# MATH 5071 - Exam 1 (100 pts)

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Name: _____

**1)** Let $f(x) = \frac{7}{x} - 3$ and $g(x) = x^2 + 3$.

**a)** What is the domain of $f(x)$? (1 pt)

**b)** Find $f^{-1}(x)$. (2 pts)

**c)** What is the domain of $f^{-1}(x)$? (1 pt)

**d)** Using your answers from parts **a** and **c**, find the ranges of both $f(x)$ and $f^{-1}(x)$. Explain your reasoning. (2 pts)

**e)** Evaluate $(f \circ g)(2)$. (2 pts)

**f)** Find $f(g(x))$. (2 pts)

**2)** Let $Y = -4 + i$ and $X = 2 + 3i$.
Perform the following operations and report your answers in $a + bi$ form.

**a)** $2Y + X$ (3 pts)

**b)** $Y \cdot (2X)$ (3 pts)

**c)** $\frac{2Y}{X}$ (4 pts)

**3)** Factor the following polynomials *completely*.

**a)** $72x - 50x^3$ (3 pts)

**b)** $x^3 - x^2 - 25x + 25$ (3 pts)

**c)** $18x^2 + 41x - 10$ (4 pts)

**4)** Simplify: $\frac{2x^2-9x-5}{3x^2-13x-10}$ (5 pts)

**5)** Combine into a single fraction and simplify: $\frac{8}{x^2+2x-3} - \frac{6}{x^2+3x}$ (7 pts)

**6)** Find the quotient using polynomial long division and simplify: $(2x^3 - 9x^2 + 15) \div (2x - 5)$ (8 pts)

**7)** Simplify. Report your answers **WITHOUT** negative exponents.

**a)** $\frac{\left(x^{-3}y^{-4}\right)^2}{(xy^3)^{-4}}$ (3 pts)

**b)** $\left[5x^{\frac{2}{3}}y^{-\frac{1}{2}}\right]^{-3} \cdot \left[16xy^{-3}\right]^{\frac{1}{2}}$ (4 pts)

**8)** Simplify fully.

**a)** $3\sqrt{242x^5y^5} + 11xy\sqrt{2x^3y^3}$ (3 pts)

**b)** $\sqrt[3]{54a^7b^4}$ (3 pts)

**9)** Let $X = 3 - \sqrt{45}$ and $Y = 2 + \sqrt{80}$.
Perform the following operations and fully simplify your answers.

**a)** $X - \frac{Y}{2}$ (3 pts)

**b)** $\left(\frac{X}{2}\right) \cdot Y$ (3 pts)

**c)** $\frac{2X}{Y}$ (4 pts)

**10)** Simplify and report your answer in $a + bi$ form: $2i^{70} - 5i^{45} + 3i^{27} - 7i^{16}$ (2 pts)

**11)** Evaluate:

**a)** $\log_3\left(\sqrt[5]{9}\right)$ (2 pts)

**b)** $\log_4(8)$ (3 pts)

**12)** Between which two consecutive integers must $\log_4(7342)$ fall? (2 pts)

**13)** Rewrite as a single logarithm: $3\log(\sqrt{x}) - \frac{1}{2}\log\left(y^4\right) - 5\log\left(\frac{1}{z}\right)$. (3 pts)

**14)** If $a = \log_2(x)$, then rewrite $\log_2(8x^6)$ as a function of $a$ only (no $x$). (4 pts)

**15)** Expand the products and simplify:

**a)** $(x+2)(x-2)(x+4)(x-4)$ (3 pts)

**b)** $(2x-1)^3$ (3 pts)

**16)** Express as a single, simplified fraction: $\frac{\frac{1}{8}-\frac{1}{2x}}{\frac{1}{x}-\frac{x}{16}}$. (5 pts)