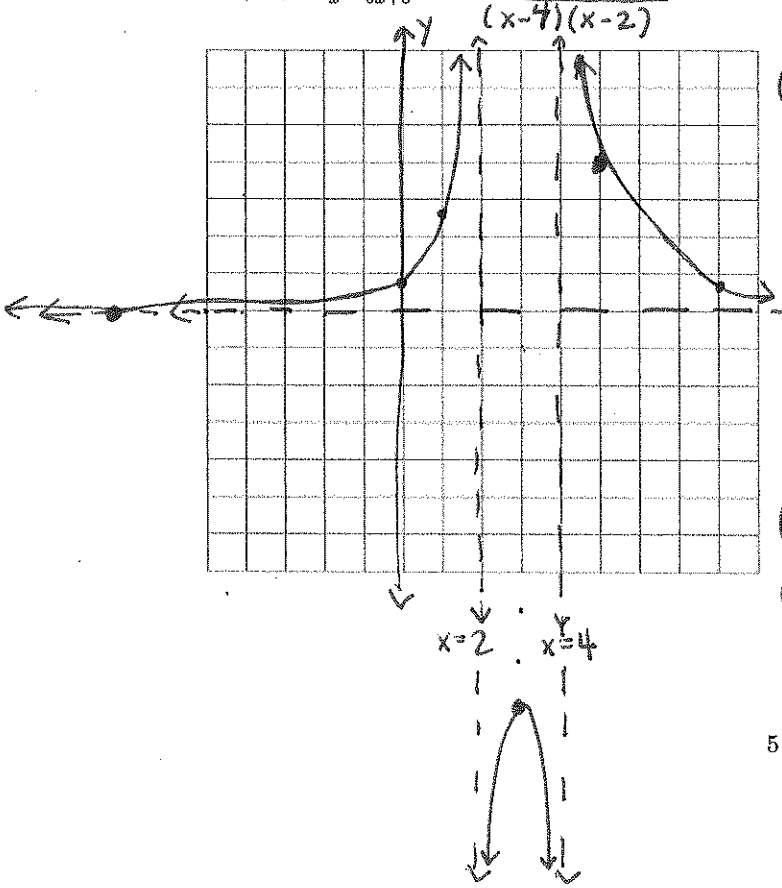
(f) $(-\infty,-7)\cup(-7,\infty)$

(g) $(-\infty,2)\cup(2,\infty)$

(h)

| x | y |
|---|---|
| -5 | -5 |
| -3 | $-\frac{3}{2}$ |
| -8 | 16 |
| -11 | 5.5 |

ii) $y = \frac{x+7}{x^2-6x+8} = \frac{(x+7)}{(x-4)(x-2)}$




(a) $x-4=0$ $\boxed{x=4}$ and $x-2=0$ $\boxed{x=2}$

(b) $\boxed{y=0}$ because degree of denom. is > degree of numerator

(c) $0=x+7$ $\boxed{x=-7}$ $\boxed{(-7,0)}$

(d) $y=\frac{0+7}{8}=\frac{7}{8}$ $\boxed{(0,7/8)}$

(e) no holes

(f) $(-\infty,2)\cup(2,4)\cup(4,\infty)$

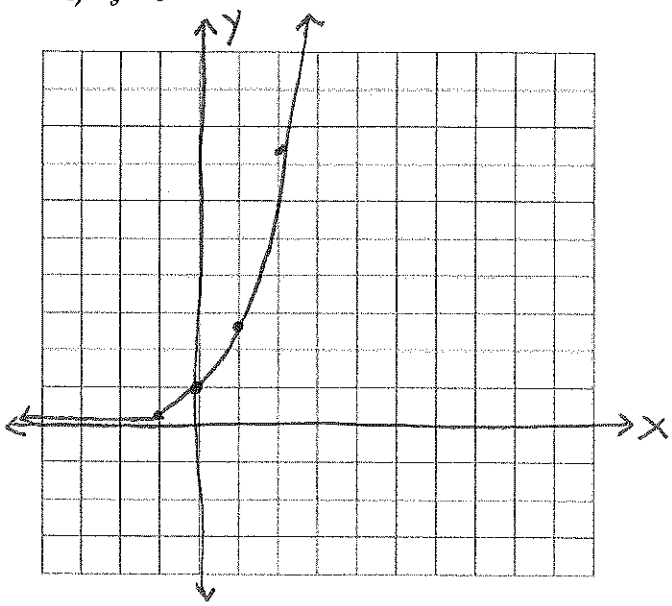
(g) $\mathbb{R}$

(h)

| x | y |
|---|---|
| 1 | $\frac{8}{3}$ |
| 3 | -10 |
| 5 | 4 |
| 8 | $\frac{5}{8}$ |

4) Make a table of values to graph the following exponential functions. Additionally, find a. any horizontal and/or vertical asymptotes, b. any x- or y- intercepts, and c. the domain and range.

i) $y = e^x$




| x | y |
|---|---|
| -1 | .368 |
| 0 | 1 |
| 1 | 2.718 |
| 2 | 7.389 |

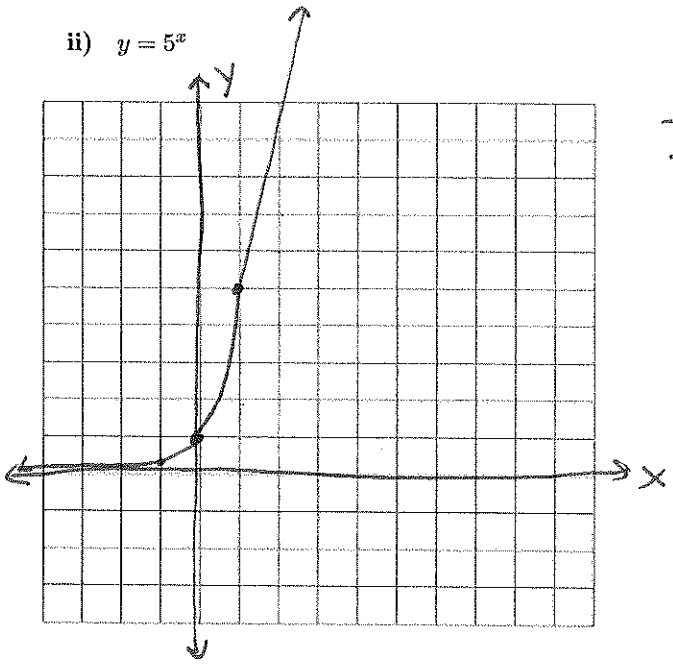
(a) $\boxed{y=0 \text{ H.A.}}$

(b) $\boxed{(0,1)}$ y-intercept

(c) D: $\mathbb{R}$

R: $(0, \infty)$

ii) $y = 5^x$




| x | y |
|---|---|
| -1 | .2 |
| 0 | 1 |
| 1 | 5 |
| 2 | 25 |

(a) $\boxed{y=0 \text{ H.A.}}$

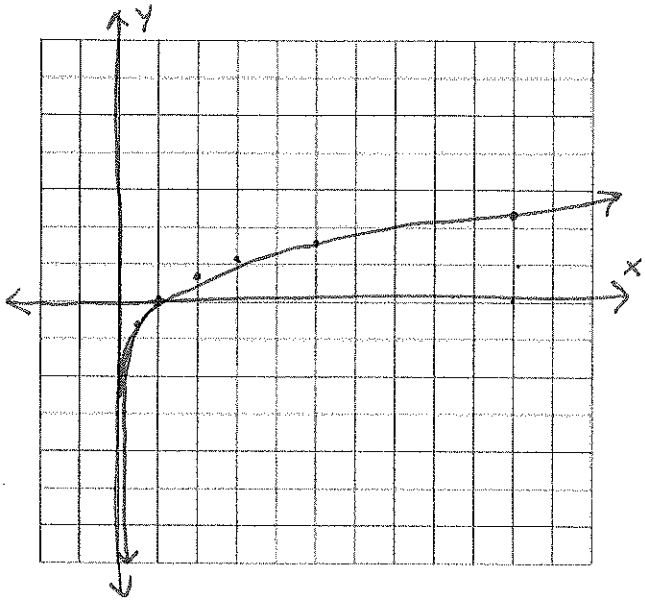
(b) $\boxed{(0,1)}$ y-intercept

(c) D: $\mathbb{R}$

R: $(0, \infty)$

**5)** Make a table of values to graph the following logarithmic functions. Additionally, find a. any horizontal and/or vertical asymptotes, b. any x- or y- intercepts, and c. the domain and range.

**i)** $y = \ln(x)$




| X | Y |
|---|---|
| -1 | undefined |
| 0 | undefined |
| .5 | -.693 |
| 1 | 0 |
| 2 | .693 |
| 3 | 1.099 |
| 5 | 1.609 |
| 10 | 2.303 |

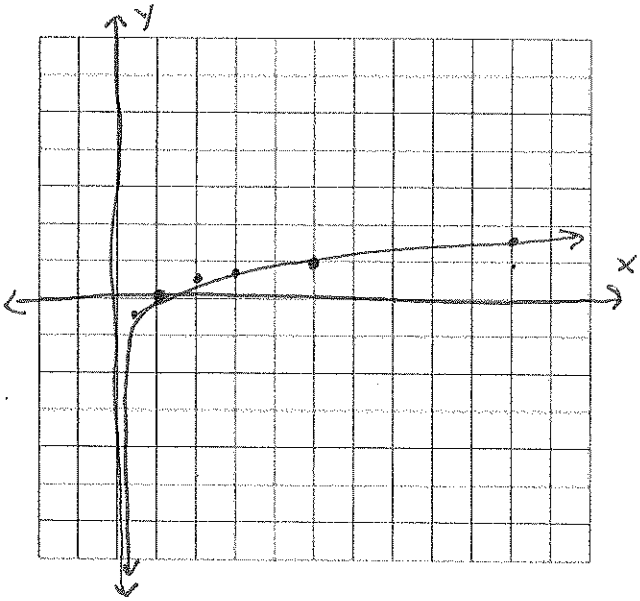
(a) $\boxed{x=0 \quad \text{V.A.}}$

(b) $\boxed{(1,0) \text{ x-intercept}}$

(c) D: $(0, \infty)$

R: $\mathbb{R}$

**ii)** $y = \log_5(x)$




| X | Y |
|---|---|
| -1 | undefined |
| 0 | undefined |
| .5 | -.43 |
| 1 | 0 |
| 2 | .431 |
| 3 | .683 |
| 5 | 1 |
| 10 | 1.431 |

(a) $\boxed{x=0 \quad \text{V.A.}}$

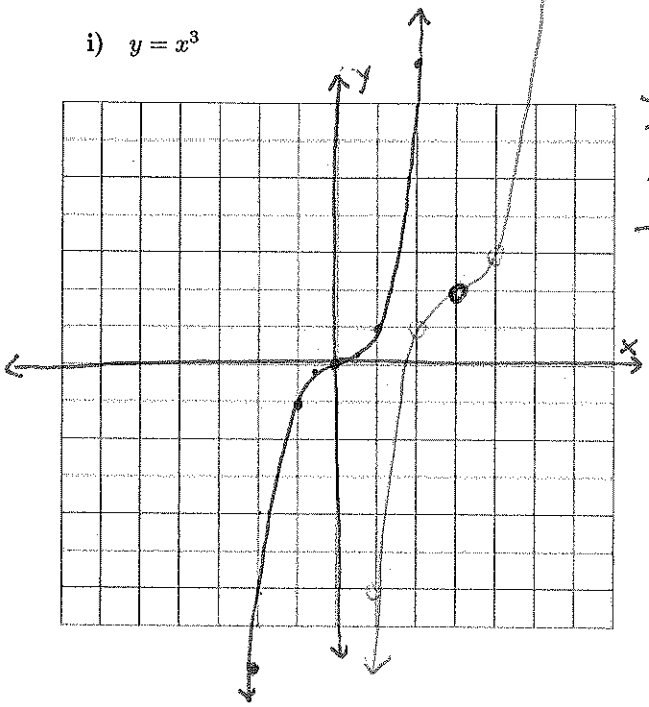
(b) $(1,0)$ x-intercept

(c) D: $(0, \infty)$

R: $\mathbb{R}$

**6)** Make a table of values to graph the following functions. Then, translate the entire function up two units and to the right 3 units. In addition, write the equation for the translated function.

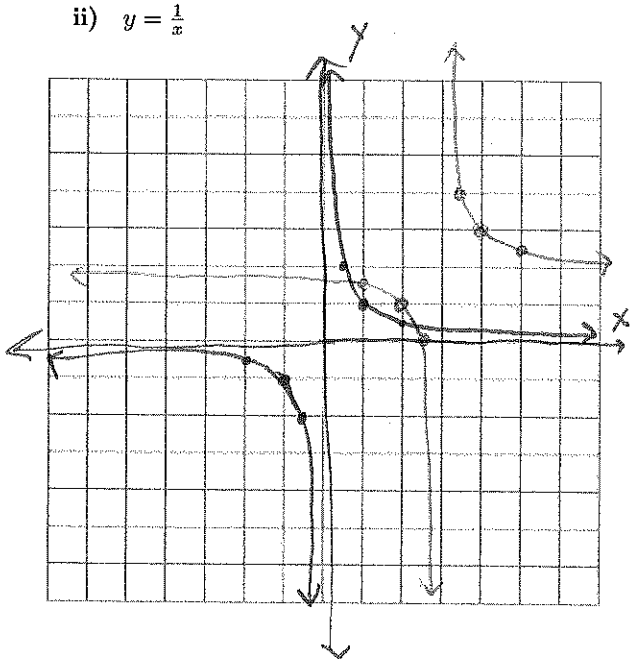
**i)** $y = x^3$




| x | y |
|---|---|
| $-2$ | $-8$ |
| $-1$ | $-1$ |
| $-\frac{1}{2}$ | $-\frac{1}{8}$ |
| $0$ | $0$ |
| $1$ | $1$ |
| $2$ | $8$ |
| $\frac{1}{2}$ | $1/8$ |

$$\boxed{y = (x-3)^3 + 2}$$

**ii)** $y = \frac{1}{x}$




| x | y |
|---|---|
| $-2$ | $-\frac{1}{2}$ |
| $-1$ | $-1$ |
| $-\frac{1}{2}$ | $-2$ |
| $0$ | undefined |
| $\frac{1}{2}$ | $2$ |
| $1$ | $1$ |
| $2$ | $\frac{1}{2}$ |

$$\boxed{y = \frac{1}{x-3} + 2}$$