MATH 5071 - Problem Set 9 - Exam 2 Review

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1) Solve the following inequalities. Report you answer with interval notation AND draw the solution set on a number line:

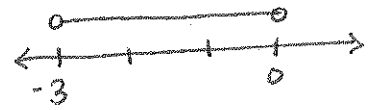
i) $1 < 1 - 2x < 7$

$1 < 1 - 2x$ $\quad$ $1 - 2x < 7$

$-1 \quad -1$ $\quad$ $-1 \quad -1$

$\frac{0}{-2} < \frac{-2x}{-2}$ $\quad$ $\frac{-2x}{-2} < \frac{6}{-2}$

$0 > x \quad x < 0$ $\quad$ $x > -3$

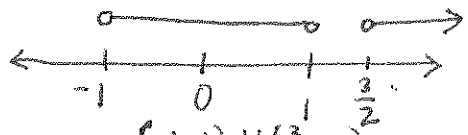


$(-3, 0)$

ii) $(x+1)(2x-3)(1-x) < 0$

$x = -1 \quad x = \frac{3}{2} \quad x = 1$

draw circles here



$(-1, 1) \cup (\frac{3}{2}, \infty)$

test values: $-2 = x$

$(-1)(-7)(3) < 0$

$21 < 0$ ✗ false

$x = -\frac{1}{2}$

$(\frac{1}{2})(-4)(\frac{3}{2}) < 0$

$-3 < 0$ ✓ true

$x = 1.25$

$(2.25)(-.5)(-.25) < 0$

$.28125 < 0$ ✗ false

$x = 2$

$(3)(1)(-1) < 0$

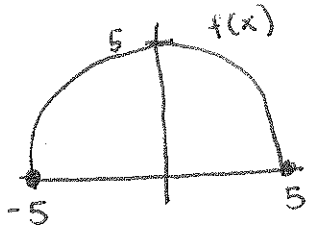
$-3 < 0$ ✓ true

2) Consider the following function: $f(x) = \sqrt{25 - x^2}$

i) Find the domain and range of $f(x)$

D: $[-5, 5]$

R: $[0, 5]$



ii) Find $f(5\sqrt{5})$

$f(5\sqrt{5}) = \sqrt{25 - (5\sqrt{5})^2}$

$= \sqrt{25 - 25(5)} = \sqrt{25 - 125} = \sqrt{-100} = \boxed{10i}$

iii) Find x when $f(x) = 7$

$7^2 = \sqrt{25 - x^2}^{\,2}$

$49 = 25 - x^2$

$+x^2 \; -49 \quad -49 + x^2$

$\sqrt{x^2} = \sqrt{-24}$ $\quad$ $\boxed{x = \pm 2i\sqrt{6}}$

3) Sonya invests \$3,000 into an account that has a 6% interest rate and is compounded monthly.

i) How much money will Sonya have after 10 years?

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

$P = 3000$
$r = .06$
$n = 12$
$t = 10$

$A = 3000\left(1 + \frac{.06}{12}\right)^{12 \cdot 10}$

$A = 3000(1.005)^{120} = \$5{,}458.19$

ii) How long will it take Sonya to double her investment?

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

$A = 2P$ (to double her money)
$r = .06$
$n = 12$

$2\cancel{P} = \cancel{P}\left(1 + \frac{.06}{12}\right)^{12t}$ $\quad 2 = (1.005)^{12t}$

$\log 2 = \log (1.005)^{12t}$

$\frac{\log 2}{12 \log(1.005)} = \frac{\cancel{12}t \log(\cancel{1.005})}{\cancel{12} \log(\cancel{1.005})}$

$t = \frac{\log 2}{12 \log(1.005)}$

$\boxed{t = 11.6 \text{ years}}$

4) Combine the following into one logarithmic expression:
$3\log(x) - \log(6) + 0.5\log(1-x) - 2\log(2x+1)$

$= \log(x^3) - \log(6) + \log(\sqrt{1-x}) - \log(2x+1)^2$

$= \boxed{\log\left[\frac{x^3\sqrt{1-x}}{6(2x+1)^2}\right]}$

5) Solve the following system of equations using any method that **doesn't** use a calculator. Check that your answer satisfies both equations. Elimination

$$\begin{cases} 3x - 4y = 4 & (1) \\ 9x + 2y = -3 & (2) \end{cases}$$

$(1) \times -3: \quad -9\cancel{x} + 12y = -12$
$(2): \quad 9\cancel{x} + 2y = -3$

$14y = -15$

$\boxed{y = \frac{-15}{14}}$

$3x - 4\left(\frac{-15}{14}\right) = 4$

$3x + \frac{4(15)}{14} = 4$

$3x + \frac{30}{7} = 4$

$3x = 4 - \frac{30}{7}$

$3x = \frac{28}{7} - \frac{30}{7}$ $\quad \frac{\cancel{3}x}{\cancel{3}} = \frac{-2}{7}\,\frac{1}{3}$

$\boxed{x = \frac{-2}{21}}$

$t = \frac{5730}{\log(.5)} \cdot \log\left(\frac{0.3}{4}\right) = \boxed{21,412.8 \text{ years}}$

6) The half-life of Carbon-14 is 5730 years. And archeologist finds a bone containing 0.3 grams of Carbon-14. If the bone originally had 4 grams of Carbon-14, how old is this bone?

$A = A_0\, b^{t/K}$

$K = 5730$

$A = 0.3\text{ g}$

$A_0 = 4\text{ g}$

$b = \frac{1}{2} = 0.5$

$\frac{0.3}{4} = \frac{\cancel{4}(.5)^{t/5730}}{\cancel{4}}$

$\log\left(\frac{0.3}{4}\right) = \log(.5)^{t/5730}$

$\log(0.3/4) = \frac{t}{5730} \cdot \log(.5)$

7) Solve by factoring: $2x^2 + 5x + 3 = 0$

$(2x+3)(x+1) = 0$

$2x + \cancel{3} = 0$
$\quad -\cancel{3} \quad -3$

$\frac{\cancel{2}x}{\cancel{2}} = \frac{-3}{2}$

$x = -\frac{3}{2}$

$x + 1 = 0$

$x = -1$

8) Solve using the quadratic equation: $2x^2 + 5x + 10 = 0$

$a = 2 \quad b = 5 \quad c = 10$

$x = \frac{-5 \pm \sqrt{(-5)^2 - 4\cdot(2)\cdot(10)}}{2\cdot(2)} = \frac{-5 \pm \sqrt{25 - 80}}{4} = \frac{-5 \pm \sqrt{-55}}{4} = \boxed{\frac{-5 \pm i\sqrt{55}}{4}}$

$x = \frac{-5}{4} + \frac{i\sqrt{55}}{4}$ And $x = \frac{-5}{4} - \frac{i\sqrt{55}}{4}$

9) Solve for x: $(x+2)^2 = (\sqrt{x+8})^2$

$(x+2)^2 = x + 8$

$x^2 + 4x + 4 = x + 8$

$x^2 + 3x - 4 = 0$

$(x-4)(x+1) = 0$

$\cancel{x = 4}$ and $\cancel{x = -1}$ check our answers

$(4+2) = \sqrt{4+8}$

$6 = \cancel{\sqrt{12}}$

$-1 + 2 = \sqrt{-1+8}$

$1 = \cancel{\sqrt{7}}$

$\boxed{\text{no solutions}}$

10) Solve by completing the square:

i) $x^2 - 10x + 16 = 0$

$x^2 - 10x = -16$

$2b = -10$

$b = -5$

$b^2 = 25$

$x^2 - 10x + 25 = -16 + 25$

$x^2 - 10x + 25 = 9$

$\sqrt{(x-5)^2} = \sqrt{9}$

$x - 5 = \pm 3$

$x = 3 + 5 = \boxed{8}$ and

$x = -3 + 5 = \boxed{2}$

ii) $x^2 + 8x + 20 = 0$

$x^2 + 8x + 16 = 20 + 16$

$8 = 2b$

$b = 4$

$b^2 = 16$

$\sqrt{(x+4)^2} = \sqrt{36}$

$x + 4 = \pm 6$

$x = 6 - 4 = \boxed{2}$

$x = -6 - 4 = \boxed{-10}$

11) The length a of the semi-major axis of a planet's elliptical orbit is directly related to the time T it takes for the planet to complete a revolution around the Sun. When T is measured in days and a in millions of kilometers, an equation modeling the situation is $\ln(T) = \frac{3}{2}\ln(a) - 1.72$. Solve this equation for a as a function of T.

$$\cancel{e^{\ln}} T = e^{\frac{3}{2}\ln(a)} e^{-1.72}$$

$$(a^{3/2})^{2/3} = (T e^{1.72})^{2/3}$$

$$\frac{T}{e^{-1.72}} = a^{3/2} \frac{e^{-1.72}}{e^{-1.72}}$$

$$a^{3/2} = \frac{T}{e^{-1.72}}$$

$$a = T^{2/3} e^{1.72 \cdot \frac{2}{3}}$$

$$\boxed{a = T^{2/3} e^{1.147}}$$

12) Find the quadratic equation that contains the following 3 points **(NO CALCULATOR)**: $(0, 21)$, $(3, 33)$, $(-5, 121)$

$$y = ax^2 + bx + c$$

$$21 = a(0)^2 + b(0) + c \qquad c = 21$$

$$33 = a(3)^2 + b(3) + c \rightarrow 33 = 9a + 3b + 21$$

$$121 = a(-5)^2 + b(-5) + c \qquad 121 = 25a - 5b + 21$$

$$12 = 9a + 3b \Rightarrow \times^{+5}(4 = 3a + b)$$

$$100 = 25a - \cancel{5b}$$

$$+20 = +15a + \cancel{5b}$$

$$\frac{120}{40} = \frac{40a}{40} \qquad \underline{\underline{a = 3}}$$

$$12 = 9(3) + 3b$$

$$12 = 27 + 3b$$

$$-27 \quad -27$$

$$\frac{-15}{3} = \frac{3b}{3} \qquad \underline{\underline{b = -5}}$$

$$\boxed{y = 3x^2 - 5x + 21}$$