MATH 5071 - Problem Set 8

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1) Solve the following equations for x:

i) $\frac{3}{x-5} + 4x = 1$

$(x-5)\cdot\frac{3}{x-5} + 4x = 1\cdot(x-5)$

$3 + 4x(x-5) = x-5$

$3 + 4x^2 - 20x = x - 5$

$4x^2 - 21x + 8 = 0$

use quad. equation

$x = 4.84$ and $x = .414$

ii) $\frac{x}{2x+1} = -\frac{4}{5-2x}$

$(5-2x)\cdot\frac{x}{2x+1} = -\frac{4}{5-2x}\cdot(5-2x)$

$\frac{(5-2x)(2x+1)}{2x+1} = -4\cdot(2x+1)$

$5-2x = -4(2x+1)$

$5-2x = -8x-4$

$9 = 6x$

$x = \frac{9}{6}$

$\boxed{x = \frac{3}{2}}$

iii) $\sqrt{3x+10} + 25 = 16$

$\sqrt{3x+10} = -9$

$3x+10 = 81$

$3x = 71$

$x = \frac{71}{3}$

plug in to check: $\sqrt{3\cdot\frac{71}{3}+10} + 25 = 16$

$\sqrt{71+10} + 25 = 16$

$\sqrt{81} + 25 = 16$

$9 + \cancel{25} = 16$

no solution

iv) $\sqrt{x+9} - \sqrt{5x+3} = 0$

$\sqrt{x+9} = \sqrt{5x+3}$

$x+9 = 5x+3$

$6 = 4x$

$x = \frac{6}{4}$

$\boxed{x = \frac{3}{2}}$

check the answ

$\sqrt{\frac{3}{2}+9} - \sqrt{5\cdot\frac{3}{2}+3} = 0$

$\sqrt{\frac{3+18}{2}} - \sqrt{\frac{15}{2}+\frac{6}{2}} = 0$

$\sqrt{\frac{21}{2}} - \sqrt{\frac{21}{2}} = 0$ ✓ works!

2) Find the distance between the following points:

distance theorem: $d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

i) $(8,1)$ and $(4,4)$

$d = \sqrt{(8-4)^2 + (1-4)^2} = \sqrt{4^2 + (-3)^2}$

$= \sqrt{16+9}$

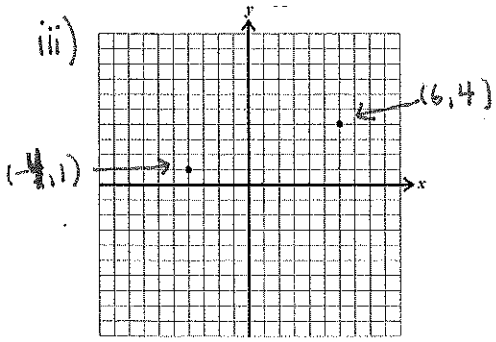
$= \sqrt{25} = 5$

ii) $(-2,1)$ and $(-4,-5)$

$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$= \sqrt{(-4+2)^2 + (-5-1)^2} = \sqrt{(-2)^2 + (-6)^2} = \sqrt{4+36} = \sqrt{40} = \boxed{2\sqrt{10}}$

iii)



$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$d = \sqrt{(6+4)^2 + (4-1)^2}$

$d = \sqrt{10^2 + 3^2}$

$d = \sqrt{100+9}$

$\boxed{d = \sqrt{109}}$

3) Graph on the complex plane:

i) $4+2i$

ii) -2

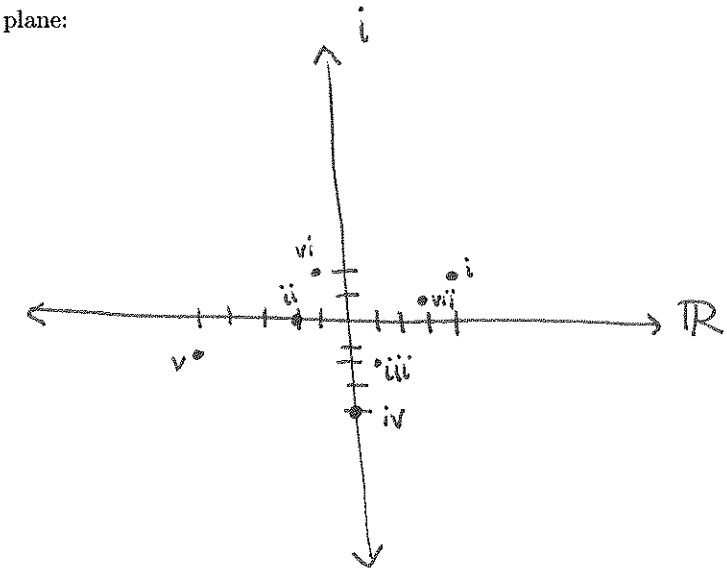
iii) $1-2i$

iv) $-4i$

v) $-5-2i$

vi) $-1+2i$

vii) $3+i$



4) Evaluate the following expressions:

i) $|4+2i|$ $a=4$ $b=2$ $|4+2i| = \sqrt{4^2+2^2} = \sqrt{16+4} = \sqrt{20} = \boxed{2\sqrt{5}}$

ii) $|-2|$ $a=-2$ $b=0$ $|-2| = \sqrt{(-2)^2+0^2} = \sqrt{4} = \boxed{2}$

iii) $|1-2i|$ $a=1$ $b=-2$ $|1-2i| = \sqrt{1^2+(-2)^2} = \sqrt{1+4} = \boxed{\sqrt{5}}$

iv) $|-4i|$ $a=0$ $b=-4$ $|-4i| = \sqrt{0^2+(-4)^2} = \boxed{4}$

v) $|-5-2i|$ $a=-5$ $b=-2$ $|-5-2i| = \sqrt{(-5)^2+(-2)^2} = \sqrt{25+4} = \boxed{\sqrt{29}}$

vi) $|-1+2i|$ $a=-1$ $b=2$ $|-1+2i| = \sqrt{(-1)^2+(2)^2} = \sqrt{1+4} = \boxed{\sqrt{5}}$

vii) $|3+i|$ $a=3$ $b=1$ $|3+i| = \sqrt{a^2+b^2} = \sqrt{3^2+1^2} = \sqrt{9+1} = \boxed{\sqrt{10}}$

5) Find the equation of the line...

$y = \underset{\text{slope}}{m}x + b \leftarrow$ y-intercept

i) that has slope $= 1$ and y-intercept $= -1$. $y = 1x - 1$ $\boxed{y = x - 1}$

ii) that passes through the points $(1,-1)$ and $(-3,7)$. $m = \frac{y_2-y_1}{x_2-x_1} = \frac{7+1}{-3-1} = \frac{8}{-4} = -2$

point-slope form:

$y - y_1 = m(x - x_1)$

$y + 1 = -2(x-1)$

$y = -2x + 2 - 1$

$\boxed{y = -2x + 1}$

6) Find the slope and y-intercept of the following lines:

i) $2x - 5y = -8$

$2x = 5y - 8$ $\quad \frac{5y}{5} = \frac{2x}{5} + \frac{8}{5}$ $\quad y = \frac{2}{5}x + \frac{8}{5}$

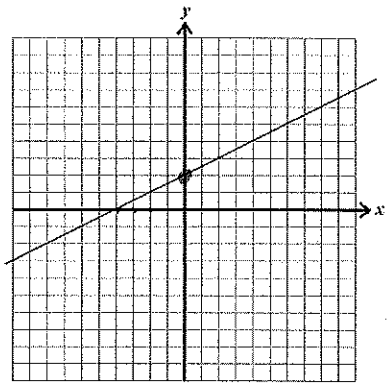
slope $= \frac{2}{5}$ $\quad$ y-intercept: $\frac{8}{5}$

ii) The line that passes through $(-2, 3)$ and $(3,1)$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-1}{-2-3} = \frac{2}{-5}$ $\quad y - y_1 = m(x - x_1)$ $\quad y - 1 = -\frac{2}{5}(x-3)$ $\quad y = -\frac{2}{5}x + \frac{6}{5} + 1$ $\quad y = -\frac{2}{5}x + \frac{11}{5}$

slope $\quad$ y-intercept

(a) iii



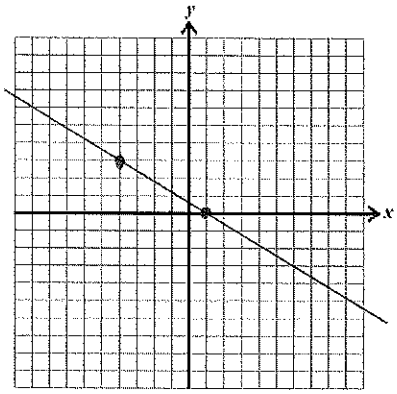
y-intercept: 2

(0,2)

x-intercept: (-4,0)

slope: $m = \frac{2-0}{0+4} = \frac{1}{2}$

(b) iv



(1,0) and (-4,3)

$m = \frac{3-0}{-4-1} = -\frac{3}{5}$

$y - 0 = -\frac{3}{5}(x-1)$

$y = -\frac{3}{5}x + \frac{3}{5}$

slope $\quad$ y-intercept

7) Solve the following systems of equations by graphing. Check that your answer satisfies both equations.

i) $\begin{cases} -x + y = -11 \\ 9x - 2y = 71 \end{cases}$

$y = x - 11$

$\frac{-2y}{-2} = \frac{-9x}{-2} + \frac{71}{-2}$

$\begin{cases} y = x - 11 \\ y = \frac{9}{2}x - \frac{71}{2} \end{cases}$

see next page →

ii) $\begin{cases} x + y = -1 \\ 6x - 7y = -45 \end{cases}$

$y = -x - 1$

$\frac{-7y}{-7} = \frac{-6x}{-7} - \frac{45}{-7}$

$\begin{cases} y = -x - 1 \\ y = \frac{6}{7}x + \frac{45}{7} \end{cases}$

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iii) $\begin{cases} -4x - 3y = 17 \\ -8x - 5y = 31 \end{cases}$

$\frac{-3y}{-3} = \frac{4x}{-3} + \frac{17}{-3}$

$\frac{-5y}{-5} = \frac{8x}{-5} + \frac{31}{-5}$

$\begin{cases} y = -\frac{4}{3}x - \frac{17}{3} \\ y = -\frac{8}{5}x - \frac{31}{5} \end{cases}$

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iv) $\begin{cases} 7x + 3y = 6 \\ 6x - 5y = 43 \end{cases}$

$\frac{3y}{3} = \frac{-7x}{3} + \frac{6}{3}$

$-5y = -6x + 43$

$y = \frac{6}{5}x - \frac{43}{5}$

$\begin{cases} y = -\frac{7}{3}x + 2 \\ y = \frac{6}{5}x - \frac{43}{5} \end{cases}$

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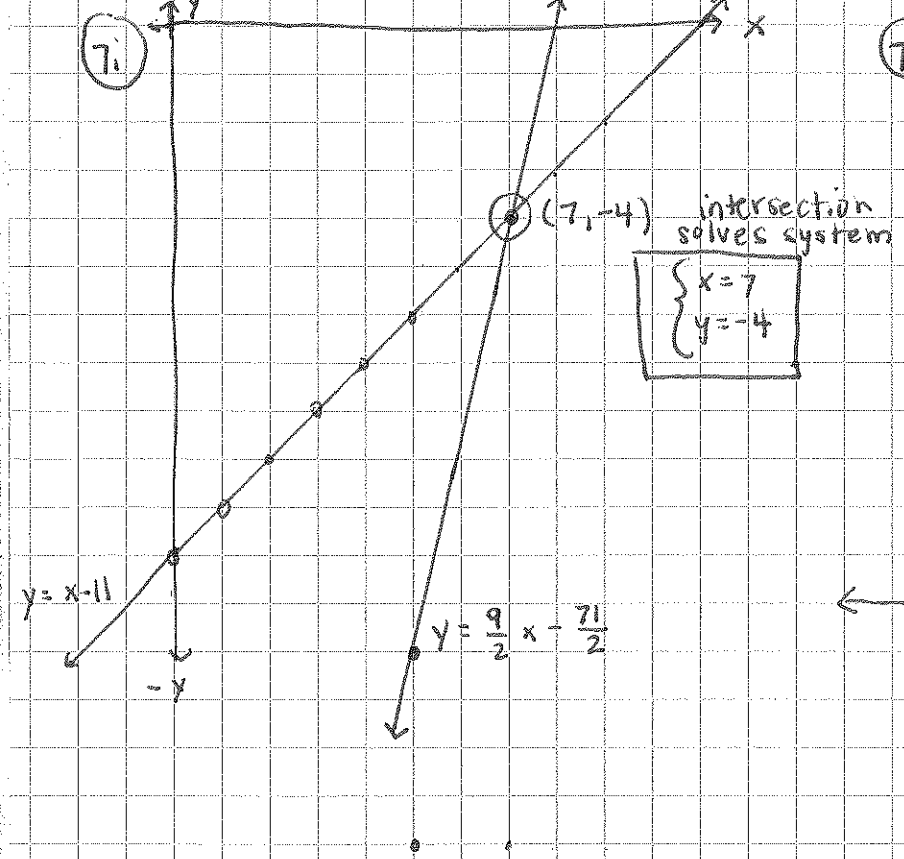
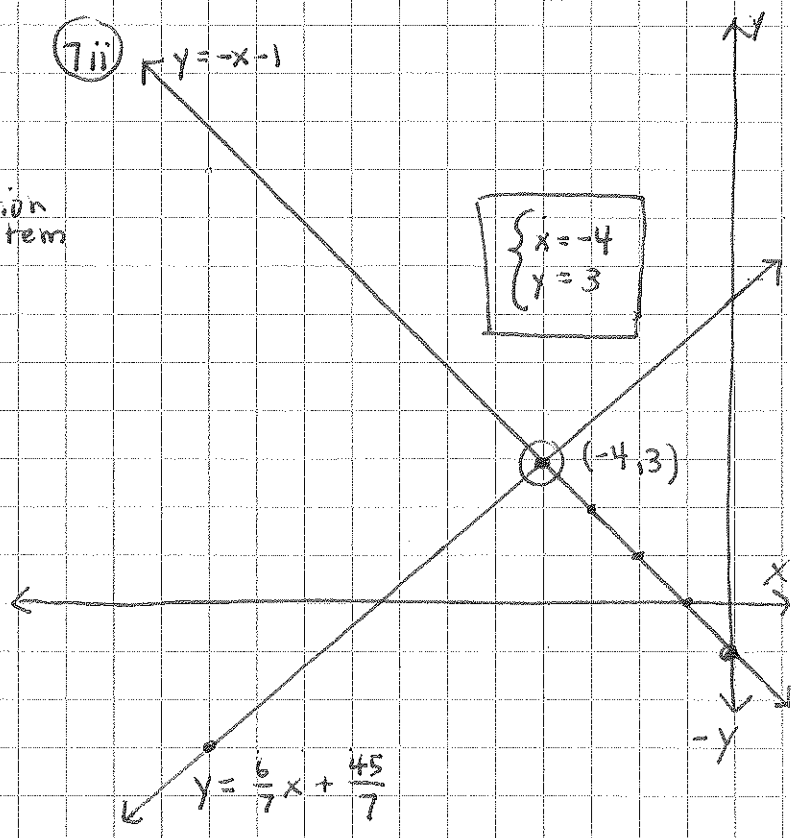
7i



intersection solves system

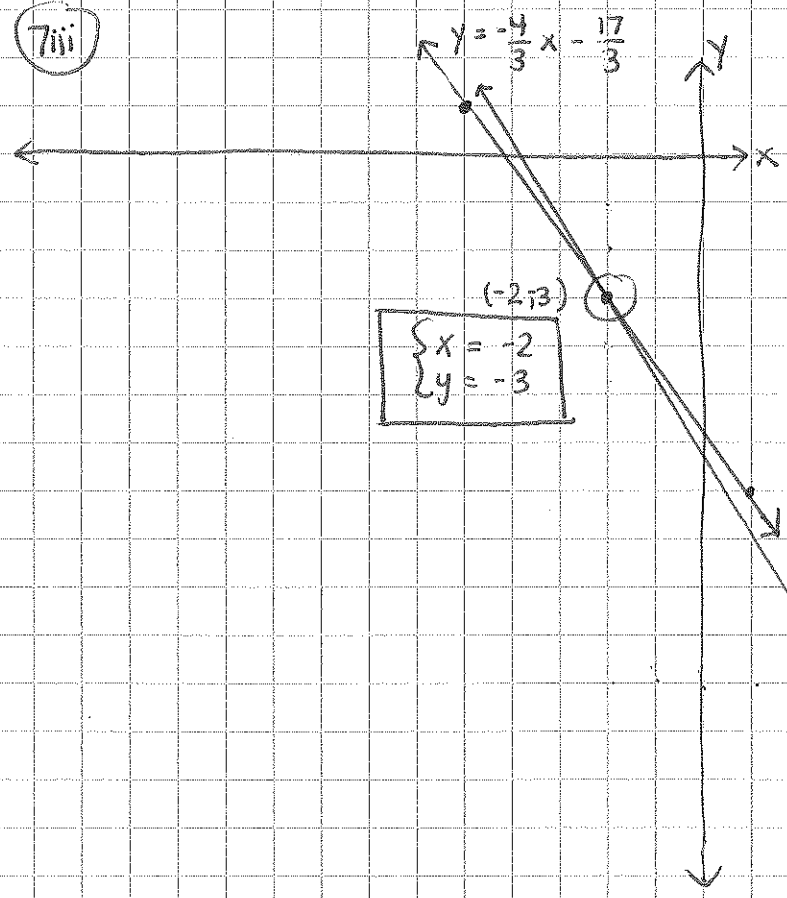
$$\boxed{\begin{cases} x = 7 \\ y = -4 \end{cases}}$$

7ii



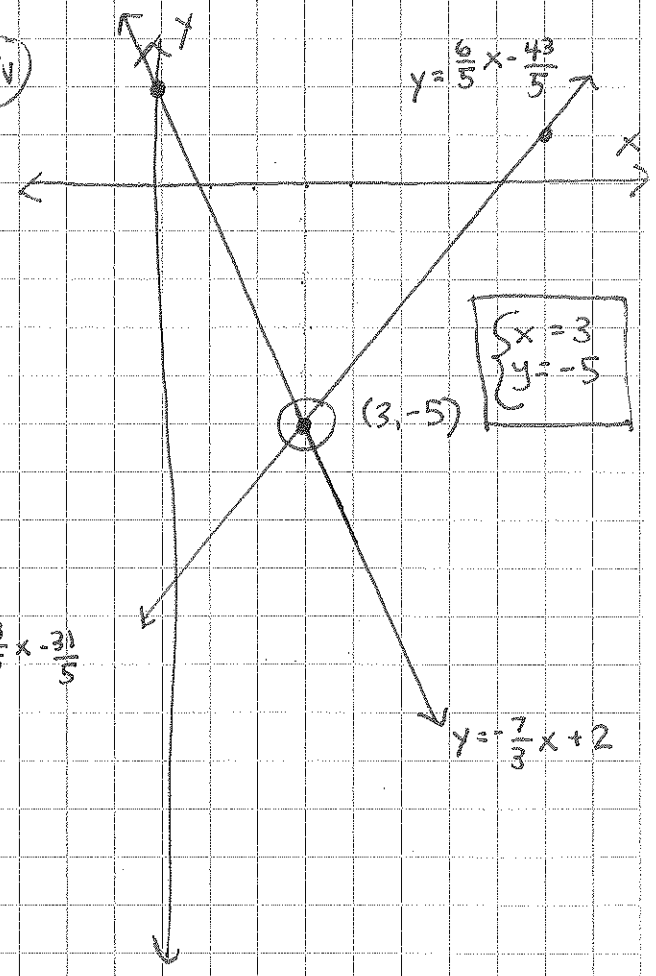
$$\boxed{\begin{cases} x = -4 \\ y = 3 \end{cases}}$$

7iii



$$\boxed{\begin{cases} x = -2 \\ y = -3 \end{cases}}$$

7iv



$$\boxed{\begin{cases} x = 3 \\ y = -5 \end{cases}}$$