MATH 5071 - Problem Set 4

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Introduction to Logarithms

1) Write the equivalent exponential equation: $y = \log_4(x)$

2) First, write the equivalent exponential expression. Then use it to find the value of the following logarithms:

i) $\log_2(32)$

ii) $\log_4(1024)$

iii) $\log_5(\frac{1}{25})$

iv) $\log_4(2)$

v) $\log_4(8)$

vi) $\log_2(\sqrt[5]{4})$

vii) $\log(100)$

viii) $\log(0.345)$

3) Between which two integers does $\log_5(167)$ lie?

4) Can the value of $\log_2(-4)$ be found? Why or why not? Please explain your reasoning in full sentences!

The Natural Logarithm

5) First, write the equivalent exponential expression. Then use it to find the value of the following logarithms:

i) $y = \ln(e^{-\frac{5}{2}})$

ii) $y = \ln(\frac{1}{e^{-3}})$

iii) $y = \ln\left(\sqrt[5]{\frac{1}{e}}\right)$

Properties of Logarithms

6) Write a combination of logarithms that is equivalent to the following:

i) $\log_3(20)$

ii) $\log\left(\frac{x^3}{\sqrt[3]{y}}\right)$

iii) $\ln\left(\frac{x^5}{e^2}\right)$

7) When rounded to the nearest hundredth, $\log_3(7) = 1.77$. Combine this information with the properties of logarithms to evaluate $\log_3(63)$.

8) Combine into a single logarithm:

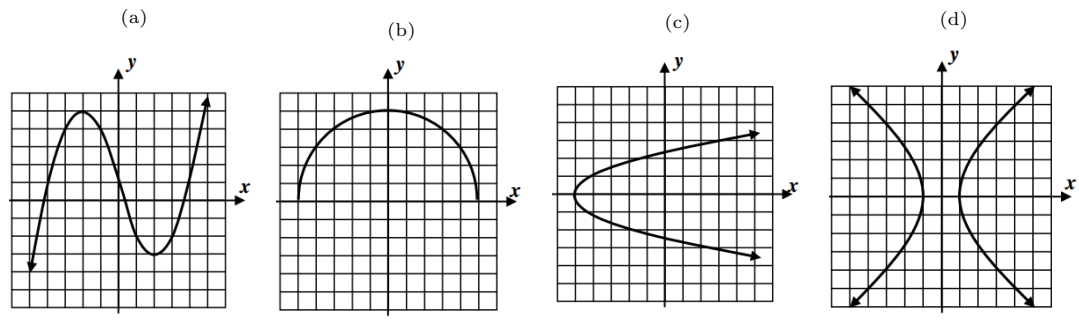
i) $\ln(x-6) - \ln(x^2 - 2x - 24)$ assuming $x \neq 6$

ii) $4\log_3(x) - \frac{1}{2}\log_3(y) + 3\log_3(z)$

9) If $k = \log_2(3)$, then what is $\log_2(48)$ in terms of k?

Introduction to Functions

10) Use the vertical line test to determine whether y is a function of x. Then, determine the domain and range for each relation.



11) In which of the following sets of ordered pairs is y *not* a function of x?

a) $\{(3,7),(5,9),(11,7)\}$

b) $\{(-3,9),(0,0),(3,9)\}$

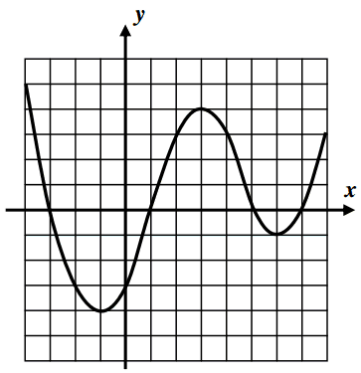
c) $\{(-2,5),(5,1),(-2,7)\}$

d) $\{(-5,-9),(1,3),(7,15)\}$

12) Evaluate each of the following, given the function definitions and input values. Then, determine the domain and range for each function.

i) $f(x) = 3x^2$; $f(-3) = ?$

ii) $h(x) = \sqrt{x-5}$; $f(54) = ?$



13) Based on the graph of the function $y = g(x)$ shown above, answer the following questions.

i) Evaluate $g(-2)$, $g(0)$, $g(3)$, and $g(7)$.

ii) What values of x solve the equation $g(x) = 0$?

iii) How many values of x solve the equation $g(x) = 2$?

14) Ian invested \$2500 in a bank account that is guaranteed to earn 4% interest compounded yearly. The amount of money, A, in his account as a function of the number of years, t, since creating the account is given by the equation $A(t) = 2500(1.04)^t$.

i) Evaluate $A(0)$ and $A(10)$.

ii) What do the two values that you found in part i) represent?

iii) Using a table and your calculator, determine, to the nearest whole year, the value of t that solves the equation $A(t) = 5000$. Justify your answer with numerical evidence.

iv) What does the value of t that you found in part iii) represent about Ian's investment?

v) Find the domain and range of $A(t)$.

Function Composition and Inverses

15) Given $f(x) = 3x - 4$ and $g(x) = -2x + 7$, evaluate:

i) $g(f(-2))$

ii) $(f \circ g)(5)$

iii) $g^{-1}(x)$ and give the domain and range

iv) $f(g(x))$

16) Given $h(x) = x^2 + 11$ and $g(x) = \sqrt{x-2}$, evaluate:

i) $g(g(x))$

ii) $(h \circ g)(38)$

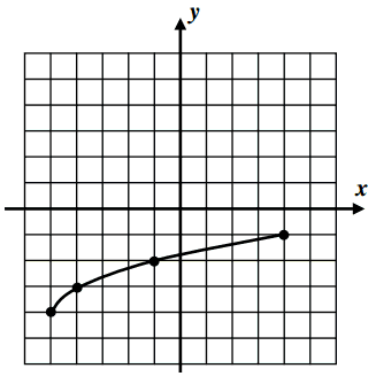
iii) $(g \circ h)(0)$

iv) $h^{-1}(x)$ and give the domain and range

17) Given $f(x) = 2x + 9$ and $g(x) = \frac{x-9}{2}$, calculate $g(f(x))$.

18) If the point $(-7, 5)$ lies on the graph of $y = f(x)$, what point must lie on the graph of $y = f^{-1}(x)$?

19) The function $y = h(x)$ is entirely defined by the graph shown below.

i) Sketch a graph of $y = h^{-1}(x)$. Create a table of values if needed.

ii) Write the domain and range of $y = h(x)$ and $y = h^{-1}(x)$ using interval notation.