Ice aspect ratio influences on mixed-phase clouds: Impacts on phase partitioning in parcel models
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The influences of evolving ice habit on the maintenance and glaciation of stratiform mixed-phase clouds are examined theoretically. Unlike most current modeling methods where a single axis length is predicted, the primary habits, or two axis lengths, are computed explicitly. The method produces a positive non-linear feedback between mass growth and crystal aspect ratio evolution. Furthermore, ice particle growth has a distinct initial-size dependence with smaller initial ice particles evolving into more extreme crystal shapes with greater overall mass. This feedback cannot be captured with simpler growth methods, leading to underestimates of ice growth and mixed-phase glaciation. Aspect ratio prediction is most critical for mixed-phase maintenance at temperatures where pronounced habits exist (dendritic growth, \( T = -15^\circ C \) and needle growth, \( -6^\circ C \)) and at ice concentrations between 1 \( L^{-1} \) and 100 \( L^{-1} \). At these temperatures and concentrations, rates of glaciation can be under-predicted by as much as an order of magnitude by equivalent density spheres. Habit prediction is less important for the maintenance of liquid at lower ice concentrations (<0.1 \( L^{-1} \)) as the time-scale for liquid depletion is relatively long (days). At higher concentrations (>100 \( L^{-1} \)) the time-scale for liquid depletion is shorter (minutes), thus predicting crystal habit has only a small impact on liquid lifetime. Updraft strength also affects mixed-phase cloud maintenance primarily at ice concentrations between 1 \( L^{-1} \) and 100 \( L^{-1} \). It is theoretically possible for vertical oscillating motions to maintain stratiform mixed-phase clouds indefinitely when temperatures are relatively high (>\( -10^\circ C \)) and ice concentrations are relatively low (<0.1 \( L^{-1} \)).


1. Introduction

Cloud-cover is an integral part of Arctic climate [Francis and Hunter, 2006; Kay et al., 2008; Kay and Gettelman, 2009; Stramler et al., 2011] and may depend on the accurate prediction of cloud feedbacks. Mixed-phase clouds are prevalent in the Arctic [e.g., Pinto, 1998], and the lifetime of these clouds exhibits a strong sensitivity to the number concentration and possibly to the habit (shape) of the ice particles [Harrington et al., 1999; Morrison et al., 2005; Fridlind et al., 2007; Avramov and Harrington, 2010]. Mixed-phase clouds are composed of supercooled liquid drops and ice crystals, a combination which is inherently unstable because the equilibrium vapor pressure for ice is lower than that of liquid. As a consequence, ice crystals can grow at the expense of the evaporating water droplets. Because ice crystal concentrations are generally much lower (1 to 100 \( L^{-1} \)) than liquid drop concentrations (100 to 1000 \( cm^{-3} \)) in mixed-phase clouds, the ice particles can grow to relatively large sizes, producing rapid precipitation. Ice growth can possibly lead to the demise of the cloud layer [e.g., Pinto, 1998] impacting the climate effects discussed by Kay et al. [2008]. In such clouds, ice growth by vapor deposition is significant, and so attention must be paid to how crystals evolve.

Korolev and Isaac [2003] explored the effects of ice growth on mixed-phase glaciation. They deduce that glaciation time, or the time it takes for ice to evaporate all liquid drops, is impacted by the environmental temperature (and thus ice supersaturation), updraft velocity, and ice number concentration. Their results indicate glaciation time is longer when the environmental temperature is higher, (\( -5^\circ C < \bar{T} < -1^\circ C \), i.e. low ice saturation ratio), the cloud has a small initial ice concentration, and has either a higher updraft velocity or an oscillatory updraft. This work was extended in [Korolev and Field, 2008] in which the dynamic impacts and theoretical constraints on mixed-phase cloud glaciation and stability were explored. Oscillating parcels with updraft speeds above a certain threshold can activate liquid water despite ice growth and never completely glaciate. Though parcel studies do not include either mixing effects (such as entrainment) or sedimentation, the results of Korolev and Field [2008] suggest that dynamic motions can maintain the liquid in mixed-phase clouds.
[4] Maintenance of liquid within mixed-phase clouds is known to be critical for their longevity. For instance, Harrington et al. [1999] suggest that enough liquid must be present for strong cloud-top long-wave radiative cooling to continue to drive the eddies that maintain the cloud. Recently, Morrison et al. [2011] show this is indeed the case using a suite of numerical models and illustrate the importance of large-scale moisture and thermal forcing for maintenance of the cloud layers. These studies and many others [e.g., Morrison et al., 2005; Fridlind et al., 2007; Prenni et al., 2007; de Boer et al., 2011] focus special attention on ice nucleation as ice concentration strongly impacts glaciation rates. While some observational cases show that mixed-phase clouds in the Arctic can persist for days even when significant concentrations of ice are observed [e.g., Morrison et al., 2005], relatively small increases in ice concentrations can lead to the complete glaciation of a mixed-phase layer in a short period of time [e.g., Harrington et al., 1999; Jiang et al., 2000]. Although the glaciation studies by Korolev and Isaac [2003] shed light on the theoretical conditions necessary for mixed-phase maintenance, a limitation of their work is that only spherical ice particles are considered. Ice particle growth is non-linearly related to the change in aspect ratio [Kuroda and Lacman, 1982; Chen and Lamb, 1994; Fukuta and Takahashi, 1999], and the possible impact on mixed-phase cloud evolution has yet to be explored in a systematic sense.

[5] Laboratory measurements show that the primary crystal habit (plate-like or column-like) depends on temperature, where plates exist in the temperature range $-10^\circ C < T < -22^\circ C$, columns in $-3^\circ C < T < -10^\circ C$, both habits in $T < -22^\circ C$, and isometric growth dominates at the transition temperatures ($\sim -10^\circ C$ and $\sim -22^\circ C$) [Fukuta and Takahashi, 1999; Bailey and Hallett, 2009]. These primary habits (plates and columns) can evolve into extreme, or secondary habits such as dendrites and needles (respectively) at high (liquid) saturations. Though these more “perfect” habits are found in the lab, many times crystals occur as irregular, as assemblages of plates and columns, or of polycrystalline structure in mixed-phase clouds [Korolev et al., 1999; Lawson et al., 2001]. Regardless of final shape, laboratory measurements provide vital data on the surface attachment physics that ultimately determines how ice crystals grow.

[6] Chen and Lamb [1994] provide a successful model that uses laboratory-measured data as input, captures the non-linear feedbacks between mass growth and habit evolution at liquid saturation, and compares well to wind tunnel data. Sheridan et al. [2009] study these non-linear feedbacks in simulations of cirrus. They find that initially smaller particles grow more quickly than initially larger particles, establishing a more pronounced habit. Consequently, distributions of ice formed on smaller ice nuclei (IN) particles may produce more extreme aspect ratios, faster growth, larger mass, and broader size spectra than when larger ice particles are initially formed.

[7] Approximating crystal vapor growth is a complex and obscure process. Current models use some form of the capacitance model, most predicting a single dimension assuming either an equivalent-density sphere or a mass-size relation. Methods such as these require a technique to approximate both the primary and secondary habits. The parameterization by Chen and Lamb [1994] is a first-order improvement over these methods in that two dimensions, the $a$- and $c$-axes, and hence the primary habit, are predicted. In this work, the method of Chen and Lamb [1994] is used to explore the consequences of evolving ice habits on the glaciation, lifetime, and phase partitioning between liquid and ice in mixed-phase clouds.

2. Vapor Depositional Growth and the Fickian-Distribution Model

[8] Like other ice crystal vapor growth studies [e.g., Harrington et al., 1995; Fukuta and Takahashi, 1999; Korolev and Isaac, 2003; Fridlind et al., 2007; Morrison et al., 2009], the Chen and Lamb [1994] method uses the capacitance model to compute the rate of vapor mass diffusion toward the surface of an ice particle. However, the Chen and Lamb [1994] method deviates fundamentally from these studies by using a mass distribution hypothesis (see below) to allow for the evolution of ice aspect ratio. This method is not purely a capacitance model, and so it has been re-termed the “Fickian-distribution method” (“Fickian” to describe the vapor mass flux toward the particle and “distribution” for how the vapor mass is distributed across the particle). Because the Chen and Lamb [1994] method has only been recently revived as a way to grow ice in cloud models [e.g., Hashino and Tripoli, 2007, 2008; Sheridan et al., 2009], it is worth explaining the physical basis of the method. Furthermore, though the capacitance model is used almost exclusively in the cloud modeling community, its physical basis and consequences for habit evolution are never explained, even in modern textbooks [e.g., Lamb and Verlinden, 2011, pp. 342–345]. As the capacitance model and the mass distribution hypothesis are physically and analytically correlated, an explanation of the mass diffusion process in the capacitance model is provided, and then that of the mass distribution hypothesis thereafter.

2.1. Mass Diffusional Growth (Capacitance Model)

[9] An explanation of the capacitance model is critical for understanding the Fickian-distribution method (i.e., the Chen and Lamb [1994] model). Through Fick’s Law, the diffusion of vapor molecules toward the surface of an ice crystal is directly proportional to the vapor gradient at the particle edge. For instance, spherically symmetric particles have the same gradient at all points on the surface, thus, the flux of vapor toward the particle is the same in all directions, growing the crystal radially and uniformly: $F_v = \frac{D_v \Delta \rho_s}{r}$, where $D_v$ is the vapor diffusivity, $\Delta \rho_s$ is the vapor density difference between the environment and the particle surface, and $r$ is the particle radius. The vapor flux rises as a function of curvature, or $\frac{1}{r}$, thus, for the same steady-state mass flow rate, a smaller area receives a larger flux. This is precisely because the smaller sphere represents a smaller sink of vapor for the same far-field mass flow rate.

[10] The above impact of local curvature on diffusive fluxes is a general result. In this work, oblate and prolate spheroids are used as a proxy for planar and columnar crystals, respectively. Though spheroids never occur in nature, they do provide a first-order approximation for the evolution of the two primary axes ($a$ and $c$) and therefore an improvement over the use of an equivalent density sphere. Moreover, unlike a pristine hexagonal plate or column, spheroids allow for the estimation of a larger variety of secondary habits.
through the use of a reduced density. Using the equations for the capacitance model [Moon and Spencer, 1961], it can be shown that the fluxes along a and c for oblate and prolate spheroids are:

\[
\begin{align*}
F_a &= \frac{D_a}{c} \Delta \rho_c f_{ob}(\phi) \\
F_c &= \frac{D_c}{a} \Delta \rho_a f_{ob}(\phi) \\
F_a &= \frac{D_a}{c} \Delta \rho_c f_{pr}(\phi) \\
F_c &= \frac{D_c}{a} \Delta \rho_a f_{pr}(\phi)
\end{align*}
\]

where \( \phi = \frac{a}{c} \) is the aspect ratio, and \( f_{ob}(\phi) \) and \( f_{pr}(\phi) \) are the respective oblate and prolate shape factors [see Chen and Lamb, 1994].

[11] Unlike spherical particles, non-spherical ice crystals do not have a uniform flux over the entire crystal as Figure 1 illustrates for a plate-like crystal. The curvature at the ends of the a- and c-axes, which are proportional to the radii of c and a respectively, is represented as circles for clarity. As equation (1) shows, the flux is greatest where the radius is smallest, thus, the flux is greatest at the particle’s tips, or along a, and weakest along c, leading to faster growth of the a-axis. This effect occurs in detailed crystals models [Libbrecht, 2005] and is non-linear: If a increases more quickly than c, the curvature along a becomes stronger relative to c in time, resulting in rapid growth. This analysis is analogous for columnar habits.

[12] The non-linearity above directly influences the mass diffusion rate because the fluxes are related to changes in the a and c-axes:

\[
\begin{align*}
\frac{F_a}{\rho_i} \frac{da}{dt} &= \frac{F_c}{\rho_i} \frac{dc}{dt}
\end{align*}
\]

Moreover, these relations allow for the derivation of the mass growth equation for a spheroid (see Appendix A):

\[
\frac{dm}{dt} = 4\pi C(c, a) G_i (S_i - 1)
\]

where \( G_i \) is defined in Appendix A, \( C(c, a) = a f_{ob}(\phi) \) for a plate and \( c f_{pr}(\phi) \) for a column, and \( S_i = \frac{e_i}{e_c} \) is the ice saturation ratio, with \( e_c \) and \( e_i \) as the ambient and ice equilibrium vapor pressures, respectively. The Fickian-distribution model uses the capacitance model to compute the mass diffusion rate. The non-linearity described above is now embodied in the capacitance as \( C \) is nonlinear relative to the aspect ratio (Figure 2). For the same volume particle, \( C \) increases as the shape (aspect ratio) deviates from spherical. This result is a quantification of the increased vapor fluxes as local radii of curvature decrease, which leads to an overall increase in mass growth. Unfortunately, the capacitance model predicts a constant aspect ratio (see below), and so the crystal shape never evolves.

Figure 1. The vapor flux incident on the a-axis, \( F_a \) (c-axis, \( F_c \)) is proportional to the vapor density gradient in that direction. The respective fluxes depend on the local curvature, indicated by the color spheres, at the end of each axis. Colored vapor density gradients correspond with their respective spheres. Far-field vapor flux is constant and uniform.

Figure 2. \( C(a, c) \) versus aspect ratio. Plates exist when \( \phi < 1 \) and columns when \( \phi > 1 \).
2.2. Aspect Ratio Evolution (Mass Distribution Hypothesis)

[13] The Fickian-distribution model deviates fundamentally from the capacitance model in the way mass is distributed along the \(a\)- and \(c\)-axes, allowing for aspect ratio evolution. By ratioing \(\frac{dV}{dt}\) and \(\frac{d\phi}{dt}\) in equation (2), combining with equation (1), and multiplying each flux by a deposition coefficient (see below), the mass distribution hypothesis based on crystal growth theory is derived:

\[
\frac{dc}{da} = \frac{\alpha_c F_c}{\alpha_a F_a} = \frac{\alpha_c}{\alpha_a} \phi = \Gamma \phi, \tag{4}
\]

where \(\alpha_c\) and \(\alpha_a\) are the mass deposition coefficients along \(c\) and \(a\), respectively. The crux of the hypothesis is the modification of the fluxes onto \(a\) and \(c\) by these deposition coefficients. The deposition coefficients are growth efficiencies for each axis direction varying from 0 to 1 and are determined from laboratory measurements [e.g., Libbrecht, 2003]. This hypothesis works in the correct physical manner: The mass distributed onto each axis should be, to first approximation, the growth efficiency multiplied by the flux onto that axis. Central to the above hypothesis is the fact that the Inherent Growth Ratio (IGR), \(\Gamma\), depends primarily on temperature at liquid saturation [e.g., Chen and Lamb, 1994].

[14] The capacitance model has deposition coefficients that are unity, and as a consequence, the aspect ratio remains constant in time (\(\Gamma = 1\)). This result occurs because the vapor density is forced to be constant over the surface of the ice crystal [McDonald, 1963], which is not the case for crystals with an aspect ratio different from unity or for faceted crystals. Though this physical consequence of the capacitance model is almost never stated, it is critical because any improvements to the model must, in some way, account for this fundamental physical inconsistency. Because the deposition coefficients multiply the fluxes, and therefore the vapor density gradients in the mass distribution hypothesis above, the method is in essence a first-order correction for the consequences of the constant-density boundary condition, allowing for aspect ratio evolution. Also, the method is flexible in the sense that it can accommodate new data as provided through laboratory and in-situ measurements. Indeed, Hashino and Tripoli [2007] have used IGR values derived for rosettes and polycrystals in their model for ice growth at low temperatures (\(T < -20^\circ\mathrm{C}\)), where the IGR from the Chen and Lamb [1994] model shows less accuracy and may not predict the correct crystal habit [Bailey and Hallett, 2009].

[15] The above hypothesis has some important consequences for mass and aspect ratio evolution not discussed by Chen and Lamb [1994]. For instance, differentiating the volume of a spheroid (\(V = \frac{4}{3}\pi r^3\)) with equation (4), and recasting in terms of the aspect ratio, it can be shown that

\[
\frac{d\phi}{\phi} = \frac{dV}{V} \left(\frac{\Gamma - 1}{\Gamma + 2}\right) \text{ or } \frac{d\phi}{\phi} = \frac{3dr}{r} \left(\frac{\Gamma - 1}{\Gamma + 2}\right) \tag{5}
\]

where it is assumed that most ice particles begin as spheres or are nearly isometric so that \(C(a, c)\) is the radius, \(r\). With the Fickian-distribution model, \(\Gamma \neq 1\), evolving the aspect ratio and allowing for non-linear feedbacks with mass diffusion through the capacitance (Figure 2). Moreover, an important size effect occurs in equation (5). Because \(d\phi \propto \frac{1}{r}\), the smaller the initial radius of the crystal, the greater the relative change in aspect ratio. Therefore more pronounced habits (either very large or small \(\phi\)) are produced for smaller initial ice crystals [e.g., Sheridan et al., 2009] ultimately leading to greater mass growth.

2.3. Density Evolution

[16] All simulated ice particles begin as solid ice spheres with a density of bulk ice (\(\rho_i = 920 \text{ kg m}^{-3}\)). As ice particles evolve, they will naturally take on shapes with branches and hollow sections. Consequently, simulated ice particles require a density reduction in time in order to represent the lower spatial density of a real crystal: The reduced density derives from data and is the observed crystal mass divided by the spheroidal volume with the same \(a\)- and \(c\)-axis lengths. As an ice crystal grows, this reduced density is accounted for by a “deposition density” that relates changes in mass to changes in volume. \(\frac{d\rho}{dV} = \rho_{dep} \frac{d\phi}{\phi}\). This density is an approximation for the development of secondary habits such as dendrites (\(T = -15^\circ\mathrm{C}\)) and hollow columns (\(T = -6^\circ\mathrm{C}\)) in time and consequently, \(\rho_{dep}\) depends on temperature and ice supersaturation [Chen and Lamb, 1994, equation (42)]: The deposition density is lowest at temperatures and saturations that promote the most extreme crystalline shapes (such as dendrites). Mass added over a time-step is assumed to have a density given by \(\rho_{dep}\) and so the ice crystal density after a time-step is estimated as a volume-weighted fraction of the prior crystal density and \(\rho_{dep}\). The weighting is given by the fractional change in volume of the crystal over the time-step.

[17] Because the deposition density is valid for ice supersaturation only, a sublimation density is defined here for ice subsaturated conditions. Using the current particle density as the sublimated mass is inaccurate because the particle density has varied over time, leading to density variation with size. As an example, during sublimation the tips of a dendrite’s branches sublimate at the most rapid rate, and so in a spheroidal representation, the density of the sublimated ice is less than that of the particle as a whole. It is assumed that the density of the crystal varies with volume following a power law: \(\rho_{dep} = \alpha V^{\beta}\). The coefficients \(\alpha\) and \(\beta\) can be derived as the initial crystal has the density of bulk ice, \(\rho_b\). These assumptions are reasonable and follow other approaches in the literature [Mitchell, 1996]. Differentiating the mass \((m = \rho_{ice} V)\) and using the ice density relationship above gives

\[
\frac{dm}{dt} = \rho_{ab} \frac{dV}{dt}, \quad \rho_{ab} = \rho_{ice}[1 + \beta] \tag{6}
\]

where \(\beta = \frac{\rho_{icem}(T)}{\rho_{icem}(T_{icem})} \frac{1}{\ln\left(\frac{V_i}{V_a}\right)}\), \(V_a = \frac{4}{3}\pi r_i^3\) is the initial crystal volume at the beginning of growth, and \(V_i\) and \(\rho_{ice}\) are the volume and density of the crystal at the beginning of each time-step, respectively. Note that \(\beta\) is a first-order correction for the variation of crystal density with size. Without this correction, particle density is assumed to be constant during the process of sublimation. While this correction is not entirely necessary, especially when the particle is large, the correc-
tion does lead to more stable solutions at the critical juncture when the particles completely sublimate.

2.4. Ventilation

When an ice particle grows to a mass large enough for its terminal velocity to become appreciable, ventilation should be considered. When falling through a cloudy environment, the particle experiences a direct, saturated flow on the face perpendicular to the fall direction and an indirect flow on the faces parallel (or nearly parallel) to the flow leading to a net increase in deposition rates. Ventilation requires particle fall speeds that are computed using the mass and the projected area of the spheroidal particles in a consistent fashion following [Mitchell, 1996]. The projected area is computed assuming particles fall with their major axis perpendicular to the flow following [Böhm, 1992]. Ventilation is accounted for by modifying the thermal and vapor diffusivities through empirical relationships connecting the Best number and the Reynolds number, which is used to compute the fall speeds of the non-spherical particles [e.g., Mitchell, 1996]. Ventilation coefficients are computed for each axis following the method prescribed by [Chen and Lamb, 1994].

In order to compare with prior results [Korolev and Isaac, 2003; Korolev and Field, 2008], ventilation is omitted in most simulations in this study. However, in order to recognize its effect, ventilation is included in a few results indicated below.

2.5. Limitations and Uncertainties

As described above, the capacitance model is limited because the aspect ratio never evolves. While the method of [Chen and Lamb, 1994] provides a first-order correction to this physical inconsistency, the method has its own limitations. When simulating the growth of ice crystals, the habit into which the particles evolve depends on the IGR. However, the values of the IGR are extracted from laboratory measurements [Lamb and Scott, 1972] and in-situ observations [Ono, 1969, 1970; Auer and Veal, 1970; Heymsfield and Knollenberg, 1972; Jayaweera and Ohtake, 1974]. In the observational cases, it is likely that ventilation indirectly affected the IGR values whereas some of the laboratory measurements included stationary particles in a controlled environment [Lamb and Scott, 1972]. Therefore, caution should be exercised when interpreting results using IGR values derived from various measurement sources. Moreover, some newer data sets do exist and have been used by [Hashino and Tripoli, 2008] to derive IGR values for temperatures below −20°C.

Though promising, it is not clear whether the Fickian-distribution method is accurate below liquid saturation where the assumption that the IGR depends only on temperature may break down. As the ice supersaturation decreases, the deposition coefficients decrease, and so the IGR should change. Additionally, a method does not exist to include surface kinetic resistance, which appears when ice supersaturations fall below a critical value at a given temperature [see Nelson and Baker, 1996; Wood et al., 2001]. Though [Lamb and Chen, 1995] provided a hypothetical method for including surface kinetic resistance, this was done for spherical particles only. In addition, there do not currently exist any studies on whether mass distribution hypothesis corrects for irregularly-shaped particles [e.g., Korolev et al, 2004; Takahashi and Mori, 2006], which occur frequently in nature, or if the method is accurate for varying temperatures and saturations. Furthermore, it is unclear whether the dependence of growth on initial size is

Figure 3. The evolution of axis length with respect to temperature in the single particle model simulations. a- (darker regions) and c- (lighter regions) axes lengths of a single crystal after 10 (grey region) and 20 (red region) minutes of growth. Initial particle radii between 1–10 μm are used and produce the shaded ranges.
physically correct. At present, there is no clear data available to test this consequence of the theory.

3. Box Model Evolution and Impact of Initial Particle Size

3.1. Single-Particle Model

[22] Feedbacks between the evolution of crystal mass and aspect ratio impact the final sizes and shapes of simulated ice crystals and therefore affect the partitioning of liquid and ice masses in mixed-phase clouds. Thus, understanding the growth of individual ice particles is essential for interpreting results presented later. To this end, ice particle growth is examined in a simplified box model, holding all parameters constant except for the particles’ size and shape, which are allowed to evolve freely.

[23] Simulations were run allowing initially spherical particles with sizes 1 \( \mu \text{m} \) and 10 \( \mu \text{m} \) to evolve for 20 minutes at each temperature in the range \(-30^\circ \text{C} < T < -3^\circ \text{C}\) (Figure 3). As expected, the \( a(c) \)-axis length is longest (shortest) within the temperature range \(-20^\circ \text{C} < T < -10^\circ \text{C}\) when planar growth is at its greatest, and the \( c(a) \)-axis length is longest (shortest) within the temperature range \(-10^\circ \text{C} < T < -3^\circ \text{C}\) when columnar growth is at its greatest. The IGR values used here predict columns at \( T < -20^\circ \text{C}\), which is inconsistent with newer data [e.g., Bailey and Hallett, 2009], though this can be corrected [Hashino and Tripoli, 2008]. However, because the focus of the current study is on mixed-phase clouds with \( T > -20^\circ \text{C}\), this inconsistency does not affect the conclusions drawn here. Maxima occur at \(-15^\circ \text{C}\) and \(-6^\circ \text{C}\) where dendritic and needle growth respectively dominate. The intersection points occur where both axis lengths are equal, and the growth is essentially isometric. The mass of the particle is also the lowest at these transition temperatures and peaks within habit-prone temperature regimes (Figure 4), following wind tunnel data [see Fukuta and Takahashi, 1999].

[24] There also exists a dependence of crystal shape evolution on initial particle size: As discussed in section 2, initially smaller particles lead to crystals with more extreme shapes. For instance, Figure 3 indicates that the particle axes lengths can vary by up to an order of magnitude based on the initial size of the particle. Initially larger ice particles (10 \( \mu \text{m} \)) evolve into thicker plates (\( \phi = 0.04 \) at \(-15^\circ \text{C}\)) and shorter columns (\( \phi = 0.07 \) at \(-6^\circ \text{C}\)), whereas initially smaller particles (1 \( \mu \text{m} \)) evolve into thin crystals resembling dendrites (\( \phi = 0.003 \) at \(-15^\circ \text{C}\)) and needles (\( \phi = 0.09 \) at \(-6^\circ \text{C}\)). This effect disappears near the transition temperatures where growth is isometric. Note that the final crystal shape depends strongly on the initial size, and this indicates that there could be influences of ice nucleation on particle shape.

[25] The initial spherical shape of an ice particle also affects the mass, predominantly at habit-prone temperatures. Figure 4 shows the mass after 20 minutes of growth for four different initial particle radii. Initially smaller particles attain larger final masses as compared to initially larger particles; the effect is most pronounced for initial sizes less than about 5 \( \mu \text{m} \). Though the effect still exists for larger nucleated particles, it is less pronounced: initially larger particles with radii greater than 10 \( \mu \text{m} \) have a final mass similar to that of a spherical particle.

[26] In order to illustrate the robust nature of the ice growth method, the results are juxtaposed with the wind tunnel cloud chamber data of Takahashi and Fukuta [1991]. Ice crystals in a wind tunnel experience ventilation effects, which are included in the simulations. The data presented by Fukuta and Takahashi [1999] showed that riming began to affect growth after about 20 minutes and so simulations are carried out no longer than this time.

[27] The single particle’s axes lengths, mass, and fall velocity are simulated for a range of temperatures and growth times (Figure 5). Model runs use a range of initial particle sizes not only because of the initial size effect, but also because the wind tunnel experiments [Takahashi and Fukuta, 1991] contain an array of initial particle sizes. As expected, the maxima and minima in the masses (Figure 5b) and axis lengths (Figure 5a) with respect to temperature follow those of the non-ventilated cases presented above, though the growth is now more rapid. While the match is not perfect with some relative errors up to 30\%, the axis lengths after 10 and 15 minutes compare well to the wind tunnel data over the ranges of initial particle sizes simulated. The model also captures the temperature-dependence and transition temperatures of the particle mass, though the predicted values are larger than the wind tunnel data. This is due primarily to the deposition density used [Chen and Lamb, 1994], which is larger than that measured by Takahashi and Fukuta [1991]. The fall speed variation predicted by the model with temperature is also physically correct: The more compact, nearly isometric particles fall with the greatest velocity. These peaks in the fall speed at the transition temperatures do not show up completely in the laboratory data, but Takahashi and Fukuta [1991] indicate that they do exist. The particle fall speeds are the lowest at habit-prone temperatures, where aspect ratios deviate substantially from unity and the particle densities are at their minimum. The fact that the model is able to capture variations in four parameters (two axis lengths, mass, and fall speed) in time and with temperature indicates first-order accuracy for growing ice crystals at liquid saturation. Comparisons between the model and laboratory data become less reliable...
at lower temperatures ($\leq -20^\circ C$): Both columns and plates can be found in this temperature range, and as the absolute saturation decreases with temperature, surface kinetic effects are likely to become more important.

3.2. Habit Influences on Distribution Evolution and Glaciation Time

[28] The temperature and initial size-dependences of ice growth and aspect ratio evolution indicate that there could be important consequences for the evolution of a distribution of ice crystals possibly affecting the phase partitioning in mixed-phase clouds. To illustrate the initial size effect, ice particle distribution evolution is examined at $T = -15^\circ C$. Ice growth maximizes here, enabling the largest possible impact of initial ice size. Ice is initiated assuming a gamma distribution of spheres that evolve into non-spheres (Figure 6). Over time, the growth of initially smaller particles results in the out-growth of initially larger particles in terms of mass and size (Figure 6a). In addition, the aspect ratio of all particles deviates further from unity with the initially smaller particles achieving the most extreme aspect ratios (Figure 6b). Initially larger particles produce thicker plates, whereas initially smaller particles produce very thin plate-like crystals with aspect ratios similar to dendrites.

[29] The temperature and initial size dependence of ice growth play a role in the glaciation time of simulated clouds. As expressed by Korolev and Isaac [2003], the time required for ice to completely evaporate the cloud liquid water (glaciation time) depends primarily on temperature and ice concentration. What Korolev and Isaac [2003] do not include are the effects of aspect ratio changes on glaciation.

[30] Including the growth of non-spherical particles generally produces much shorter glaciation times as compared to spherical particles, depending on the temperature. At the most habit-prone temperatures ($-15^\circ C$ and $-6^\circ C$), crystals with extreme aspect ratios are produced, the ice growth is rapid, and so liquid depletion occurs quickly. Glaciation tends to be the slowest at the transition temperatures between habits ($-10^\circ C$ and $-22^\circ C$) because growth is isometric and therefore slower. Similar to the findings of Korolev and Isaac [2003], the glaciation time for non-spherical ice is highly dependent on the ice concentration (Figure 7). At relatively low ice concentrations ($i.e., N = 10^6 \ L^{-1}$), ignoring aspect ratio evolution can underestimate the glaciation time by an order of magnitude. Higher ice concentrations glaciate the cloud more quickly and so aspect ratio evolution is less important, and the glaciation times are closer to those of spherical particles ($\sim 1$ minute versus $0.2$ minutes for non-spherical par-

Figure 5. Comparison between single particle model simulations and Takahashi and Fukuta [1991] wind tunnel data (dots and circles). (a) $a$ (darker regions) and $c$ (lighter regions) axes lengths, (b) mass, and (c) fall velocity of a single crystal after 10 (grey region) and 15 (red region) minutes of growth. Initial particle radii between 5–10 $\mu m$ are used and produce the shaded range. [Takahashi and Fukuta, 1991] data are after 10 (black dots and circles) and 15 (red dots and circles) minutes of growth. All simulations include ventilation.
Evolution of the ice size distribution at a temperature of \(-15°C\) and liquid saturation. Particles are initiated within a distribution with shape, \(\nu = 4\), and a mean radius of 10 \(\mu m\). (a) Equivalent volume spherical radius \((r)\) and (b) aspect ratio are shown. Line 1: time = 30s; Line 2: time = 60s; Line 3: 90s; Line 4: time = 110s; Line 5: time = 140s; Line 6: time = 170s; Line 7: time = 200s.

Figure 6. Evolution of the ice size distribution at a temperature of \(-15°C\) and liquid saturation. Particles are initiated within a distribution with shape, \(\nu = 4\), and a mean radius of 10 \(\mu m\). (a) Equivalent volume spherical radius \((r)\) and (b) aspect ratio are shown. Line 1: time = 30s; Line 2: time = 60s; Line 3: 90s; Line 4: time = 110s; Line 5: time = 140s; Line 6: time = 170s; Line 7: time = 200s.

4. Parcel Model

It is worth mentioning that these results are specific to the IGR data derived by Chen and Lamb [1994] and predict only certain habit types. The results presented would certainly change if different crystalline forms were predicted. For instance, although irregular and poly-crystalline forms regularly occur in clouds [e.g., Korolev et al., 1999], the quasi-spherical shapes of many of these particles suggest that the equivalent volume sphere results may provide an appropriate estimate of particle growth. Nevertheless, the goal of this study is not to replicate every possible crystal type that can exist in a cloud, but to illustrate the possible habit impacts on glaciation if aspect ratios are allowed to evolve freely.

4. Parcel Model

The box model analysis shows that aspect ratio evolution impacts the rate at which liquid is depleted through ice growth, indicating possible influences on mixed-phase cloud lifetimes. However, the studies of the prior section were static, so those analyses are extended to more realistic scenarios using a Lagrangian parcel model. Parcel models are advantageous because the simplified framework allows for the study of detailed microphysical processes that cannot be examined with other models (e.g., kinematic or fully-dynamical cloud models). The parcel model used here is similar to that of Lebo et al. [2008] and Sheridan et al. [2009] in that differential equations for the parcel’s environmental variables which include temperature, pressure, water vapor, saturation, and height are solved for in time. The parcel begins at a prescribed temperature, pressure, and in a subsaturated (with respect to liquid) environment. As the parcel rises (according to a pre-determined updraft velocity profile that is either constant or sinusoidal) an initial cloud condensation nuclei (CCN) population swells following Köhler theory until liquid saturation is exceeded, at which point cloud drops above their Köhler curve maxima grow continuously. Following recent studies showing that many times ice first appears only after liquid is formed [e.g., de Boer et al., 2011], a population of small, initially isometric ice crystals is initiated assuming a gamma size spectrum once liquid saturation is achieved. This method avoids the complications of including ice nucleation mechanisms, as the focus of this work is vapor growth. Ice nucleation effects are the subject of a companion study [Ervens et al., 2011]. It is important to note that ventilation is excluded from many simulations for consistency with prior results [Korolev and Isaac, 2003; Korolev and Field, 2008]. However, ventilation is included in a subset of simulations to indicate its importance. The differential equations for the environmental variables along with 200 differential equations for both liquid and ice are solved with VODE (Variable Order Differential Equation Solver; see Appendix B for details) [Brown et al., 1989].

A distinct difference between the box model and the parcel model simulations is the evolution of the liquid supersaturation \((S_l = \xi - 1)\) in time. Mixed-phase persistence depends primarily on the available vapor in the environment, which in turn depends on the updraft speed \((w)\) and the rates of condensational and depositional growth [e.g., Korolev and Isaac, 2003] (see equation (B17)). To first order, a faster updraft speed results in a greater supersatura-
tion production rate, and if production is greater than consumption by ice growth, then liquid and ice can co-exist. The sink term due to ice growth can be greater than the source, inducing drop evaporation, and thus glaciating the cloud.

[35] It is important to note that one of the major limitations of the parcel model is that liquid drops and ice particles cannot sediment into or out of the parcel. This includes precipitation above and below the cloud, as well as large-scale fluxes of vapor and thermal energy. However, introducing processes such as sedimentation into a parcel model framework is not only problematic, it could be argued that it is physically incompatible with the parcel framework itself as no reliable method of communicating ice particles to other parcels exists. Ignoring sedimentation produces overestimations in ice water contents (IWCs) and underestimations in liquid water contents (LWCs), and therefore parcel model results are an upper limit for ice amounts and a lower limit for cloud glaciation times. Omitting large-scale fluxes also creates uncertainties in parcel mass partitioning and the overall structure of the cloud: Large-scale advective fluxes can substantially impact mixed-phase cloud lifetime [Stramler et al., 2011].

[36] In addition, entrainment and detrainment of the parcel with the surrounding environment are not considered leading to a more homogenous parcel environment than would likely occur. Radiative cooling and warming are also ignored, which should alter the parcel temperature. While the consequences of these uncertainties are re-iterated below, it is important to keep them in mind when interpreting the results.

4.1. Single Oscillatory Updraft

[37] Following Korolev and Isaac [2003] and Korolev and Field [2008], a cosine function with a maximum (minimum) speed of $+(-)0.5 \text{ m s}^{-1}$ is used to mimic the up and downdrafts that compose a typical eddy in a mixed-phase cloud with depths chosen to approximately vary between 150 m–600 m. [38] As discussed by Korolev and Isaac [2003], an oscillating updraft lengthens the glaciation time, especially when ice concentrations are low. At low ice concentrations the deposition rate is slow enough so that the updraft, and at times the downdraft, environment remains supersaturated with respect to liquid, maintaining the cloud as mixed-phase.

[39] Using an ice concentration of $10 \text{ L}^{-1}$ along with an initial temperature of $T=\pm 15^\circ C$, as in the work of Korolev and Isaac [2003], both liquid and ice are able to persist within the updraft for part of the cycle. However, even with a typical updraft speed, the parcel eventually glaciates with the non-spherical evolution producing lower LWC and IWC in comparison to the spherical results (Figures 8a and 8b). Moreover, habit evolution produces a reduction in super-cooled liquid cloud depth by approximately 200 m when initial ice sizes are smaller (2 $\mu m$). However, a larger initial size (10 $\mu m$) produces a depth similar to that of spheres.

[40] Initial parcel temperature also plays a role in determining the lifetime of the liquid. Figures 8c and 8d compare temperatures of $\mp 10^\circ C$ and $\mp 15^\circ C$ for initial ice particle radii of 2 $\mu m$ (Figure 8c) and 10 $\mu m$ (Figure 8d). Figures 8c and 8d show that even when the parcel is initialized at a transition temperature ($\pm 10^\circ C$) where particles grow nearly isometrically, they do not grow in the same fashion as spherical particles. Therefore, neglecting crystal habit produces an underestimation of the glaciated cloud IWC, and inaccurately predicts the liquid cloud depth. This result occurs because temperature is not constant, forcing the ice particles to transition into different growth regimes altering their growth characteristics, which cannot be captured with the spherical theory.

4.2. Multiple Oscillatory Updrafts

[41] A single eddy cycle, while demonstrative of the main physical impacts of habits, is not quite realistic. Cloud-scale circulations produce numerous eddies with many over-
turning cycles. While this model cannot capture turbulent effects that occur in real clouds, the effects of eddy cycling by using multiple oscillations in a similar fashion in the work of Korolev and Field [2008] can be illustrated using \( w = w_{\text{max}} \cos \left( \frac{t}{t_{\text{osc}}} \right) \) where \( t_{\text{osc}} \) is an oscillation time scale that depends on cloud depth and the vertical motion and \( t \) is the increasing time throughout the simulation. However, it is important to keep in mind that the lack of sedimentation will lead to an overestimation of ice crystal growth.

[42] Figures 9, 10, and 11 show changes in water content, average ice particle and drop diameters, and supersaturation as a result of dynamic recirculation for both spherical and non-spherical ice particles. Spherical and non-spherical effects are examined first (Figure 9) followed by ice concentration impacts at different initial temperatures (Figures 10 and 11). The results presented here match the general trends that were exhibited so thoroughly in the work of Korolev and Isaac [2003]. The evolution of the LWC, IWC, average particle sizes, and supersaturations oscillate following the updraft cycle (Figure 9) and depend on ice number concentration (Figures 10 and 11). The results shown in Figure 9 for spherical particles are similar to those illustrated by Korolev and Isaac [2003]. While the LWC oscillates, it decreases over time as ice crystals grow continuously in both up and downdrafts. This recycling is, in part, artificial because a fraction of the ice particles would sediment from the parcel. Ice particles never completely sublimate, and their sizes remain relatively large because of the continued vapor growth. Predicting non-spherical particles leads to complete liquid depletion (glaciation) by about 18,000 seconds and larger ice particle sizes (\( \sim 3000 \) versus \( 1000 \) mm for spherical growth). It is important to keep in mind that the ice particle concentration is held constant. During sublimation, ice particles that become smaller than their initial size are effectively removed by deactivating that bin. These particles then act as passive-tracers and are permitted to recirculate and grow once the parcel becomes ice supersaturated. This method avoids feedbacks with ice nucleation, allowing a focus on vapor growth alone.

[43] Figures 9a, 10a, and 10b illustrate the effects of ice concentration and temperature on parcel evolution; note that the habit effect is pronounced at the lowest ice concentrations (Figure 9a), as earlier box model results show. As the ice concentration is increased, non-spherical particle aspect ratios deviate less from unity (not shown), and so the spherical and non-spherical results become similar, as the sequence of Figure 9a, Figure 10a, and Figure 10b shows. It is important to note that mixed-phase clouds can persist at low ice concentrations, but whether they do so depends on the temperature: Cloud parcels with an initial temperature of \(-15^\circ C\) cannot maintain liquid even when \( N_{\text{ice}} = 0.1 \) L\(^{-1}\), whereas at \(-10^\circ C\), liquid can be maintained for about 20,000 seconds (or 5.5 hours) even when particles grow non-spherically (Figure 11).

[44] A second physical effect that can be gleaned from Figure 9 is the steady-state obtained for the average axes
lengths, but not the liquid drop average diameters. This difference is an effect of the fluctuating supersaturation. The liquid drops evaporate in the downdraft, returning them to haze particles, and maintaining the environment supersaturated with respect to ice, allowing for the continued growth of the ice particles, although at a reduced rate. The haze particles grow back into liquid drops in subsequent updrafts. However, given that the pre-existing ice crystals have relatively large sizes, the rate of evaporation is stronger than that of condensation. The cycle continues until the ice particles deplete the available vapor, eventually leading to cloud glaciation. This process also occurs for spherical particles, but at a slower rate as the figures show. In addition, if ice particle sedimentation were to be considered, this recirculation effect would be less pronounced. Consequently, glaciation would occur more slowly than these estimates suggest.

[45] Initial temperature also affects the partitioning of mass between liquid and ice. This is shown in Figure 11 for a cloud base temperature of $T = -10^\circ C$ as opposite to $-15^\circ C$ in the previous two figures. While the temperature is not constant throughout the simulation, ice particles will tend to grow, at least to first-order, with the general shape that is similar to the temperature at which the particles nucleate. Because $T = -10^\circ C$ is near the transition temperature between columns and plates, nearly isometric habits form and grow. Here, differences in IWC between the spherical and non-spherical results are never greater than 0.05 g m$^{-3}$ compared to differences of up to 0.2 g m$^{-3}$ at $-15^\circ C$.

[46] Following prior studies [e.g., Korolev and Isaac, 2003], the above simulations do not include ventilation. Nevertheless, simulations with ventilation indicate where the effect is likely most prominent. The influence of ventilation on ice growth is most evident within certain temperature regimes and ice particle concentrations (Figure 12). At higher temperatures ($T_{base} = -5^\circ C$), ice growth is slower due to the lower difference in liquid and ice equilibrium vapor pressures, and therefore the effect of ventilation is only noticeable at higher ice concentrations (10 L$^{-1}$). At lower cloud-base temperatures ($T_{base} \lesssim -10^\circ C$), the equilibrium vapor pressure differences are larger, ice growth is greater, and thus the ventilation effect is more prominent at lower ice concentrations (1 L$^{-1}$). At habit-prone temperatures ($T_{base} = -15^\circ C$) when ice growth is substantial, the ventilation effect is muted at most ice concentrations due to rapid ice crystal growth and subsequent glaciation. It is important to note that, although temperature varies with height in these simulations, the depth of mixed-phase stratus is relatively shallow (320 m); thus the temperature variation from cloud base to cloud top is small and so the particles will grow within a single habit regime.

[47] Interestingly, the IWCs in the non-ventilated and ventilated cases asymptote in unison to the same steady state. A steady-state is reached because the oscillating cloud...
Multiple cloud parcel oscillations of spherical - = 0.5 N = 1 0 L ð Þ - = 0 . 1 ms f G Q m ms < 1 0 0 N [2008] describe for spherical ice, that there are; and Korolev; ffi [2008], where stable states of oscillating parcels are showed because T f to 0.9 and T - a dependencies on ice concentration, particle habit, and internal of idealized mixed (black) water contents for (solid) and non (dashed) ice (red) and liquid (black) water contents for T_pase = −10°C, initial ice particle concentration (a) N_ice = 0.1 L−1, (b) N_ice = 1 L−1 and (c) N_ice = 10 L−1, and updraft velocity w_max = 0.5 ms−1.

Figure 11. Multiple cloud parcel oscillations of spherical (solid) and non-spherical (dashed) ice (red) and liquid (black) parcel is always supersaturated with respect to ice. While ventilated particles take up vapor more quickly, the vapor source is limited to the total mass of water initially available, so the ice can only take up a certain maximum amount. It can be shown analytically that when the simulated parcel glaciates and reaches a steady state, the ice supersaturation becomes proportional to the inverse of the ventilation coefficients, canceling the ventilation coefficients in the mass growth equation:

$$\frac{dW_{C}}{dt} \cong 4\pi N_{C}(c, a) G(T, \overline{T}, \overline{f}, \overline{\tau}) S_{eq} \cong \frac{Q_{1\pi}}{Q_{2}}$$

(7)

where S_{eq} = \frac{Q_{1\pi}}{Q_{2\pi}N_{C}(c, a)T_{f}}, Q_{1} and Q_{2} are given in Appendix B, and the over-bars indicate averaged quantities. This indicates that both ventilated and non-ventilated growth reach the same steady-state value once the liquid is depleted.

4.3. Mixed-Phase Cloud Longevity

The simulations in the previous section paint a picture of idealized mixed-phase cloud evolution and its interlinked dependencies on ice concentration, particle habit, and internal dynamics. As the results suggest, mixed-phase stratus lifetime may not only depend on ice concentrations as some have speculated [e.g., Harrington et al., 1999; Fremini et al., 2007], but the temperatures at which the cloud resides due to the influences of ice aspect ratio evolution.

4.3.1. Summary of Results

The results from Figures 9–11 suggest, as Korolev and Field [2008] describe for spherical ice, that there are particular combinations of updraft speed, ice concentration, and temperature that determine the phase partitioning in mixed-phase clouds and whether the clouds will glaciate. Rather than following the method shown by Korolev and Field [2008], where stable states of oscillating parcels are depicted, the main effects of ice concentration and habit are illustrated on a diagram similar to Figure 7. Many of the various ingredients for determining mixed-phase cloud lifetime are encapsulated in Figure 13. Similarly to Figure 7, Figure 13 shows the glaciation time with respect to temperature, except now for multiple parcel oscillations. Simulations are completed for the most critical physical factors that appear to determine the co-existence of liquid and ice in mixed-phase clouds: a range of maximum vertical motions, ice concentrations, and two typical cloud depths (150 m and 320 m). The glaciation time is determined when the maxima in the LWC oscillations disappears (i.e. around 18,000 seconds for non-spherical growth shown in Figure 9a). The shaded regions represent the range of glaciation times for a particular ice concentration when the updraft velocity is varied over a range typical of layered mixed-phase stratus [Shupe et al., 2008] from 0.1 ms−1 to 0.9 ms−1. It is immediately apparent that the greatest sensitivity to updraft exists at ice concentrations 1 L−1 < N_ice < 100 L−1. Above this range, the ice concentration is large enough that the available liquid water is depleted rapidly, and the effect of the updraft speed on glaciation time is minimal. Below this range, the effect of varying the updraft velocity diminishes due to the fact that the ice concentration is so low relative to the available vapor that liquid and ice can co-exist for long periods even when the updraft velocity is very slow (comparable to the box model). These results show that sensitivity to in-cloud oscillations and updraft velocity varies depending on ice concentration, as was discussed by Korolev and Field [2008] for spherical particles.

Similar to the box model, differences occur between the non-spherical and spherical results (dotted lines). Simulations with spherical particles produce the slowest possible growth and therefore act as an upper limit for glaciation time at a given updraft velocity. Only 0.1 ms−1 is shown because the influence of varying the updraft speed is qualitatively similar to that of non-spherical particles. Most importantly, the effect of habit on glaciation time is most pronounced at habit-prone temperatures (−15°C and −6°C) as the ice concentration decreases. At these low ice concentrations, the glaciation time can be reduced from the order of days to the order of hours because of non-spherical growth effects. This result is especially pronounced near −15°C, which also happens to be near the temperature at which many field studies have taken place (discussed below). To note, it is likely that within the time allotted for parcel glaciation that large-scale surface heat and moisture fluxes could affect the outcome of the parcel lifetime, providing a source of vapor, and potentially altering the growth of the ice particles through thermal variation.
Similar to the results of Korolev and Field [2008], a region of parameter space exists where glaciation appears to never occur (where the lines and shades stop). In these simulations, mixed-phase conditions can be maintained if $T = -10^\circ C$ and $N_{ice} \lesssim 1 L^{-1}$. While Korolev and Field [2008] show that a broader range of parameter space can produce mixed-phase clouds, larger updraft speeds and deeper clouds would be required for non-spherical particles. It is important to keep in mind that this conclusion is drawn based on the range of updraft speeds, the cloud depth considered typical of layered mixed-phase stratus, and for continually oscillating parcels. Consequently, if the updraft speeds are strong enough, deeper mixed-phase clouds (such as cumulus) could have regions of parameter space where glaciation never occurs, even at low temperatures and high ice concentrations [Korolev, 2007].

It is interesting to note that Figure 13 shows a relatively small sensitivity to a reduction in cloud depth. The shaded regions represent a cloud-depth of 319 m while the black lines represent a cloud-depth of 150 m (only for an updraft velocity of 0.1 ms$^{-1}$). The glaciation time is faster than those for deeper clouds, but the effect is relatively small. Though there is a smaller amount of total liquid mass in the shallower cloud, the time required for a parcel to move through the cloud is shorter for a given updraft, reducing the growth time for the ice, and partially compensating for the lower liquid mass. Nevertheless, the glaciation time is shorter for the shallower cloud because the cloud contains less liquid mass. Overall, the effects are similar to those associated with the thicker cloud.

### 4.3.2. Comparisons With Observational Cases

Observations of mixed-phase cloud cases such as May 1–10, 1998 of SHEBA (Surface Heat Budget of the Arctic Ocean) [Lawson et al., 2001], October 9–11 of MPACE (Mixed-Phase Arctic Cloud Experiment) [McFarquhar et al., 2007], April 8 of ISDAC (Indirect and Semi-Direct Aerosol Campaign) [McFarquhar et al., 2011], Case 18 of BASE (The Beaufort and Arctic Storms Experiment) [Pinto, 1998], and CLEX-5 (Complex Layered Cloud Experiment) [Larson et al., 2006] show that cloud lifetimes range from a couple of hours to days [Pinto, 1998; Morrison et al., 2005; Prenni et al., 2007; Larson et al., 2006]. Curiously, many of these clouds maintain supercooled liquid for days despite the inherent instability associated with ice growth and precipitation. The ovals placed on Figure 13 represent the approximate glaciation times for ranges of temperatures and ice concentrations.
for particular cloud cases that were observed in the various field experiments. Many of the Arctic cloud cases contain relatively low average ice concentrations, and consequently should have relatively long glaciation time-scales (∼12 hours) even at habit-prone temperatures (ISDAC, BASE, SHEBA). As has been pointed out [e.g., Harrington et al., 1999; Fridlind et al., 2007; Morrison et al., 2011], other mechanisms must maintain the cloud supersaturations otherwise the cloud will eventually dissipate. Given the long glaciation times for clouds with lower ice concentrations, processes that maintain the saturation can act on relatively slow time-scales (perhaps days). Clouds with higher ice concentrations, such as MPACE, need external processes that act more rapidly (hours or less). In MPACE strong surface fluxes resupplied the vapor. In the altostratus case of CLEX-5, the clouds dissipated, and simulations of Larson et al. [2006] with an LES produced lifetimes of approximately 120 minutes which are similar to the lifetime estimates given in Figure 13.

5. Conclusions and Broader Context

[54] The lifetime of Arctic clouds may have a substantial impact on the existing sea ice extent, possibly affecting the Arctic climate as a whole [e.g., Francis and Hunter, 2006; Kay et al., 2008; Kay and Gettelman, 2009; Stramler et al., 2011]. Supercooled liquid water critically impacts the surface radiative budget, reducing surface radiative energy by 20–50 W m⁻² [Stramler et al., 2011]. The prevalence of mixed-phase clouds in the pan-Arctic region suggests that the lifetime of these clouds is highly dependent on the liquid-ice partitioning, glaciation occurring when the liquid mass is depleted via the Bergeron process. Previous studies of mixed-phase stratus lifetime have shown that a strong sensitivity to temperature and ice concentrations exists within these clouds [e.g., Korolev and Isaac, 2003]. However, these studies have neglected the effect of non-spherical ice particle growth, and that the evolution of particle aspect ratios can greatly modify the liquid and vapor supply.

[55] In agreement with prior studies, the results presented here show that the Bergeron process is greatest when temperatures are ≲ −5°C (ice and liquid saturation differences are greater) and when ice concentrations are higher (>10 L⁻¹). Non-spherical ice growth modifies this picture with plates and columns primarily growing within specified temperature regimes and isometric ice particles dominating the transitional temperatures. The non-linear feedback between particle mass and the evolution of particle aspect ratio proves to enhance ice particle growth especially at habit-prone temperatures (−15°C and −6°C) and when ice concentrations are lower (<100 L⁻¹), further depleting the local vapor supply. Further studies suggest that there also exist other factors complementing the nonlinearity of ice particle habit growth: (1) initial ice crystal size when the particle is <5 μm, (2) ventilation, enhancing the phase partitioning to second-order, and (3) dynamic motions, allowing for increased supercooled liquid lifetime, and possibly promoting regions of parameter space in which the cloud ceases to glaciate. Through the results presented, it is clear that the detailed growth of ice crystals exists within the complex web of microphysical and dynamical interactions of mixed-phase clouds affecting phase partitioning, cloud glaciation time, and cloud lifetime.

Figure 13. Glaciation time with respect to temperature for different ice concentrations and multiple parcel oscillations. Shaded regions represent maximum updraft velocities ranging from 0.1–0.9 ms⁻¹, with a cloud depth of 319 m. Black solid lines are simulations for a cloud depth of 150 m and a maximum updraft velocity of 0.1 ms⁻¹. Spherical results (small dotted lines) use an updraft speed of 0.1 ms⁻¹. Regions where the shades and lines end abruptly at the highest temperatures indicate that glaciation does not occur within the allotted simulation time. Observational cases (shaded ovals) are placed within the approximate temperature and ice concentration region the experiments took place. Differing shading colors are for clarity.
Despite the microphysical instability of the liquid-ice coexistence within mixed-phase clouds, it is puzzling to find that often these clouds are observed to persist for long periods of time. It has been known for some time that the persistence of mixed-phase clouds is likely due to low ice concentrations, as suggested by Pinto [1998] for BASE. Moreover, a number of modeling studies have suggested that cloud dynamic-microphysical interactions are critical for maintaining mixed-phase clouds [e.g., Harrington et al., 1999; Korolev and Field, 2008; Morrison et al., 2011; Solomon et al., 2011].

When ice crystal aspect ratios evolve in time, ice growth can be increased by a large amount, indicating that evolving habits may have a substantial impact on the evolution of mixed-phase stratus. As an example, consider that during M-PACE (Oct 9-11 case) many of the ice crystals were small spheres, but irregular shaped crystals were also observed along with columns and branched crystals [McFarquhar et al., 2007]. The irregular crystals tend to grow in a more spherical fashion (given their quasi-spherical shapes) and so the M-PACE clouds likely had particles with greater fall speeds and longer glaciation time-scales, allowing for liquid persistence despite relatively large ice concentrations (1 L\(^{-1}\) to 10 L\(^{-1}\)). The May 7 case from SHEBA, which has been the focus of many modeling studies [e.g., Morrison et al., 2005; Fridlind et al., 2007; Morrison et al., 2011], also had many hexagonal plates and plate assemblages which grow relatively slowly. The lower temperatures and weaker growth lead to longer glaciation times (c.f. Figure 13). While particle habits and ice concentrations are not well known for BASE, Pinto [1998] reports relatively low concentrations which would produce glaciation times that are relatively long. During ISDAC [McFarquhar et al., 2011], the ice concentrations were lower on average (0.1 L\(^{-1}\) to 4 L\(^{-1}\)), with primarily dendritic crystals (G. McFarquhar, personal communication, 2011). Consequently, it is possible that the cloud persisted because of the lower ice concentration due to the strong dendritic growth. Speculation suggests that had the concentrations been higher (closer to the MPACE values), these clouds may have dissipated. The type of ice particle habit that evolves within a mixed-phase layer may have a relatively strong control of the longevity of the layer. The impacts could be substantial given that many works indicate that predicting the amount of supercooled liquid over the Arctic basin is critical for capturing the correct longwave radiative forcing at the surface [Prenni et al., 2007; Kay and Gettelman, 2009; Stramler et al., 2011].

Given that mixed-phase clouds maintain a precarious balance between processes that deplete supercooled liquid (ice crystal growth and precipitation loss) and processes that effectively resupply moisture (radiative cooling, turbulent fluxes from above and below, and large scale forcing), it appears that concentrating effort on understanding ice habit effects on cloud evolution is of paramount importance. While the model presented cannot capture every detail of ice crystal evolution, it is a first-order improvement over earlier models in that particle shapes can evolve in time and as it is based on physical processes observed and refined in the laboratory. In the future, the hope is to test and refine this habit modeling technique, including the development of improved models that can be used within a cloud modeling framework.

Appendix A: Derivation of the Mass Growth Equation for a Spheroid

As discussed in section 2.1, the mass growth rate for a non-spherical particle (equation (3)) can be derived by first differentiating the mass of a spheroid \( m = \frac{4}{3} \pi a^3 c \rho_i \):

\[
\frac{dm}{dt} = \frac{4}{3} \pi \rho_i \left( 2ac \frac{da}{dt} + a^2 \frac{dc}{dt} \right).
\]  

(A1)

Then using equation (2) it is found that,

\[
\frac{dm}{dt} = \frac{4}{3} \pi \rho_i \left( 2ac \frac{D_i \Delta \rho_i f_{ob}(\phi)}{c} + a^2 \frac{D_i \Delta \rho_i f_{ob}(\phi)}{a} \right) = \frac{4}{3} \pi D_i \Delta \rho_i f_{ob}(\phi)(2a + a)
\]  

(A2)

and by using equation (1),

\[
\frac{dm}{dt} = 4\pi a f_{ob}(\phi) D_i \Delta \rho_i = 4\pi C(c, a)(\phi) D_i \Delta \rho_i.
\]  

(A3)

This derivation is analogous for a prolate spheroid. Following the procedure given by Prenni et al. [2007], one arrives at the classical diffusion equation for a non-spherical particle,

\[
\frac{dm}{dt} = 4\pi C(c, a) G_i (S_i - 1)
\]  

(A4)

where \( G_i = \left( \frac{K_T c_T L_i T_a}{K_T L_i T_a - 1} \right) \), \( L_i \) is the latent heat of sublimation, \( K_T \) is the thermal diffusivity in air, \( D_i \) is the water vapor diffusivity in air, \( T_a \) is the ambient temperature, \( R_n \) is the water vapor gas constant, and \( e_i \) is the equilibrium vapor pressure over ice.

Appendix B: Parcel Model Equations

As noted in section 4, a parcel model is used to study evolving ice particles in mixed-phase clouds. The model was configured for 200 bins of both liquid drops (\( nb_l = 200 \)) and ice particles (\( nb_i = 200 \)). VODE was then used to solve the set of equations [Brown et al., 1989] provided below.

For ice growth, the equivalent volume radius, \( r_{ei} \), is solved for over a time-step (1 second),

\[
\frac{dr_{ei}}{dt} = \frac{G_i (S_i - 1) q_i}{r_{ei} \rho_{dep}}
\]  

(B1)

where \( j \) is the bin number and \( q_i = \frac{C_{i(c,a)}}{r_{ei}} \) is the particle shape factor. When the environment becomes ice saturated \( \rho_{dep} \) [Chen and Lamb, 1994, equation (42)] is to be substituted...
with $\rho_{\text{sub}}$ (equation (6)). Note that $r_{ij}$ is related to the spheroidal volume by
\[
\frac{4}{3} \pi r_{ij}^3 = \frac{4}{3} \pi a_j^2 c_j. \tag{B2}
\]

The model solves for $r_{ij}$ assuming $q_j$ is constant over a time-step. This procedure avoids solving separate differential equations for $q_j$ and $c_j$, which are diagnosed from $r_{ij}$ in the following manner: equation (5) is integrated over a time-step ($\Delta t$) to find the change in aspect ratio as a function of the change in volume,
\[
\phi(t + \Delta t) = \phi(t) \left( \frac{\Gamma(t + \Delta t)}{\Gamma(t)} \right)^{\gamma}, \tag{B3}
\]
where $\gamma = \frac{n}{n-1}$. The new value of $a_j$ is determined by first finding the new volume by solving equation (B1), and then inverting the spheroidal volume equation,
\[
a_j = \left[ \frac{3}{4\pi} \frac{V(t + \Delta t)}{\phi(t + \Delta t)} \right]^{1/3}, \tag{B4}
\]
and $c_j$ is then simply determined from the aspect ratio and the new value of $a_j$,
\[
c_j = \phi(t + \Delta t) a_j. \tag{B5}
\]

This method of evolving ice aspect ratio is accurate when $\Delta t < 10$ seconds and is numerically stable, unlike solving for $a_j$ and $c_j$ simultaneously as was done by Sheridan et al. [2009].

[62] For liquid growth, the radii evolve following the standard diffusion equation for a solution drop [Pruppacher and Klett, 1997]:
\[
\frac{dr_i}{dt} = G_i \left( S_i - 1 - \left( \frac{a_i}{r_i} - \frac{b_i}{r_i} \right) \right), \tag{B6}
\]
where the functions for $G_i$, $A$, $B$, and the liquid saturation ratio, $S_i$, are as follows:
\[
G_i = \frac{L_v}{K_i T_v} \left( \frac{L_v}{R_v T_v} - 1 \right) + \frac{R_v T_v}{D_i c_i}, \tag{B7}
\]
\[
A = \frac{2 M_a \sigma}{R^* T \rho_l}, \tag{B8}
\]
\[
B = \frac{3 H m_{\text{sub}} M_a}{4 \pi M_a \rho_l}, \tag{B9}
\]
and
\[
S_i = \frac{e_v}{e_f}. \tag{B10}
\]

where $e_v$ and $e_f$ are the ambient and liquid equilibrium vapor pressures, respectively, $L_v$ is the latent heat of vaporization, $M_a$ is the molar mass of water, $\sigma$ is the surface tension, $R^*$ is the universal gas constant, $\rho_l$ is the density of liquid water, $H$ is the Van’t Hoff Factor, $m_{\text{sub}}$ is the mass of solute, and $M_c$ is the molar mass of solute.

[63] The prescribed updraft velocity is given by
\[
\frac{dz}{dt} = w \tag{B11}
\]
where $z$ is the height and $w$ is the vertical motion. The respective pressure and temperature equations are
\[
\frac{dp}{dt} = -\rho_0 g \frac{dz}{dt} \tag{B12}
\]
and
\[
\frac{dT}{dt} = -\frac{g}{c_p} \frac{dz}{dt} + \frac{L_v}{c_p} \frac{d\omega_i}{dt} + \frac{L_v}{c_p} \frac{d\omega_l}{dt}, \tag{B13}
\]
where the ice and liquid mixing ratio derivatives are given by
\[
\frac{d\omega_i}{dt} = \frac{1}{\rho_i} \sum_{j=1}^{n_j} \left[ N_i 4 \pi r_j^2 \rho_{ep}\frac{r_j}{c_j} + m_i \frac{N_i}{\rho_i} \frac{d\rho_j}{dt} \right], \tag{B14}
\]
and
\[
\frac{d\omega_l}{dt} = \frac{1}{\rho_l} \sum_{j=1}^{n_j} \left[ N_i 4 \pi r_j^2 \rho_{ep}\frac{r_j}{c_j} + m_i \frac{N_i}{\rho_i} \frac{d\rho_j}{dt} \right], \tag{B15}
\]
respectively, where $N_i$ is the ice concentration, $m_{ij}$ is the ice crystal mass, $N_l$ is the liquid concentration, $m_{ij}$ is the liquid mass, and $\rho_l$ is the liquid density. The air density variation is given by,
\[
\frac{d\rho_a}{dt} = \frac{1}{R_d T} \frac{dP}{dt} - \frac{P}{R_d T^2} \frac{dT}{dt}, \tag{B16}
\]
where $R_d$ is the gas constant for dry air. The evolution of ambient saturation ratio ($S_i = \frac{e_v}{e_f}$) is,
\[
\frac{dS_i}{dt} = Q_1 w - Q_2 \frac{d\omega_i}{dt} - Q_3 \frac{d\omega_l}{dt}, \tag{B17}
\]
where each term in the above equation is
\[
Q_1 w = S_{ig} \left( \frac{L_v}{R_v T c_p} - \frac{1}{R_d T} \right)^w \tag{B18}
\]
is the production of supersaturation resulting from vertical motion where $c_p$ is the specific heat capacity at a constant pressure, and
\[
Q_2 \frac{d\omega_i}{dt} = S_i \left[ \frac{L_v^2}{R^* T c_p} + \left( \frac{d\omega_i}{dt} \right)^{-1} \right] \frac{d\omega_i}{dt} \tag{B19}
\]
and
\[
Q_3 \frac{d\omega_l}{dt} = S_i \left[ \frac{L_v L_c}{R^* T c_p} + \left( \frac{d\omega_l}{dt} \right)^{-1} \right] \frac{d\omega_l}{dt} \tag{B20}
\]
are the consumption of supersaturation by the growth of liquid drops and ice particles, respectively. Finally, to ensure mass conservation,

\[
0 = \frac{dw_i}{dt} + \frac{dw_j}{dt} + \frac{dw_k}{dt}.
\]  

(B21)


Moon, P., and D. Spencer (1961), Field Theory for Engineers, 530 pp., Van Norstrand, Princeton, N. J.


