Problem Set 2 : Solutions

① $6a-20 = 2\cdot 3\cdot a - 2\cdot 2\cdot 5$

$\boxed{= 2(3a-10)}$

② $35x^4y^2 + 21x^3y^3 - 56x^5y^7$

$= 5\cdot 7\cdot x\cdot x\cdot x\cdot x\cdot y\cdot y + 3\cdot 7\cdot x\cdot x\cdot x\cdot y\cdot y\cdot y - 7\cdot 2\cdot 2\cdot 2\cdot x\cdot x\cdot x\cdot x\cdot x\cdot y\cdot y\cdot y\cdot y\cdot y\cdot y\cdot y$

$\boxed{= 7x^3y^2(5x+3y-8x^2y^5)}$

③ $4x^3(3y+5)^3 - 20x^2(3y+5)^2 - 16x^4(3y+5)^4$

$= 2\cdot 2\cdot x\cdot x\cdot x\cdot (3y+5)\cdot(3y+5)\cdot(3y+5) - 2\cdot 2\cdot 5\cdot x\cdot x\cdot(3y+5)\cdot(3y+5)$
$- 2\cdot 2\cdot 2\cdot 2\cdot x\cdot x\cdot x\cdot x\cdot(3y+5)\cdot(3y+5)\cdot(3y+5)\cdot(3y+5)$

$\boxed{= 4x^2(3y+5)^2[-4x^2(3y+5)^2 + x(3y+5) - 5]}$

④ $-3x^3+4x^2+9x-12 = x^2(-3x+4) - 3(-3x+4)$

$\boxed{= (x^2-3)(-3x+4)}$

⑤ $12x^2+18xy+2x+3y = 12x^2+2x+18xy+3y$

$= 2x(6x+1) + 3y(6x+1) \boxed{= (6x+1)(2x+3y)}$

⑥ $-4x^2-2x+10x+5 = -2x(2x+1)+5(2x+1)$

$\boxed{= (2x+1)(-2x+5)}$

⑦ $-15x^2-12y+6xy+30x = -15x^2+30x+6xy-12y$

$= -15x(x-2)+6y(x-2) = \boxed{(x-2)(-15x+6y)}$

⑧ $3y^4+9y^2-6y^3-18y$

$3y^2(y^2+3)-6y(y^2+3) = (y^2+3)(3y^2-6y)$

$= (y^2+3)3y(y-2)$

$\boxed{= 3y(y^2+3)(y-2)}$

⑨ $y^2+7y+10 \boxed{= (y+2)(y+5)}$

⑩ $t^2+2t-15 = \boxed{(t+5)(t-3)}$

⑪ $x^2-7xy-30y^2 = \boxed{(x+3y)(x-10y)}$

⑫ $a^2+16ab+28b^2 = \boxed{(a+14b)(a+2b)}$

⑬ $2c^2f-18cdf+36d^2f$

$= 2f[c^2-9cd+18d^2]$

$= 2f[(c-6d)(c-3d)]$

$\boxed{= 2f(c-6d)(c-3d)}$

⑭ $3s^2+12st-63t^2 = 3[s^2+4st-21t^2]$

$\boxed{= 3(s+7t)(s-3t)}$

⑮ $6x^2+10x-4 = 2(3x^2+5x-2)$

$\boxed{= 2(3x-1)(x+2)}$

⑯ $81k^2-180k+100$ $\quad (a-b)^2 = a^2-2ab+b^2$

$\sqrt{a^2} = \sqrt{81k^2}$ $\quad \sqrt{b^2} = \sqrt{100}$

$a = 9k$ $\quad b = 10$

$\boxed{(9k-10)^2}$

⑰ $25m^2-64$ $\quad a^2-b^2 = (a+b)(a-b)$

$\sqrt{25m^2} = \sqrt{a^2}$ $\quad \sqrt{64} = \sqrt{b^2}$

$5m = a$ $\quad 8 = b$

$\boxed{= (5m+8)(5m-8)}$

⑱ x^4-1 $\quad a^2-b^2 = (a+b)(a-b)$

$\sqrt{x^4} = \sqrt{a^2}$ $\quad \sqrt{1} = \sqrt{b^2}$

$x^2 = a$ $\quad 1 = b$

$\boxed{= (x^2+1)(x^2-1)}$

⑲ a^6-8 $\quad a^3-b^3 = (a-b)(a^2+ab+b^2)$

$\sqrt[3]{a^6} = \sqrt[3]{a^3}$ $\quad \sqrt[3]{8} = \sqrt[3]{b^3}$

$a^2 = a$ $\quad 2 = b$

$\boxed{= (a^2-2)(a^4+2a^2+4)}$

⑳ $[u^2-2uv+v^2]-w^2$ $\longrightarrow a^2-2ab+b^2 = (a-b)^2$

$(u-v)^2 - w^2$ $\quad a^2-b^2 = (a+b)(a-b)$

$\sqrt{(u-v)^2} = \sqrt{a^2}$ $\quad \sqrt{w^2} = \sqrt{b^2}$

$u-v = a$ $\quad w = b$

$= [(u-v)+w][(u-v)-w] \boxed{= (u-v+w)(u-v-w)}$

PS2 solutions (continued)

(21) $[4a^2+12ab+9b^2] - m^2+8mn-16n^2$

$a^2+2ab+b^2=(a+b)^2$

$\sqrt{a^2}=\sqrt{4a^2}$ $\quad \sqrt{b^2}=\sqrt{9b^2}$

$a=2a$ $\quad b=3b$

$(2a+3b)^2 - [m^2-8mn+16n^2]$

$a^2-2ab+b^2=(a-b)^2$

$\sqrt{m^2}=\sqrt{a^2}$ $\quad \sqrt{16n^2}=\sqrt{b^2}$

$a=m$ $\quad 4n=b$

$(2a+3b)^2-(m-4n)^2$

$a^2-b^2=(a+b)(a-b)$

$\sqrt{(2a+3b)^2}=\sqrt{a^2}$ $\quad \sqrt{(m-4n)^2}=\sqrt{b^2}$

$a=2a+3b$ $\quad b=m-4n$

$=[(2a+3b)+(m-4n)][(2a+3b)-(m-4n)]$

$\boxed{=(2a+3b+m-4n)(2a+3b-m+4n)}$

(22) $9x^4+45x^2+14$ $\quad a=9 \quad b=45 \quad c=14$ $\quad m\cdot n=126$ $\quad m=42$

$a\cdot c=9\cdot 14=126$ $\quad m+n=45$ $\quad n=3$

now split our b term into two terms added together

$=9x^4+3x^2+42x^2+14=3x^2(3x^2+1)+14(3x^2+1)=\boxed{(3x^2+1)(3x^2+14)}$

(23) $15a^6+31a^3-24$ $\quad a=15 \quad b=31 \quad c=-24$ $\quad m\cdot n=-360$ $\quad m=40$

$a\cdot c=-360$ $\quad m+n=31$ $\quad n=-9$

now split our b term into the sum of m+n

$=15a^6-9a^3+40a^3-24=3a^3(5a^3-3)+8(5a^3-3)=\boxed{(5a^3-3)(3a^3+8)}$