Answers

MATH 5071 - Problem Set 4

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Introduction to Logarithms

1) Write the equivalent exponential equation: $y = \log_4(x)$ $\quad 4^y = x$

2) First, write the equivalent exponential expression. Then use it to find the value of the following logarithms:

i) $\log_2(32)$ $\quad 2^x = 32 \quad \boxed{x = 5}$

ii) $\log_4(1024)$ $\quad 4^x = 1024 \quad \boxed{x = 5}$

iii) $\log_5(\frac{1}{25})$ $\quad 5^x = \frac{1}{25} \quad 5^x = \frac{1}{5^2} = (5^2)^{-1} = 5^{-2} \quad \boxed{x = -2}$

iv) $\log_4(2)$ $\quad 4^x = 2 \quad x = \frac{1}{2}$ because $(2^2)^x = 2^1 \quad 2^{2x} = 2^1 \quad 2x = 1 \quad \boxed{x = \frac{1}{2}}$

v) $\log_4(8)$ $\quad 4^x = 8 \quad (2^2)^x = 2^3 \quad 2^{2x} = 2^3 \quad 2x = 3 \quad \boxed{x = \frac{3}{2}}$

vi) $\log_2(\sqrt[5]{4})$ $\quad 2^x = \sqrt[5]{4} \quad 2^x = \sqrt[5]{2^2} = 2^x = (2^2)^{1/5} = 2^{2/5} \quad \boxed{x = \frac{2}{5}}$

vii) $\log(100)$ $\quad 10^x = 100 \quad \boxed{x = 2}$

viii) $\log(0.345)$ $\quad 10^x = 0.345$ plug into calc. $\boxed{x = -.462}$

3) Between which two integers does $\log_5(167)$ lie?
between 3 + 4 $\quad 5^x = 167 \quad 5^1 = 5 \quad 5^2 = 25 \quad 5^3 = 125 \quad 5^4 = 625$ (167)

4) Can the value of $\log_2(-4)$ be found? Why or why not? Please explain your reasoning in full sentences!

No. $\quad 2^x = -4$ You cannot ~~ge~~ raise 2 to any power to get a negative answer.

The Natural Logarithm

5) First, write the equivalent exponential expression. Then use it to find the value of the following logarithms:

i) $y = \ln(e^{-\frac{5}{2}})$ $\quad e^y = e^{-5/2}$ $\quad \boxed{y = -5/2}$

ii) $y = \ln(\frac{1}{e^{-3}})$ $\quad e^y = \frac{1}{e^{-3}}$ $\quad e^y = (e^{-3})^{-1} = e^3$ $\quad \boxed{y = 3}$

iii) $y = \ln\left(\sqrt[5]{\frac{1}{e}}\right)$ $\quad e^y = \sqrt[5]{\frac{1}{e}}$ $\quad e^y = \sqrt[5]{e^{-1}} = (e^{-1})^{1/5} = e^{-1/5}$ $\quad \boxed{y = -1/5}$

Properties of Logarithms

6) Write a combination of logarithms that is equivalent to the following:

i) $\log_3(20) = \boxed{\log_3(10) + \log_3(2)}$

ii) $\log\left(\frac{x^3}{\sqrt[3]{y}}\right) = \log(x^3) - \log(\sqrt[3]{y}) = 3\log(x) - \log(y^{1/3}) = \boxed{3\log(x) - \frac{1}{3}\log(y)}$

iii) $\ln\left(\frac{x^5}{e^2}\right) = \ln(x^5) - \log(e^2) = 5\overset{\ln}{\cancel{\log}}(x) - 2\overset{\ln}{\cancel{\log}}e = \boxed{5\ln(x) - 2}$

7) When rounded to the nearest hundredth, $\log_3(7) = 1.77$. Combine this information with the properties of logarithms to evaluate $\log_3(63)$.

$\log_3(63) = \log_3(9) + \log_3(7) = \log_3(3^2) + \log_3(7)$
$= 2\log_3(3) + \log_3(7)$
$= 2(1) + 1.77$
$= 2 + 1.77 = \boxed{3.77}$

8) Combine into a single logarithm:

i) $\ln(x-6) - \ln(x^2 - 2x - 24)$ assuming $x \neq 6$

$= \ln(x-6) - \ln[(x-6)(x+4)]$
$= \ln\left[\frac{\cancel{(x-6)}}{\cancel{(x-6)}(x+4)}\right]$
$= \boxed{\ln\left[\frac{1}{x+4}\right]}$ or (8i)

$= \ln 1 - \ln(x+4)$
$= 0 - \ln(x+4)$
$= \boxed{-\ln(x+4)}$

ii) $4\log_3(x) - \frac{1}{2}\log_3(y) + 3\log_3(z)$

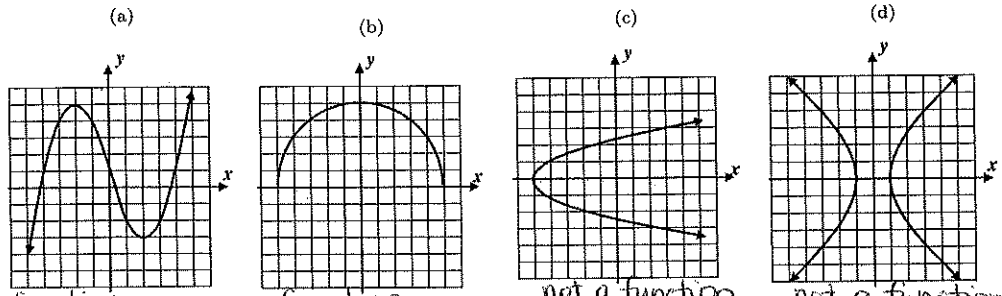
(8ii)
$= \log_3(x^4) - \log_3(y^{1/2}) + 3\log_3(z^3)$
$= \cancel{\log_3\left(\frac{x^4 z^3}{\sqrt{y}}\right)}$
$= \log_3\left(\frac{x^4 z^3}{\sqrt{y}}\right)$

9) If $k = \log_2(3)$, then what is $\log_2(48)$ in terms of k?

$\log_2(48) = \log_2(3) + \log_2(16)$
$= k + \log_2(2^4)$
$= \boxed{k + 4}$

Introduction to Functions

10) Use the vertical line test to determine whether y is a function of x. Then, determine the domain and range for each relation.



function
D: $\mathbb{R}$ R: $\mathbb{R}$

function
D: $[-5, 5]$
R: $[0, 5]$

not a function
D: $[-5, \infty)$
R: $\mathbb{R}$

not a function
D: $(-\infty, -1] \cup [1, \infty)$
R: $\mathbb{R}$

11) In which of the following sets of ordered pairs is y *not* a function of x?

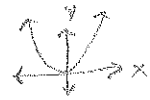
a) $\{(3,7),(5,9),(11,7)\}$ function

b) $\{(-3,9),(0,0),(3,9)\}$ function

c) $\{(-2,5),(5,1),(-2,7)\}$ not a function. $x=-2$ has more than one y-value

d) $\{(-5,-9),(1,3),(7,15)\}$ function

12) Evaluate each of the following, given the function definitions and input values. Then, determine the domain and range for each function.



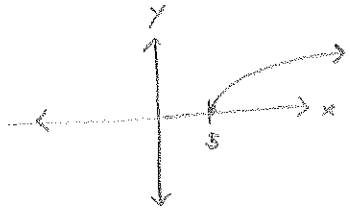
i) $f(x) = 3x^2$; $f(-3) = ?$ $f(-3) = 3(-3)^2 = 27$ D: $\mathbb{R}$ R: $[0, \infty)$

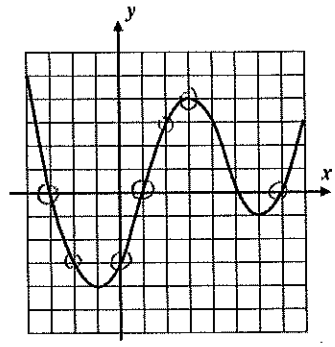
ii) $h(x) = \sqrt{x-5}$; $f(54) = ?$

$h(54) = \sqrt{54-5} = \sqrt{49} = 7$

D: $[5, \infty)$

R: $[0, \infty)$





13) Based on the graph of the function $y = g(x)$ shown above, answer the following questions.

i) Evaluate $g(-2)$, $g(0)$, $g(3)$, and $g(7)$.

$g(-2) = -3$ $\quad g(3) = 4$

$g(0) = -3$ $\quad g(7) = 0$

ii) What values of x solve the equation $g(x) = 0$? $y = 0$ when $\boxed{x = -3, 1, \text{ and } 7}$

iii) How many values of x solve the equation $g(x) = 2$? there are 3 x-values that correspond to $y = 2$

14) Ian invested \$2500 in a bank account that is guaranteed to earn 4% interest compounded yearly. The amount of money, A, in his account as a function of the number of years, t, since creating the account is given by the equation $A(t) = 2500(1.04)^t$.

i) Evaluate $A(0)$ and $A(10)$.

$A(0) = 2500(1.04)^0 = 2500 \cdot (1) = 2500$

$A(10) = 2500(1.04)^{10} = 3700.61$

ii) What do the two values that you found in part i) represent?

The amount of money in his account in the beginning (\$2500 at 0 years) and after 10 years (\$3700.61 after 10 years).

iii) Using a table and your calculator, determine, to the nearest whole year, the value of t that solves the equation $A(t) = 5000$. Justify your answer with numerical evidence.

$5000 = 2500(1.04)^t$ $\quad 2 = 1.04^t$ $\quad \boxed{t = 17.67 \text{ years}}$

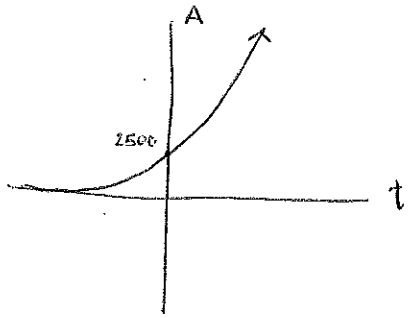
iv) What does the value of t that you found in part iii) represent about Ian's investment?

it means that it will take Ian 17.67 years to double his investment.

v) Find the domain and range of $A(t)$. => Graph it!

Domain: $\mathbb{R}$

Range: $[0, \infty)$



approaches 0 but never reaches it

Function Composition and Inverses

15) Given $f(x) = 3x - 4$ and $g(x) = -2x + 7$, evaluate:

i) $g(f(-2))$ $\quad f(-2) = 3(-2) - 4 = -6 - 4 = -10 \quad g(f(-2)) = g(-10) = -2(-10) + 7 = 20 + 7 = \boxed{27}$

ii) $(f \circ g)(5) = f(g(5))$ $\quad g(5) = -2(5) + 7 = -10 + 7 = -3 \quad f(g(5)) = f(-3) = 3(-3) - 4 = -9 - 4 = \boxed{-13}$

iii) $g^{-1}(x)$ and give the domain and range $\quad x - 7 = -2y \quad y = \frac{x-7}{-2}$

$y = -2x + 7 \quad x = -2y + 7 \quad \boxed{g^{-1}(x) = \frac{x-7}{-2}} \quad D = \mathbb{R} \quad R = \mathbb{R}$

iii) $f(g(x))$

$f(g(x)) = 3\,g(x) - 4 = 3(-2x + 7) - 4 = -6x + 21 - 4 = \boxed{-6x + 17 = f(g(x))}$

16) Given $h(x) = x^2 + 11$ and $g(x) = \sqrt{x - 2}$, evaluate:

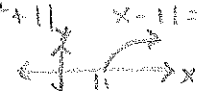
i) $g(g(x)) = \sqrt{g(x) - 2} = \boxed{\sqrt{\sqrt{x - 2} - 2}}$

ii) $(h \circ g)(38) = h(g(38))$ $\quad g(38) = \sqrt{38 - 2} = \sqrt{36} = 6 \quad h(g(38)) = h(6) = 6^2 + 11 = 36 + 11 = \boxed{47}$

iii) $(g \circ h)(0) = g(h(0))$ $\quad h(0) = 0^2 + 11 = 11 \quad g(h(0)) = g(11) = \sqrt{11 - 2} = \sqrt{9} = \boxed{3}$

iii) $h^{-1}(x)$ and give the domain and range $\quad y = x^2 + 11 \quad x = y^2 + 11 \quad x - 11 = y^2 \quad y = \sqrt{x - 11}$

$h^{-1}(x) = \sqrt{x - 11} \quad D: [11, \infty) \quad R: [0, \infty)$



17) Given $f(x) = 2x + 9$ and $g(x) = \frac{x-9}{2}$, calculate $g(f(x))$.

$g(f(x)) = \frac{f(x) - 9}{2} = \frac{2x + 9 - 9}{2} = \frac{2x}{2} = \boxed{x}$

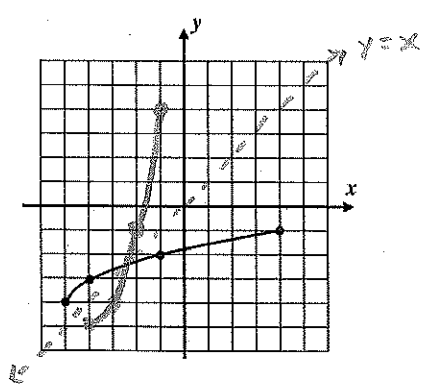
18) If the point $(-7, 5)$ lies on the graph of $y = f(x)$, what point must lie on the graph of $y = f^{-1}(x)$?

$\boxed{(5, -7)}$

19) The function $y = h(x)$ is entirely defined by the graph shown below.

i) Sketch a graph of $y = h^{-1}(x)$. Create a table of values if needed.

ii) Write the domain and range of $y = h(x)$ and $y = h^{-1}(x)$ using interval notation.



i

x	$h(x)$	x	$h^{-1}(x)$
4	-1	-1	4
-1	-2	-2	-1
-4	-3	-3	-4
-5	-4	-4	-5

$D = [-5, 4] \quad D = [-4, -1]$

$R = [-4, -1] \quad R = [-5, 4]$