SUMMARY OF WAVES

Review of Basic Wave Properties (from Physics 213)

• Types of Waves

Ia. transverse traveling wave:
\[ y(x, t) = f(x \pm vt) \]

Ib. longitudinal traveling wave:
\[ s(x, t) = g(x \pm vt) \]

II. harmonic traveling wave:
\[ y(x, t) = A \sin(k x - \omega t + \phi) \]

III. spherical harmonic traveling wave:
\[ \phi(r, t) = \frac{s_0}{r} \sin(k r \mp \omega t) \]

IV. plane transverse harmonic travelling wave on a string:
\[ \eta(x, t) = A \sin(k x - \omega t) \]

V. standing waves on a string:
\[ y(x, t) = 2A_0 \sin(k x) \cos(\omega t) \]
VI. beats:

\[ y(x, t) = 2A_0 \cos \left\{ 2\pi \left( \frac{f_1 - f_2}{2} \right) t \right\} \cos \left\{ 2\pi \left( \frac{f_1 + f_2}{2} \right) t \right\} \]

\( f_b = f_1 - f_2 \)

- Relationship between wave quantities

\[ v = \lambda f = \frac{\lambda}{T} \quad k = \frac{2\pi}{\lambda} \quad \omega = \frac{2\pi}{T} = 2\pi f \]

\( v = \sqrt{\frac{T}{\mu}} \) (for waves on a string), \( v = \sqrt{\frac{E}{\rho}} \) (for sound waves in air),

\( v = \sqrt{\frac{T}{\rho}} \) (for sound waves in solids) \( (\rho \text{ is the density of the solid}) \)

- Power transmitted by a harmonic travelling wave on a string:

\[ P = \frac{1}{2} \mu \omega^2 A^2 v \]

- Power transmitted by a harmonic travelling sound wave:

\[ P = \frac{1}{2} \rho A v (\omega s_{max})^2 \]

- Intensity of a spherical wave:

\[ I = \frac{P}{A} \propto \frac{1}{r^2} \]

A Famous Relationship between Path Difference and Phase Angle

\[ \frac{\delta}{\lambda} = \frac{\phi}{2\pi} \]
Standing Waves on a String Fixed at Both Ends

- frequency of normal modes:
  \[ f_n = \frac{n}{2L} v \quad (n = 1, 2, 3 \ldots) \]
- frequency of normal modes:
  \[ f_n = \frac{2L}{n} \quad (n = 1, 2, 3 \ldots) \]

Standing Waves in Air Columns

- frequency of normal modes:
  \[ f_n = \frac{n}{2L} v \quad (n = 1, 2, 3 \ldots) \] (open at both ends)
- frequency of normal modes:
  \[ f_n = \frac{n}{4L} v \quad (n = 1, 2, 3 \ldots) \] (closed at both ends)

Reflection and Transmission of Waves on a String

- reflection amplitude
  \[ A_{ref} = \frac{\sqrt{\mu} - \sqrt{\mu'}}{\sqrt{\mu} + \sqrt{\mu'}} A_{incident} \]
- amplitude of transmitted wave
  \[ A_{trans} = \]

for wave transmission and reflection across a barrier between two strings of differing linear mass densities \( \mu \) and \( \mu' \)

- The Wave Equation (one dimensional)
  \[ \frac{\partial^2}{\partial x^2} f(x,t) - \frac{1}{v^2} \frac{\partial^2}{\partial t^2} f(x,t) = 0. \]

- Exercise: Show \( f(x,t) = 2A \sin(kx) \cos(\omega t) \) is a solution of the wave equation with \( v = \frac{\omega}{k} \). 