PS #3

Simplifying Rational Expressions

1) $\dfrac{xy+3x}{6x^2+xy} = \dfrac{\cancel{x}(y+3)}{\cancel{x}(6x+y)} = \dfrac{y+3}{6x+y}$

2) $\dfrac{x^2+10x+25}{x^2-25} = \dfrac{\cancel{(x+5)}(x+5)}{\cancel{(x+5)}(x-5)} = \dfrac{x+5}{x-5}$

3) $\dfrac{2}{5x+10} \cdot \dfrac{10x+20}{x+1} = \dfrac{2}{5\cancel{(x+2)}} \cdot \dfrac{10\cancel{(x+2)}}{(x+1)} = \dfrac{20}{5(x+1)} = \dfrac{4}{x+1}$

4) $\dfrac{x^2}{12x+6} \div \dfrac{x}{2x+1} = \dfrac{x^2}{12x+6} \cdot \dfrac{2x+1}{x} = \dfrac{x^{\cancel{2}}\cancel{(2x+1)}}{\cancel{x}\cdot 6\cancel{(2x+1)}} = \dfrac{x}{6}$

5) $\dfrac{3x-1}{4x+12} \div \dfrac{4x-4}{x+3} = \dfrac{3x-1}{4x+12} \cdot \dfrac{x+3}{4x-4} = \dfrac{3x-1}{4\cancel{(x+3)}} \cdot \dfrac{\cancel{(x+3)}}{4(x-1)} = \dfrac{3x-1}{16(x-1)}$

6) $\dfrac{2x^2+3x-9}{6x^2-7x+1} \cdot \dfrac{2x^2+3x-5}{x^2+2x-3} \cdot \dfrac{6x^2+17x-3}{2x^2+7x-15}$

$\dfrac{\cancel{(2x-3)}\cancel{(x+3)}}{\cancel{(6x-1)}(x-1)} \cdot \dfrac{(2x+5)\cancel{(x-1)}}{\cancel{(x+3)}\cancel{(x-1)}} \cdot \dfrac{\cancel{(6x-1)}(x+3)}{\cancel{(2x-3)}(x+5)} = \dfrac{(2x+5)(x+3)}{(x-1)(x+5)}$

Adding / Subtracting Rational Expressions

7) $\dfrac{4}{x} + \dfrac{7}{4x} + 1 = \dfrac{4}{4}\cdot\dfrac{4}{x} + \dfrac{7}{4x} + \dfrac{4x}{4x}\cdot 1 = \dfrac{16}{4x} + \dfrac{7}{4x} + \dfrac{4x}{4x} = \dfrac{16+7+4x}{4x} = \dfrac{23+4x}{4x}$

8) $\dfrac{a}{b} + \dfrac{b}{a} = \dfrac{a}{a}\cdot\dfrac{a}{b} + \dfrac{b}{b}\cdot\dfrac{b}{a} = \dfrac{a^2}{ab} + \dfrac{b^2}{ab} = \dfrac{a^2+b^2}{ab}$

9) $\frac{6}{x+2} - \frac{11}{2x-7} = \frac{(2x-7)}{(2x-7)} \cdot \frac{6}{(x+2)} - \frac{(x+2)}{(x+2)} \cdot \frac{11}{(2x-7)}$

$= \frac{6(2x-7) - 11(x+2)}{(2x-7)(x+2)} = \frac{12x - 42 - 11x - 22}{(2x-7)(x+2)}$

$= \frac{x - 64}{(2x-7)(x+2)}$

10) $\frac{2x^2+2}{x+1} - \frac{7}{x^2-1} = \frac{2(x^2+1)}{(x+1)} - \frac{7}{(x+1)(x-1)} = \frac{(x-1)\cdot 2(x^2+1)}{(x-1)\cdot(x+1)} - \frac{7}{(x+1)(x-1)}$

$= \frac{2(x-1)(x^2+1) - 7}{(x+1)(x-1)} = \frac{(2x-2)(x^2+1) - 7}{(x+1)(x-1)} = \frac{2x^3+2x-2x^2-2-7}{(x+1)(x-1)} = \frac{2x^3-2x^2+2x-9}{(x+1)(x-1)}$

11) $\frac{x-3}{x^2+5x+4} + \frac{2x+1}{x^2+3x+2} = \frac{(x-3)}{(x+4)(x+1)} + \frac{(2x+1)}{(x+2)(x+1)} = \frac{(x+2)(x-3)}{(x+2)(x+4)(x+1)} + \frac{(x+4)(2x+1)}{(x+4)(x+2)(x+1)}$

$= \frac{(x+2)(x-3) + (x+4)(2x+1)}{(x+4)(x+1)(x+2)} = \frac{x^2-3x+2x-6+2x^2+x+8x+4}{(x+4)(x+1)(x+2)}$

$= \frac{3x^2+8x-2}{(x+4)(x+1)(x+2)}$

Simplifying Radical Expressions

12) (i) 3 (ii) 1 (iii) 2 (iv) 3

13) (i) $4^{3/2} = (4^{1/2})^3 = 2^3 = \boxed{8}$ (ii) $64^{2/3} = (64^{1/3})^2 = 4^2 = \boxed{16}$

14) (i) $\sqrt{18} = \sqrt{2\cdot 9} = \sqrt{2}\cdot\sqrt{9} = \boxed{3\sqrt{2}}$ (ii) $\sqrt{72} = \sqrt{36\cdot 2} = \sqrt{36}\cdot\sqrt{2} = \boxed{6\sqrt{2}}$

(iii) $\sqrt{12x^4} = \sqrt{3\cdot 2^2\cdot x^2\cdot x^2} = \boxed{2x^2\sqrt{3}}$ (iv) $\sqrt{200x^{12}y^7} = \sqrt{2\cdot 100\cdot x^{12}\cdot y^6\cdot y} = \boxed{10x^6y^3\sqrt{2y}}$

B3 continued

(15) (i) $\frac{4}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{4\sqrt{2}}{2} = \boxed{2\sqrt{2}}$

(ii) $\frac{3x}{3-\sqrt{5}} \cdot \frac{(3+\sqrt{5})}{(3+\sqrt{5})} = \frac{9x+3x\sqrt{5}}{9+\cancel{3\sqrt{5}}-\cancel{3\sqrt{5}}-5}$

$\boxed{= \frac{9x+3x\sqrt{5}}{4}}$

(16) $5\sqrt{2}-3\sqrt{2} = \boxed{2\sqrt{2}}$

(17) $73\sqrt{r^2x}+33\sqrt{r^2x} = 106\sqrt{r^2x} = \boxed{106r\sqrt{x}}$

(18) $9\sqrt{20}+2\sqrt{45}-3\sqrt{5} = 9\sqrt{4\cdot5}+2\sqrt{9\cdot5}-3\sqrt{5}$

$= 9\cdot2\sqrt{5}+2\cdot3\sqrt{5}-3\sqrt{5} = 18\sqrt{5}+6\sqrt{5}-3\sqrt{5}$

$\boxed{= 21\sqrt{5}}$

(19) $\sqrt{5}\cdot\sqrt{11} \boxed{= \sqrt{55}}$

(20) $\sqrt{35x^3}\cdot\sqrt{7x} = \sqrt{7\cdot5\cdot7\cdot x^4} = \sqrt{5\cdot7^2\cdot x^4}$

$\boxed{= 7x^2\sqrt{5}}$

(21) $(2+\sqrt{7})(1-\sqrt{6}) = \boxed{2-2\sqrt{6}+\sqrt{7}-\sqrt{42}}$

(22) $(a+\sqrt{b})(a-\sqrt{a})(b+\sqrt{b})$

$(a+\sqrt{b})(a-\sqrt{a}) = a^2-a\sqrt{a}+a\sqrt{b}-\sqrt{ab}$

$(a^2-a\sqrt{a}+a\sqrt{b}-\sqrt{ab})(b+\sqrt{b})$

$= a^2+a^2\sqrt{b}-ab\sqrt{a}-a\sqrt{ab}+ab\sqrt{b}+ab-b\sqrt{ab}-b\sqrt{a})$

$= a^2+(a^2+ab)\sqrt{b}-(ab+b)\sqrt{a}-(a+b)\sqrt{ab}+ab$

$= a^2+a(a+b)\sqrt{b}-b(a+1)\sqrt{a}-(a+b)\sqrt{ab}+ab$

(23) $i^58 = i\cdot i^4\cdot8 = \boxed{8i}$

(24) (i) $\sqrt{-9} = \sqrt{-1\cdot9} = \sqrt{-1}\cdot\sqrt{9} = \boxed{3i}$

(ii) $\sqrt{-72} = \sqrt{-1\cdot9\cdot8} = \sqrt{-1\cdot36\cdot2} = \sqrt{-1}\cdot\sqrt{36}\cdot\sqrt{2}$

$= i6\sqrt{2}$

$\boxed{= 6i\sqrt{2}}$

(25) (i) $(6-2i)+(3-5i)$

$(6+3)+(-2-5)i = \boxed{9-7i}$

(ii) $(5+3i)-(-3i-8) = 5+3i+3i+8$

$= (5+8)+(3+3)i = \boxed{13+6i}$

(26) (i) $(4-2i)(4+2i) = 16+\cancel{8i}-\cancel{8i}-4i^2$

$= 16-4i^2 = 16+4 = \boxed{20}$

(ii) $(3+5i)(8+i) = 24+3i+40i+5i^2$

$= 24+43i-5 = \boxed{19+43i}$

(27) (i) $\frac{(1+i)\cdot(1+i)}{(1-i)\cdot(1+i)} = \frac{1+i+i+i^2}{1+\cancel{i}-\cancel{i}-i^2} = \frac{1+2i-1}{1+1} = \frac{\cancel{2}i}{\cancel{2}} = \boxed{i}$

(ii) $\frac{(2+3i)\cdot(6-7i)}{(6+7i)\cdot(6-7i)} = \frac{12-14i+18i-21i^2}{36-\cancel{42i}+\cancel{42i}-49i^2}$

$= \frac{12+4i+21}{36+49} = \boxed{\frac{33+4i}{85}}$