Persistent Contrails and Contrail Cirrus. Part II: Full Lifetime Behavior

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ABSTRACT

More than 200 large-eddy simulations of long-lived contrails from several-seconds age until their demise have been performed and their lifetime-integrated behavior has been analyzed. The simulations employ size-resolved microphysics and include variations of effective ice crystal number emission index, temperature, relative humidity with respect to ice, stratification, shear, supersaturated-layer depth, uplift/subsidence, and coupled radiation. Basic scaling behaviors are analyzed for contrail lifetime, width, ice mass, and surface area. Lifetimes exceeding 40 h, widths exceeding 100 km, and ice masses exceeding 50 kg m$^{-1}$ of flight path were sometimes encountered. Distinct behavior regimes produced by radiative forcing are identified and found to be predicted by a simplified model. The lifetime-integrated ice crystal surface area per length of flight path $S_S$ is used as an approximate metric of contrail significance, and a simple, physically based model is derived. Over much of the parameter space, $S_S$ is found to vary approximately simply as the product of the maximum contrail depth and the effective number of ice crystals per flight path; other parameters have their impact on $S_S$ dominantly through their effects on these two quantities. Model and simulation results highlight the importance of crystal number loss mechanisms, the interaction between shear and ice sedimentation, the depth of the supersaturated layer below flight level, and the potential integrated significance of “cold” subvisible contrails. The results can aid in estimating the effects of more complex contrail scenarios or mitigation strategies and in understanding some aspects of natural-cirrus dynamics.

1. Introduction

Aircraft contrails represent a unique form of cloud initiation. Starting out highly localized both spatially and temporally, they may sometimes spread over hours to form “natural looking” contrail cirrus where natural cirrus might not arise—for example, in quiescent conditions with negligible vertical velocities. Given projections for increased air traffic in the coming decades it is a concern whether contrails\(^1\) might significantly affect climate. Models designed for assessing effects of total air traffic over long periods such as the Contrail Cirrus Prediction Tool (CoCIP; Schumann 2012) must necessarily treat individual contrails in highly simplified fashion. A more detailed understanding of contrail dynamics would help in the refinement of such models and also illuminate some of the dynamics of natural cirrus in a simpler (at least in some respects) setting.

A companion paper (Lewellen et al. 2014, hereafter Part I) described an efficient large-eddy-simulation (LES) model for treating contrails from shortly behind the aircraft to their demise hours later, including size-resolved ice microphysics and coupled radiation. Results from a sample set of simulations were described, highlighting some of the dynamics involved and the requirements for simulating them. Even given reliable simulations, however, contrail behavior depends on many variables in complex fashion. Considering only individual parameter variations away from a chosen base state can be misleading. In this work a large parameter space, including significant wind shear, is treated with over 200 simulations. The emphasis is on aging contrail dynamics, determining factors for the lifetime, and lifetime-integrated effects. Analyzing a multidimensional parameter space of simulations presents a challenge. To help in the organizing and understanding of the results we introduce an approximate lifetime-integrated metric and develop a simple

\(^1\) For simplicity, we will use the term “contrail” to denote both linear contrails and contrail-induced cirrus over its full lifetime.

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physically based model for it. Given its simplicity, the model succeeds surprisingly well in predicting the simulation results over much of the parameter space.

The potential impact of a simulated contrail on Earth’s radiation balance may be computed a posteriori from the ice evolution, but if radiative emission/absorption by the contrail strongly affects its own dynamics, this must be coupled into the simulation. The coupled radiation effects are considered here in some detail, as they can vary significantly in different regimes in both sign and magnitude. For example, Chlond (1998) finds a small longwave cooling of the contrail driving negligible dynamics while Jensen et al. (1998) find strong radiative warming driving significant updrafts. Other variations considered include effective ice crystal number emission index, ambient temperature, relative humidity, stratification, wind shear, supersaturated-layer depth, and uplift/subsidence. There are many simplifications in the simulation set, however: ice nucleation within the exhaust jets is not simulated, only a single aircraft is treated, and the interaction between contrails within the exhaust jets is not simulated, only a single contrail results to illustrate the broad range of behaviors encountered and for later reference when different dynamical elements are considered. The complexity seen motivates the need for simpler models to help organize the dynamics; these are developed in section 3 with an emphasis on contrail-integrated summary metrics. The simulation results are then compared against these simpler models at different levels of approximation, successively introducing some of the complexities involved. Tables at the end of section 3 summarize the principal physical dependencies found, with the reduced set of intermediate variables used in formulating the simple models employed to help organize the results. Limitations of the simple models and simulation set are considered in section 4. An analysis of the different regimes encountered when radiation couples with the contrail dynamics is given in section 5, again introducing a simplified model to help interpret the simulation results. Section 6 includes a summary and some concluding remarks. Further details specifying the simulation set and a table with some summary results for sample coupled radiation runs are provided in the appendix.

2. Full-lifetime simulations

a. Simulation cases

A large simulation set was performed to study the dynamics of aging contrails/contrail cirrus until demise. Given the many dynamical mechanisms involved, a large parameter space is required. Table 1 summarizes the principal variations considered, with additional details of viewing of a representative sampling of full-lifetime contrail results to illustrate the broad range of behaviors encountered and for later reference when different dynamical elements are considered. The complexity seen motivates the need for simpler models to help organize the dynamics; these are developed in section 3 with an emphasis on contrail-integrated summary metrics. The simulation results are then compared against these simpler models at different levels of approximation, successively introducing some of the complexities involved. Tables at the end of section 3 summarize the principal physical dependencies found, with the reduced set of intermediate variables used in formulating the simple models employed to help organize the results. Limitations of the simple models and simulation set are considered in section 4. An analysis of the different regimes encountered when radiation couples with the contrail dynamics is given in section 5, again introducing a simplified model to help interpret the simulation results. Section 6 includes a summary and some concluding remarks. Further details specifying the simulation set and a table with some summary results for sample coupled radiation runs are provided in the appendix.

<table>
<thead>
<tr>
<th>$E_{i, \text{iceno}}$</th>
<th>$10^{15}$</th>
<th>$10^{14}$</th>
<th>$10^{13}$</th>
<th>$10^{12}$ per kilogram of fuel</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>205</td>
<td>218</td>
<td>225</td>
<td>229 K</td>
</tr>
<tr>
<td>$R_{H_i}$</td>
<td>110%</td>
<td>130%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma$</td>
<td>2.5</td>
<td>10 K km$^{-1}$</td>
<td></td>
<td></td>
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<tr>
<td>$dz/d\sigma$</td>
<td>2</td>
<td>4</td>
<td>8 m s$^{-1}$ km$^{-1}$</td>
<td></td>
</tr>
<tr>
<td>$z_{dn}$</td>
<td>500</td>
<td>1000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$z_{up}$</td>
<td>0</td>
<td>500</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$w_{up}$</td>
<td>0</td>
<td>±1, ±2, ±4 cm s$^{-1}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Radiation</td>
<td>None, longwave, longwave + solar</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{sfc}$</td>
<td>258</td>
<td>278</td>
<td>298 K</td>
<td></td>
</tr>
</tbody>
</table>
the simulation specifications given in the appendix. An attempt was made to at least roughly sample the physical parameter space most relevant for long-lived contrails, though somewhat idealized case specifications were chosen to clarify the analysis. The choice of “input coordinates” for this parameter space as defined in Table 1 are of course neither unique nor exhaustive, but all those selected have a significant effect on contrail dynamics over their lifetimes in at least some regimes. The potential radiative impact of a contrail is clearly influenced by the total number of ice crystals (for which EI\text{iceno} provides an input lever), the local level of ambient moisture which is available for crystal growth [dependent on temperature $T$ and relative humidity with respect to ice (RH$_i$)], and the potential growth of the contrail volume in the vertical (affected in potentially different ways by the depth of the supersaturated layer below, $z_{dn}$, and above, $z_{up}$, flight level) and in the horizontal (affected by $du/dz$). Further, the $T$ and RH$_i$ experienced by the contrail may be altered over time, either in a steady fashion by an imposed uplift or subsidence $w_{up}$ or in a more complex fashion coupled to the properties of the contrail itself via radiative effects (varied here through the choice of radiation scenario or $T_{sfc}$). Several of these factors (e.g., crystal number, contrail spread, and response to radiation) are affected by $γ$. The presence of either significant wind shear or coupled radiation in these simulations can themselves produce significant levels of turbulent diffusion in and around the contrail, so we have not chosen to vary the ambient-turbulence level in these simulations. The values of $T$, RH$_i$, $du/dz$, $γ$, and supersaturation layer depths chosen are all within the ranges in which persistent contrails have been observed (e.g., Iwabuchi et al. 2012; Duda et al. 2004). In the simulations this often leads (cf. Figs. 8 and 16) to contrail lifetimes exceeding 10 h and widths of many tens of kilometers, consistent with some contrail observations (e.g., Minnis et al. 1998; Iwabuchi et al. 2012).

Most simulations were performed using multiple grids with only modest numbers of downstream grid points in the “Q3D” mode described in Part I. This mode allows a limited range of scales of fully 3D eddies to be resolved along with the accompanying turbulent mixing and energy transfer, giving a significant improvement over a pure 2D representation. As shown in Part I, Q3D and fully 3D LES results match well for contrail-averaged quantities. A few fully 3D simulations with small shear were also included here. Only a fraction of the total matrix of runs implied by the choices in Table 1 were simulated all the way to contrail demise, of order $\sim$200 in total. Further details are given in the appendix. The LES and simulation procedures are otherwise as described in Part I.

**b. Sample results**

Figures 1–5 sample the variety of behavior encountered in the simulations, varying $T$ and RH$_i$ in Fig. 1, EI\text{iceno} and $du/dz$ in Fig. 2, $w_{up}$ in Fig. 3, and radiation and $γ$ in Figs. 4 and 5. Different aspects will be discussed as they arise below. The metrics introduced in Part I are used [the evolving total ice crystal number $N(t)$, mass $M(t)$, and surface area $S(t)$, all per meter of flight path] along with the evolving mean crystal altitude relative to flight level $Z_N(t)$, which gives an idea of the vertical trajectory of the contrail core. Here $S(t)$ provides a metric for contrail radiative properties as a whole, independent of definitions of contrail width, orientation, etc. Given the direct relation between crystal surface area and both crystal mass and fall speed (for small crystals), $S(t)$ also
provides useful guidance for interpreting the observed ice mass and number evolution. For spherical ice crystals it represents twice the cross-stream-integrated vertical optical depth in the approximation taking the extinction coefficient $Q_{\text{ext}}$. It is approximately twice the total extinction $E$ considered in UG10a and UG10b.

Isolated parameter variations about selected conditions such as those in Figs. 1–5 do not give a complete picture and can sometimes be misleading. A wide range of behaviors are encountered resulting from a combination of dynamical processes. Some of these were considered in Part I, and others are taken up below. In section 3 a simplified model is introduced to help in organizing the results. Figures 6 and 7 provide sample cross sections of an evolving contrail to illustrate some of the assumptions behind the simplified model as well as effects of sizable shear. In caricature there are two distinct regions of the contrail: a core near flight level with larger number densities and a much more sparsely populated precipitation plume below with larger crystals. As discussed in Part I, the initial formation of the precipitation plume depends on a positive feedback: larger crystals fall more rapidly into the supersaturated air below the contrail core, leading to increased crystal growth and hence greater fall speed and access to supersaturated air. Only a small fraction of the total crystal population is involved and the plume growth is more universal, depending strongly on only $T$ and $\text{RH}_i$. The process is self-limiting, however: the falling population removes

\[ Q_{\text{ext}} \approx 2. \]

Any more precise determination of optical depth would necessarily be dependent on the wavelength of radiation, the shape of the contrail, and the angle of passage through it, as well as the crystal size spectrum.

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**Fig. 2.** Time evolution for sample cases varying $E_{\text{iceno}} = 10^{12}$, $10^{13}$, $10^{14}$, $10^{15}$ kg$^{-1}$ (black, red, green, and blue lines, respectively) and $du/dz = 2$ (dashed) and $8$ m s$^{-1}$ km$^{-1}$ (solid). In each case, $z_{dn} = 1000$ m, $z_{up} = 500$ m, $\text{RH}_i = 130\%$, $T = 218$ K, $\gamma = 2.5$ K km$^{-1}$, $w_{up} = 0$ cm s$^{-1}$, and there is no coupled radiation. Panels as in Fig. 1.

**Fig. 3.** Time evolution for sample cases varying uplift $w_{up} = 0, 1, 2, 4$ cm s$^{-1}$ (black, red, green, and blue solid lines, respectively) or subsidence $w_{up} = -1, -2, -4$ cm s$^{-1}$ (red, green, and blue dashed lines, respectively). In each case, $z_{dn} = z_{up} = 500$ m, $du/dz = 4$ m s$^{-1}$ km$^{-1}$, $\gamma = 2.5$ K km$^{-1}$, $\text{RH}_i = 110\%$, $T = 218$ K, $E_{\text{iceno}} = 10^{15}$ kg$^{-1}$, and there is coupled longwave radiation with $T_{\text{sl}} = 278$. Panels as in Fig. 1.
moisture from the region it falls through, inhibiting growth and sedimentation from depleting the remaining crystals in and above that region. These crystals must be mixed horizontally to grow and moved horizontally (e.g., by shear) to get access to supersaturated air below. The core crystal population is largely shielded from the supersaturated layer below by the sheared precipitation plume; its subsequent development is typically one of slower growth and gradual sedimentation. As seen in Figs. 1–3, contrail demise is usually correlated with this core population falling below $z_{dn}$. Note that the core crystal population develops spatially quite differently from a passive tracer (Fig. 7) because of the shear–precipitation interaction.

As Figs. 1–5 show, many parameters critically impact a contrail’s magnitude during its evolution; however, there are often compensating effects when considering the full evolution. For example, increasing $T$ (for fixed $RH_i$) tends to increase peak contrail magnitude by increasing ice water content but decrease contrail lifetime by speeding sedimentation (cf. Fig. 1). We have purposely simulated contrails out to demise so that we can consider lifetime-integrated quantities and the dynamics determining the contrail lifetime itself. The time integral of $S(t)$ over the full contrail lifetime $t_{life}$,

$$S_Σ = \int_0^{t_{life}} S(t) \, dt,$$

represents a metric for overall contrail significance, being essentially twice the lifetime and cross-stream-integrated optical depth (per length of flight path). This provides a useful figure of merit for gauging contrail radiative impact, though clearly no single parameter can capture all effects; for example, while both shortwave backscatter and longwave absorption by the contrail will approximately scale with optical depth, the balance between the two will obviously depend as well on time of day. Since $S(t)$ represents a surface area density that is integrated over the contrail cross section and averaged over its length, the dimensions of $S_Σ$ are $(\text{area}) \times (\text{time})(\text{length})$. Note that $S_Σ$ includes the (sometimes substantial) contribution from subvisible contrail cirrus that is sometimes neglected when defining contrail widths and optical depths.

Figures 8 and 9 plot $S_Σ$ versus $t_{life}$ for the full simulation set, giving an indication of the ranges in each encountered as well as of the composition of the simulation set. While some clear tendencies are evident in the figures [e.g., low $EI_{iceno}$ leading to low $S_Σ$ (cf. also Fig. 2), cold contrails tending to be longest lived (cf. Fig. 1) or subsidence reducing $t_{life}$ (cf. Fig. 3)], the parameter
dependencies are generally complex. We delve into these more below after first introducing a simplified model to aid in the process.

3. Organizing long-lived contrail-cirrus dynamics

a. A simplified model for $S_z$

We consider a simple caricature of contrail-cirrus evolution, not intending it to be universally valid but to provide a framework for organizing different parameter dependencies so that elements can be considered in terms of the deviations they produce away from this baseline. After the wake dynamics subsides, the contrail may be considered as a population of small ice crystals localized within a supersaturated layer. These crystals typically grow as a result of a variety of effects, increasing their sedimentation velocity until they eventually fall out of the supersaturated layer and sublimate away. For a single spherical crystal in the Stokes flow regime the evolving fall velocity $v_x$ and surface area $S_x$ may be related to the crystal mass $m_x$:

$$S_x(t) \approx (36\pi \rho_i^{-2})^{1/3} m_x^{2/3},$$

and

$$v_x(t) \approx \frac{2\rho g}{9\mu} \left(\frac{3m_x}{4\pi \rho_i}\right)^{2/3},$$

with $\rho_i$ the ice density (taken as fixed at 917 kg m$^{-3}$ in this work), $g$ the gravitational acceleration, and $\mu$ the air viscosity. Because the time dependence in both is solely through $m_x^{2/3}(t)$ the integrated surface area over the crystal lifetime $t_f$ may be exactly related (in this approximation) to the integrated velocity (i.e., the total fall distance):

$$\int_0^{t_f} S_x(t) \, dt = \alpha \int_0^{t_f} v_x(t) \, dt; \quad \alpha = 18\pi \frac{\mu}{g\rho_i}. \quad (4)$$

Summing this over all crystals, which may involve quite different $m_x(t)$ histories for each, then gives for our simplified model

$$S_z \approx \alpha N D,$$

where $N$ is the total crystal number per length of flight path and $D$ is the number-averaged fall distance for the crystals before they sublimate away. The coefficient $\alpha$ is approximately constant in the regime of interest, varying only with the temperature dependence of $\mu$ (amounting to a $\pm$ 5% variation from $T = 205$ to 225 K). The model is remarkably simple, depending essentially on only two variables, $N$ and $D$, in each case linearly. The dependence on crystal growth histories or vertical distributions has dropped out. The increase of integrated contrail impact with crystal number is larger than simple considerations of the optical depths of

Fig. 6. Sample cross sections of ice surface area (cm$^2$ m$^{-3}$) at simulation times 2, 4, 6, 8, 10, and 12 h. The vertical coordinate is scaled by a factor of 10 relative to the horizontal; maximum plume widths shown are of order 50 km. The case is that of Fig. 3 with $w_{up} = 0$.

Fig. 7. Cross sections of (top)–(bottom) passive exhaust tracer, ice crystal number (cm$^{-3} 10^{-1}$), and ice water content (mg m$^{-3}$) at 6-h time for the case of Fig. 6. The vertical coordinate is scaled by a factor of 10 relative to the horizontal; the tracer plume width is of order 60 km.
young contrails would lead one to believe ($\sim N$ versus $\sim N^{1.5}$) because of the longer lifetime that results.

b. Comparison with simulations

The simplest application of (5) is to take $N \approx N_{\text{init}}$, the initial number of ice crystals introduced as given by $\text{EI}_{\text{ICENO}}$ and the fuel flow rate, and $D \approx d_c$ the distance from the flight level to where the ambient atmospheric layer becomes subsaturated below. Figure 10 shows the results applied to the full simulation set. Given the four choices of $\text{EI}_{\text{ICENO}}$ and two choices of $z_{dn}$ in Table 1 there are effectively eight predicted values at this level of approximation.

This application of (5) is limited for two main reasons: some of the crystals do not survive long enough to reach the subsaturated layer, so $N \approx N_{\text{init}}$ is generally an overestimate, and $D$ can for some conditions grow to exceed $d_c$. A significant fraction of crystals are lost by means other than precipitation out of the supersaturated layer: subsaturation forced by adiabatic heating as a result of downward motion from wake dynamics (Sussmann and Gierens 1999; Lewellen and Lewellen 2001), ambient subsidence, or radiative cooling; in situ crystal loss from large crystals scavenging smaller because of Kelvin effects, and turbulent mixing of the contrail plume with a subsaturated air mass (e.g., driven by radiatively induced buoyant uplift into a dry layer aloft). At least in the simulation set considered, the in situ loss tends to dominate (Part I; Lewellen 2012) and is enhanced by high number densities, long times, lower $\text{RH}_{i}$, adiabatic heating from downward motions, or mixing with subsaturated air. A large fraction of the losses are during the wake-dominated regime over the first few minutes; the remainder occurs at slower, decreasing rates over longer times. For low enough $\text{EI}_{\text{ICENO}}$ the number densities are small enough that there are no such losses and generally $N \approx N_{\text{init}}$; increasing $\text{EI}_{\text{ICENO}}$ leads to larger fractions of crystals lost. In the wake-dominated regime increased $\text{RH}_{i}$, increased $\gamma$ (through decreased wake descent) and decreased $T$ (increasing the relative importance of water from fuel burning, effectively increasing local $\text{RH}_{i}$) all reduce the crystal losses (cf. Fig. 1). At later times increased $du/dz$ (reducing number densities in the contrail core), increased $w_{\text{up}}$, and increased parcel rise from radiative heating all

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig8.png}
\caption{Lifetime-integrated total crystal surface area $S_{\Sigma}$ vs lifetime for complete simulation set. Symbol shape $+$, $\circ$, $\times$, and * indicate $T = 205, 218, 225, 229$ K, respectively; symbol size indicates $\text{RH}_{i} = 110\%$ (small) and $130\%$ (large). For this and subsequent figures $t_{\text{life}}$ is operationally defined as the age at which $M(t)$ drops below 1\% of the peak value encountered in the evolution.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig9.png}
\caption{As in Fig. 8, but with symbol shapes $\ast$, $+ \times$, and $\circ$ indicating $\text{EI}_{\text{ICENO}} = 10^{12}, 10^{13}, 10^{14}$, and $10^{15}$ kg$^{-1}$, respectively, and small symbols indicating $z_{dn} = 500$ m, while large symbols indicate $z_{dn} = 1000$ m.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig10.png}
\caption{Contrail lifetime-integrated total crystal surface area $S_{\Sigma}$ vs prediction from (5) with $N_{\text{tot}} = N_{\text{init}}$ and $D = d_c$. Symbols $\ast = \text{no radiation}$ and $\times = \text{coupled radiation}$; small size = no uplift/subsidence, large = uplift or subsidence.}
\end{figure}
reduce the losses. Meanwhile \( D \) may be increased from \( d_i \) primarily (for the simulation set) through radiation produced buoyancy moving the contrail plume upward through the supersaturated layer well above flight level (cf. Fig. 4a) or general uplift significantly raising RH levels leading to crystal precipitation surviving well below the original subsaturation level (cf. Fig. 3). The estimate \( D = d_i \) also neglects the offset between the flight level and the average crystal height after the wake-dominated regime (a 10%–20% reduction across the simulation set), which depends on the wake dynamics and would assume more relative importance for shallower moist layers.

For both issues we consider alternate metrics from the simulations. In the simulation set, crystals are found to sublimate away completely primarily in two distinct regions: in a peak around flight altitude or in a peak \( z_{dn} \) or more below flight altitude. The total number in the latter class, which we denote as \( N_{\text{prec}} \), provides a useful estimate for the effective \( N_\ast \) because the crystals lost before precipitating out of the supersaturated layer are generally lost relatively early in the evolution without having ever grown very large and so contribute relatively little to \( S^\Sigma \). Figure 11 confirms the expectation that the fraction \( N_{\text{prec}}/N_{\text{init}} \) drops when \( EI_{\text{iceno}} \) becomes large enough and that much of that loss occurs in the wake-vortex regime. As a simple alternative choice for \( D \) we consider the maximum total depth achieved during the contrail’s evolution, \( D_{\text{max}} \). Figure 12 shows that \( D_{\text{max}} \) tends to differ appreciably from \( d_i \) only for larger \( EI_{\text{iceno}} \) in the presence of coupled radiation or uplift, as expected. From Figs. 11 and 12 it is not surprising that the agreement between simple model and prediction in Fig. 10 is best for simulations with lower \( EI_{\text{iceno}} \) and neither coupled radiation nor uplift.

Given the complexity involved we do not try to quantitatively parameterize either \( N_{\text{prec}} \) or \( D_{\text{max}} \) here, but simply diagnose them from the simulations. Figure 13 compares the prediction [(5)] versus simulation results with the approximations \( N \approx N_{\text{prec}} \) and \( D \approx D_{\text{max}} \). This shows a remarkably good correlation for such a simple model, albeit involving quantities \( N_{\text{prec}} \) and \( D_{\text{max}} \) determined from the simulations themselves. The four “outliers” in the figure are the cases with an imposed subsidence leading crystals to be lost when the full
layer is forced to subsaturation, making \( N_{\text{prec}} \) both poorly determined and a poor estimate for \( N \).

c. Simulation and scaling results for other quantities

It is useful to consider scaling relations for other summary quantities, particularly \( t_{\text{life}} \) and maximum values of \( S(t) \), \( M(t) \), and contrail depth encountered during the evolution, denoted respectively \( S_{\text{max}}, M_{\text{max}} \), and \( x_{\text{wmax}} \). The simple approximation found for \( S \) rests on the crystal mass evolution canceling out via (4). The approximations for these other quantities will necessarily be rougher given the many (sometimes competing) dynamical effects involved and the lack of an analogous cancellation. We retain \( N \) and \( D \) as basic crystal number and contrail depth scales and define \( \rho_c \equiv \rho q_s(RH_i - 1) \), representing the available water density above ice supersaturation in the ambient, with \( q_s \) the saturation mixing ratio. We then expect at least rough scalings:

\[
M_{\text{max}} \sim c_1 \rho_c D x_{\text{wmax}}, \quad (6)
\]

\[
S_{\text{max}} \sim c_2 \beta_b (N M_{\text{max}}^2)^{1/3}; \quad \beta_b = (36 \pi \rho_i^2)^{1/3}, \quad (7)
\]

\[
S_{\Sigma} \sim c_3 S_{\text{max} t_{\text{life}}}, \quad \text{and} \quad (8)
\]

\[
x_{\text{wmax}} \sim c_4 du/dz D_{t_{\text{life}}}. \quad (9)
\]

Equation (6) assumes contrail ice mass per flight path to scale with \( \rho_c \) times some characteristic cross-sectional plume area. The form of (7) is just the geometrical expectation for a monodispersed ice crystal spectrum. The \( c \) values are expected to be positive constants less than 1. Figures 14–16 show how these scalings fare on the simulation set evaluated using the initial ambient value at flight level for \( \rho_c, N \approx N_{\text{prec}} \), and \( D \approx D_{\text{max}} \). The outliers in Fig. 14 are those cases already mentioned from Fig. 13 where \( N_{\text{prec}} \) is poorly determined. The coefficient in (7) is largely dependent on the shape of the ice-size distribution. It is approximately correlated with EIiceno, as is barely evident in the figure (on closer inspection \( c_2 \approx 0.7 \) for EIiceno = \( 10^{14} \sim 10^{15} \) kg m\(^{-1}\), 0.8 for \( 10^{13} \) kg m\(^{-1}\), and 0.9 for \( 10^{12} \) kg m\(^{-1}\)); EIiceno strongly affects the fraction of crystals in the contrail core relative to those in the precipitation plume so it has a major influence on the contrail-integrated size-spectrum shape. In Fig. 16 (and Fig. 17) the 3D cases without an imposed mean shear have been excluded given the absence of a well-defined \( du/dz \).

Compared with the simulation results, (6) is by far the roughest of the scalings (6)–(9). It ignores several complications including variation of \( \rho_c \) within the contrail (or in time) and variation in the time scale for the growth of the plume depth relative to that for establishing the plume width. By definition, \( \rho_c \) depends directly on RH\(_i\) and, through \( q_s \), on \( T \). It changes with altitude (even for fixed RH\(_i\)) as a result of temperature stratification. It can also change in time as a result of temperature changes—for example, from radiative heating/cooling or adiabatic cooling from large-scale or buoyantly forced uplift. We have deferred a more sophisticated treatment here in large part because \( \rho_c \), conveniently, does not affect \( S \) to lowest order.

Employing (5) in addition to (6)–(9) the latter can be solved in the form

\[
M_{\text{max}} \sim \left( \frac{\alpha \rho_c}{\beta_b} \frac{du}{dz} \right)^{3/5} N^{2.5} D^{0.5}, \quad (10)
\]
\[ S_{\text{max}} \sim \left( \frac{\alpha}{\beta_b} \right)^{2/5} \left( \rho c \frac{\text{d}u}{\text{d}z} \right)^{2/5} \left( \beta_b N \right)^{3/5} D^{6/5}, \]  
\[ t_{\text{life}} \sim \left( \frac{\alpha}{\beta_b} \right)^{2/5} \left( \rho c \frac{\text{d}u}{\text{d}z} \right)^{-2/5} N^{2/5} D^{-1/5}, \]  
\[ x_{\text{wmax}} \sim \left( \frac{\alpha}{\beta_b} \right)^{3/5} \left( \rho c \frac{\text{d}u}{\text{d}z} \right)^{-2/5} N^{2/5} D^{4/5}, \]  

where overall coefficients depending on the constants \( c \) have been dropped. Figure 17 compares these scalings with the simulation results. As expected, the correlations are much rougher than that in Fig. 13. The competing dynamics found across the simulation set cannot be completely absorbed into just the two parameters \( N \) and \( D \). For example, increasing \( D \) via uplift tends to reduce \( t_{\text{life}} \) by speeding sedimentation, while increasing \( D \) by lowering the base of the supersaturated layer must modestly increase \( t_{\text{life}} \). Nonetheless, (10)–(13) provide useful general guidance on which variations strongly affect these quantities and which only weakly. Table 2 summarizes the dependencies of these metrics on the primary variables \( N, D, \rho_c, \) and \( \text{d}u/\text{d}z \).

d. Effects of different parameter variations

The variables in Table 1 were chosen because they were all found to strongly influence contrail evolution in at least some regions of parameter space (e.g., Figs. 1–5). To lowest order, however, the effects of most can be viewed as arising through their effects on the smaller set of parameters \( \text{d}u/\text{d}z, \rho_c, N, \) and \( D \), with only the latter two strongly impacting the integrated significance of persistent contrails, as estimated by \( S_{\Sigma} \).

FIG. 16. Comparison of peaks encountered during lifetime of (a) contrail ice mass \( M_{\text{max}} \) and (b) contrail width \( x_{\text{wmax}} \) vs their rough scaling forms from (6) and (9). The fitted lines correspond to \( c_1 = 0.08 \) and \( c_2 = 0.25 \), respectively. Symbols as in Fig. 10.

FIG. 17. Simulation values for (a) peak ice mass, (b) peak width, (c) peak surface area, and (d) lifetime plotted vs the rough scaling forms of (10)–(13) evaluated using \( N \approx N_{\text{prec}}, D \approx D_{\text{max}}, \) and the ambient initial value at flight level for \( \rho_c \). Symbols as in Fig. 10.
some of the inferences given in UG10a and UG10b are different regimes of parameter space considered here, reached their maxima. Because of this, along with the here were found previously in UG10a and UG10b, in-hand, several of the parameter dependencies discussed on radiative effects (see section 5 below). On the other

The principal influences of the parameters in Table 1 on $\rho_c$, $N$, and $D$ as seen in the simulations are summarized in Table 3, where $du/dz$ is considered as an independent input. The dependencies are frequently not monotonic, with the direction of the sensitivity varying with the location in parameter space because many competing physical mechanisms are involved. The most complicated dependencies arise through effects of coupled radiation, discussed in section 5.

The sensitivities of contrail lifetime-integrated and peak values to changes in different parameters have been assessed with a broad simulation set for the first time here. The only prior simulation study of aged contrails to cover a significant parameter space—that of UG10a and UG10b—did not simulate contrails out to demise or in many cases (including all those with significant shear) even out to the time where $S(t)$ or $M(t)$ reached their maxima. Because of this, along with the different regimes of parameter space considered here, some of the inferences given in UG10a and UG10b are not supported by the present study, at least in some parts of parameter space—for example, that RH$_i$ is the dominant parameter controlling total extinction [their $E(t)$, essentially $S(t)/2$ here], that shear has a lesser effect on $E(t)$, that the crystals in the fall streaks are radiatively unimportant; that the time at which $E(t)$ peaks is about half the contrail lifetime, that the dominant parameter determining that time is temperature, and some findings on radiative effects (see section 5 below). On the other hand, several of the parameter dependencies discussed here were found previously in UG10a and UG10b, including some of the effects of coupled radiation, that a reduction in EI$_{iceno}$ increases sedimentation and therefore decreases lifetime, and that $E(t)$ tends to increase with increasing RH$_i$, $T$, and EI$_{iceno}$. There are some quantitative differences (e.g., the present simulations do not support the UG10a and UG10b conclusion that mean crystal masses tend to be independent of humidity level because changes in total ice mass balance changes in crystal losses). A major source of these seems to be the different crystal-loss rates found by spectrally resolved versus bulk microphysics models, as already noted in Unterstrasser and Sölch (2010).

### Table 2. Dominant contrail parameter dependencies seen from the simulations and scalings (10)–(13):

<table>
<thead>
<tr>
<th>$M_{\text{max}}$</th>
<th>$S_{\text{max}}$</th>
<th>$x_{\text{wmax}}$</th>
<th>$t_{\text{life}}$</th>
<th>$S_E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>↑</td>
<td>↑</td>
<td>↑</td>
<td>↑</td>
</tr>
<tr>
<td>$D$</td>
<td>↑</td>
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<td>↑</td>
</tr>
<tr>
<td>$\rho_c$</td>
<td>↑</td>
<td>↑</td>
<td>↓</td>
<td>↓</td>
</tr>
<tr>
<td>$du/dz$</td>
<td>↑</td>
<td>↑</td>
<td>↑</td>
<td>↓</td>
</tr>
</tbody>
</table>

The simplicity of (5) follows because the assumption of spherical crystals and a Stokes fall velocity leads to the details of the crystal growth history dropping out of (4). For small-enough spheres there are Brownian motion (Cunningham) corrections to the Stokes result; for large-enough spheres there are high Reynolds number corrections. The Stokes fall velocity agrees with the result with both corrections included to within ±20% for crystal diameters between 1 and 130 $\mu$m (with a fall velocity range between 5 × 10$^{-6}$ and 0.5 m s$^{-1}$). The Stokes approximation is therefore reasonable here:

### Table 3. Dominant contrail parameter dependencies seen from the simulations: up or down arrows or neither indicate whether an increase in the variable in the first column tends to increase, decrease, or leave unchanged the respective contrail property listed in the other column headings. Superscripts indicate some of the dominant physical mechanisms involved: a—smaller crystals increase lifetime giving possible uplift mechanisms more time to act, b—via change in relative importance of engine water affecting in situ crystal loss, c—via effect on radiative heating/cooling and buoyant rise/fall, d—via decreasing in situ crystal loss in wake-vortex stage, e—larger crystals decrease lifetime giving possible uplift mechanisms less time to act, f—via potentially reducing mixing with subsaturated air above or below saturated layer, g—via reducing wake-vortex descent and so possibly in situ loss, h—via reduced mixing sometimes increasing late-time in situ loss, i—via larger crystals surviving longer in subsaturated layer, j—via increased mixing with saturated air reducing in situ loss, k—possible shear thinning, l—via adiabatic cooling, m—adiabatic cooling possibly reducing in situ loss, and n—radiation effects depend on regime as discussed in section 5.

| EI$_{iceno}$ | $T$ | RH$_i$ | $\xi_{dn}$ | $\xi_{up}$ | $\gamma$ | $du/dz$ | $w_{up}$ | rad$^a$ | $\rho_c$ | $N$ | $D$ | $S_E$ |
|--------------|-----|--------|----------|----------|--------|---------|--------|--------|------------------|-------|--------|--------|--------|
| ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ |
smaller crystals contribute negligibly to $S_Z$, and larger ones are not generally encountered except within quite thick supersaturated layers. This is supported by Fig. 13 since both corrections are included in the simulations.

Of more concern are nonspherical crystal shapes (which are not included in the simulation set here). There is some observational evidence that at least within the high-number-density cores of contrails out to 1-h age the crystals are predominantly small (few-microns’ radius) and nearly spherical (e.g., Schröder et al. 2000). As the crystals grow larger, however, large deviations from spherical are seen (e.g., Lawson et al. 1998). The cross-sectional area is then not simply one quarter the total surface area, but depends on orientation. The projected area averaged over all orientations is still one quarter the total surface area, however, at least for convex solids (Vouk 1948). Terminal velocity in the Stokes flow regime remains proportional to the surface area as in (3) but with a shape-dependent coefficient. If the shape were fixed as the crystal grew, (4) would still hold for nonspheres, but with a different coefficient $\alpha$. In actuality the shape will change with crystal mass during the evolution; nonetheless, (4) still proves useful because the shape sensitivity of $\alpha$ is modest. Consider as an example hexagonal columns. Defining $\alpha_{\text{hc}}$ to be the ratio of surface area to fall speed for columns of aspect ratio (length/width) $A_r$ and using the result of Westbrook (2008) for the fall speed we find

$$\alpha_{\text{hc}} = 1.16\pi(4\sqrt{3} + 3/A_r)(1 + 0.95A_r^{3/4}) \frac{\mu}{\rho_i}. \quad (14)$$

For $A_r = 1$ this gives a value $\approx 1.25 \alpha$. If we further assume that the aspect ratio grows as found in Heymsfield and Iaquinta (2000) for small columns, $A_r \sim m_x^{0.18}$, then a 1000-fold increase in $m_x$ produces only a 37% increase in $\alpha_{\text{hc}}$. Thus for the rough scaling purposes intended, (5) should remain a useful guide even for nonspherical crystals.

**b. Determination of $N$ and $D$**

The correlation in Fig. 13 is limited in its predictive value since it employs the simulation measured estimates $N_{\text{prec}}$ and $D_{\text{max}}$ for $N$ and $D$. Since many dynamical ingredients contribute to the determination of both quantities, general predictive parameterizations would necessarily be complex. For $N$, there are also uncertainties in some of the underlying physics (e.g., the accommodation coefficient as discussed in Part I), so the simulation results, while likely reliable in the qualitative trends seen for crystal loss, should be considered quantitatively less certain. Despite these limitations, there are at least simple reliable approximations for both quantities in useful physical limits \(N \approx N_{\text{init}}\) for small enough $E_{\text{IC}}$; \(D \approx d_i\) in the absence of significant vertical motions) and rough bounds available in others (e.g., the full depth of the supersaturated layer in the absence of persistent uplift).

**c. Effects of subsidence**

It is clear (e.g., from Fig. 3) that subsidence can potentially dramatically reduce a contrail’s integrated radiative impact if it persists over long enough times and is not offset by other factors such as radiative cooling. When the entire moist layer is driven to subsaturation over modest times, results such as (5) assuming a mechanism of crystals falling out of a supersaturated layer are no longer applicable. Such situations should be considered separately, but this is a regime where smaller $S_Z$ are to be expected. If subsidence does not drive the layer to subsaturation, either because of shorter duration (e.g., driven by passing gravity waves) or balanced by other factors, then (5) remains relevant.

**d. Neglect of ambient cirrus and nucleation**

Ambient cirrus has not been included in the simulations here, whether present when the contrail forms or formed later through ice nucleation. The cases considered with persistent $w_{\text{up}} > 0$, in particular, are highly idealized because at later times the increases in RH$_i$ it drives are sufficient that natural cirrus should nucleate. We have simulated cases of contrails interacting with natural cirrus but defer presentation of those results to elsewhere. Flying through thin cirrus with supersaturation still permitting persistent contrails, the integrated $S_Z$ from the contrail behaves essentially as has been considered, but there is an additional impact: the removal of natural-cirrus crystals by the wake dynamics over a modest volume. More complex are the effects of nucleation of ambient cirrus after contrail initialization, which can effectively change the depth and horizontal extent of the supersaturated layer and the effects of vertical motion or radiation on the contrail plume. When natural cirrus is included the contrail impact should be measured relative to what the natural cirrus would have contributed in its absence.

**e. Absence of horizontal inhomogeneities**

The simulations here used an initially horizontally uniform background atmosphere. The simple model for $S_Z$ predicts that the effects on $S_Z$ of many horizontal variations would drop out to lowest order, though large changes in $t_{\text{life}}, S_{\text{max}},$ or $x_{\text{wmax}}$ might occur. An exception is that significant vertical wind shear (downstream or cross stream) plus limited lateral extent of a supersaturated region will eventually lead to shear thinning of the
supersaturated layer over time [e.g., as in Jensen et al. (2011)]. This would lead to a reduction in $D$ over time and therefore a predicted reduction in integrated contrail significance.

5. Effects of coupled radiation

Appendix B (Table B1) provides sample simulation results of the effects of coupled radiation on contrail metrics. Along with Figs. 4, 5, and 18 they show that coupled radiation can lead to significant increases or decreases in contrail significance with different behavior in different regimes depending on $T$, $T_{sfc}$, and $\gamma$.

a. An approximate model

We can estimate the direction and magnitude of the coupled radiation effects as follows. For simplicity we first neglect the solar absorption by the contrail and consider only the infrared emission/absorption (which generally dominates) in a two-stream approximation with effective broadband emissivities (e.g., Stephens 1984). Assuming contrail temperature $T_c$, base and top heights $z_b$ and $z_t$, ice water content IWC, and upwelling flux at cloud base $F^\uparrow_b$, the upwelling broadband flux and emissivity in the cloud may be approximated as

$$F^\uparrow(z) \approx F^\uparrow_b - \epsilon(z_b, z)(F^\uparrow_b - \sigma T^4_c),$$

and analogously for the downwelling components. Here $\beta = 1.66$ is the diffusivity factor. For the effective mass absorption coefficient we follow Ebert and Curry (1992) in the central IR spectral range and take (for crystal effective radii $r_e$ between 13 and 130 $\mu$m) $\kappa^\infty = \gamma_{ec}/r_e$ with $\gamma_{ec} \approx 1.2$ m$^2$g$^{-1}$m$\mu$m. Then in the simple approximation of a thin uniform cloud (constant IWC, $\kappa$ and $\epsilon^\infty$ small compared to 1) we have to leading order in IWC

$$\frac{\partial T}{\partial t} = \frac{1}{\rho c_p} \left( \frac{\partial F^\downarrow - F^\uparrow}{\partial z} \right) \approx \frac{\beta \gamma_{ec}}{\rho c_p} \frac{\text{IWC}}{r_e}$$

$$\times \left[ F^\uparrow_b + F^\uparrow - 2\sigma T^4 + 4(z_t + z_b - 2z)\sigma T^4c \frac{\partial T}{\partial z} \right]$$

$$\approx \frac{\beta \gamma_{ec}}{\rho c_p} \frac{\text{IWC}}{r_e} (F^\uparrow_b + F^\uparrow - 2\sigma T^4c),$$

where, unless the cloud is very deep, the $\partial T/\partial z$ term can be ignored relative to the others as in (18). The equation can be recognized as simply an energy budget for a homogeneous, isothermal cloud layer of infinite horizontal extent and low optical depth. Because of the $r_e^{-1}$ dependence the radiative impact is predominantly in the contrail core initially around flight level rather than the precipitation plume that forms below, so we restrict attention to there. In the present cases IWC $\sim \rho c$ and $F^\uparrow_b \sim \sigma T^4c$.

The impact of this heating/cooling on the contrail over time depends strongly on the potential temperature lapse rate $\gamma$. Except for very stable conditions the direct temperature change from radiation is smaller than the indirect buoyant response of opposite sign. Radiative heating causes the contrail to buoyantly rise and thus cool as a result of adiabatic expansion, increasing ice growth; conversely, radiative cooling causes the contrail to sink and thus heat because of adiabatic compression, leading to ice loss. Dimensionally, an increment of radiative temperature change leads to a vertical rise of order $\Delta z_{rad} \sim \Delta T_{rad}/\gamma$, so that the total temperature change is of order$^3$

$$^3$$ An analogous expression applies in assessing the importance of latent heating on contrail dynamics. It is generally negligible except at the highest temperatures considered here or given barely stable lapse rates.
\[
\Delta T_{\text{total}} = \Delta T_{\text{rad}} + \Delta T_{\text{indirect}} + \Delta T_{\text{uplift}} \\
= \Delta T_{\text{rad}}(1 - \Gamma_d / \gamma) - w_{\text{up}} \Delta \Gamma_d',
\]

where \( \Gamma_d = g/c_p \approx 9.8 \, \text{K} \, \text{km}^{-1} \) is the dry adiabatic temperature lapse rate and the contribution from a uniform uplift over time \( \Delta t \) has been added. For this simple estimate we have ignored mixing of ambient air into the parcel.

From (18) and (19) coupled radiative effects may be small even for persistent contrails if IWC is small, \( \gamma \) approaches \( \Gamma_d \), or \( T_c \) is near the crossover point \( 2\sigma T_c^2 \approx F_{b,1} + F_{I,1} \). The latter will depend on the surface temperature, underlying clouds, water vapor profiles, etc.; a nominal \( F_{b,1} + F_{I,1} \approx 300 \, \text{W m}^{-2} \) gives a crossover temperature \( T_c \approx 227 \, \text{K} \), which is on the high end of the expected contrail range (i.e., contrails more commonly radiatively heat than cool). The observations of Atlas et al. (2006) may provide an example near the crossover temperature: the temperature at flight level is relatively high, \( T_c \approx 229 \, \text{K} \), and despite large ice content and only a slightly stable lapse rate there is no apparent buoyant rise of the contrail tops above the original flight level.

Equation (18) only includes the mean longwave radiative heating/cooling relative to the ambient atmosphere. Two secondary effects are worth noting. There is longwave emission/absorption from species other than ice (principally water vapor) that affects the layer as a whole. This leads to modest temperature changes in the entire layer,\(^4\) but without causing any buoyant forcing on the contrail. This affects the layer in essentially the same way as a small uplift; it could be balanced by a small imposed subsidence. Second, in addition to the contrail mean heating/cooling there are variations due to temperature and ice gradients across the contrail. These can drive additional turbulent mixing that, in a supersaturated ambient layer, can produce modest additional ice growth.

### b. Behavior regimes

Much of the behavior seen in the simulations can be understood at an approximate level from (18) and (19) and the physical feedbacks involved. The equations suggest, and the simulations confirm, at least five distinct behavior regimes, considered now in turn:

1) **Contrail cooling and modest stratification:** For sufficiently high \( T \) and low \( T_{\text{sfc}} \) (18) predicts contrail cooling relative to the ambient. The contrail core becomes negatively buoyant, is driven downward, and for \( \gamma < \Gamma_d \) adiabatically heats, causing loss of ice crystal number and mass (e.g., cases with \( T_{\text{sfc}} \approx 258 \, \text{K}, \gamma = 2.5 \, \text{K} \, \text{km}^{-1} \), and \( T = 225 \, \text{K} \) in Table B1 and Fig. 4). There is a negative feedback as the ice loss diminishes the cooling rate; nonetheless contrail lifetime is significantly reduced, particularly for shallow moist layers. For fixed \( \text{RH} \), these tend to be contrails with large \( M_{\text{max}} \), because \( \rho_c \) increases with \( T \). Despite this, \( \mathcal{N} \), and to a lesser extent \( \mathcal{D} \), are reduced by the radiative dynamics, significantly reducing \( S_E \).

2) **Contrail cooling and \( \gamma \approx \Gamma_d \):** If \( \gamma \) is increased in the previous case to \( \gamma \approx \Gamma_d \) the temperature change of a parcel from radiative cooling is now only just balanced by the adiabatic heating from the induced drop of that parcel, inducing no crystal losses. The contrail descends but persists for longer time (e.g., cases with \( T_{\text{sfc}} = 258 \, \text{K}, \gamma = 10 \, \text{K} \, \text{km}^{-1} \), and \( T = 225 \, \text{K} \) in Table B1 and Fig. 5); \( \mathcal{N} \) is not reduced by the radiative forcing and so neither is \( S_E \). Further increase in \( \gamma \) would lead to a true cooling of the contrail core in this case, tending to increase \( \rho_c, \mathcal{N}, \mathcal{D} \), and \( S_E \).

3) **Contrail heating and modest stratification:** For lower \( T \) and/or higher \( T_{\text{sfc}} \) (18) predicts contrail radiative heating relative to the ambient. This is the more common situation for the parameter space simulated here. The contrail core becomes positively buoyant, is driven upward, and adiabatically cools; \( \rho_c \) increases, ice mass increases, in situ crystal losses are reduced or eliminated, and \( \mathcal{D} \) increases from the buoyant rise (e.g., cases with \( T_{\text{sfc}} = 298 \, \text{K}, \gamma = 2.5 \, \text{K} \, \text{km}^{-1} \) in Table B1 and Figs. 4 and 18), all tending to increase \( S_E \). There is a strong positive feedback as increased \( \rho_c \) drives increased radiative heating (which may be partly offset by a decrease in \( F_{b,1} + F_{I,1} \) from the rise in altitude). The feedback quickly turns negative if the contrail core is driven above the supersaturated layer: radiatively driven turbulence then mixes subsaturated air into the contrail core, reducing \( \rho_c \) and the radiative heating, effectively shutting down the process (e.g., cases with \( T_{\text{sfc}} = 298 \, \text{K}, \gamma = 2.5 \, \text{K} \, \text{km}^{-1} \), \( z_{\text{up}} = 0 \, \text{m} \) in Table B1 and Fig. 19). The process can also shut down if the core is driven upward into a region of stronger stratification as discussed below.

4) **Contrail heating and \( \gamma \approx \Gamma_d \):** If the stratification is increased in the previous case to \( \gamma \approx \Gamma_d \) then the adiabatic cooling from the parcel rise only just offsets the direct radiative heating. There is no positive feedback, resulting in little buoyant rise against the stronger stratification. The effects of the coupled radiation are small and likely dominated by the small

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\(^4\) Generally cooling at rates of 0.1 K h\(^{-1}\) or less for the chosen simulation set; water vapor increases with \( \text{RH} \), or with \( T \) for fixed \( \text{RH} \), and thus also depends on \( \gamma \).
cooling rates in the moist layer itself rather than that from the ice cloud (cf. Figs. 5 and 18).

5) Little contrail heating relative to the ambient: If $T$ and $T_{sfc}$ are such that averaged over the contrail core a whole there is a near cancellation between radiative heating from below and radiative cooling to space above, then there is no buoyant rise or fall, little feedback, and the effects of the radiation are generally small even for large $\rho_c$. What remains is mainly the radiative cooling of the layer as a whole (not associated with the cloud) and the differential heating due to gradients in the cloud (e.g., cooling above and heating below), which will drive some turbulent mixing. Both tend to modestly enhance the cloud but shorten its life; $N^*$, $D$, and $S_x$ are left essentially unchanged.

In all five cases the radiative effects tend to be decreased by decreased RH$_t$ or decreased $E_{iceno}$. In the latter case this is partly from the increase in $r_c$ but mostly from the reduced $t_{life}$ limiting the integrated heating or cooling; the radiative effects prove negligible for the lowest $E_{iceno}$ considered. Low $T$ also reduces $\rho_c$ and hence radiative heating rates, but they also tend to increase $t_{life}$ allowing integrated radiative effects to still be significant in some cases (cf. Fig. 18).

The coupled radiation studies in UG10a and UG10b seem to have been confined to regime (3), but did identify the importance of $\gamma$ as well as of the upwelling radiation (varied with $T_{sfc}$ here). Some of their conclusions, however, are not completely supported across the full parameter space here; for example, we find that radiative heating often reduces the contrail lifetime.

c. Incoming solar radiation

Equation (18) includes only the effects on the contrail of longwave radiation. The effects of incoming solar radiation are expected to be less since those wavelengths are more scattered than absorbed. Since there is absorption but no emission the direct effect is always heating. This will either augment or oppose the longwave contribution, depending on the longwave regime. The basic behavior regimes encountered are still as discussed above for the purely longwave contribution, and the chief effect is to shift some of the regime boundaries. The diurnal variation tends to dilute the solar effects in the average over long enough times; for the simulations here we have chosen a midmorning fly-through time in an attempt to maximize the effects of solar absorption on the contrail. The simulation results generally support the conclusion that the effects of the solar radiation on the contrail dynamics are of secondary importance to those from the longwave (cf. Table B1, Fig. 4); the results in UG10a and UG10b seem consistent with this conclusion as well. The largest relative changes are seen where the longwave contribution is modestly cooling the contrail core; the shortwave can then govern whether the contrail is heating or cooling overall and thus buoyantly rising or falling. Note that the longwave heating of the contrail is often only in a relative sense (i.e., actually cooling the contrail but less so than the surrounding air) while the shortwave absorption is always heating.

6. Summary and concluding remarks

Using an efficient LES with size-resolved microphysics and coupled radiation described in a companion paper (Part I), a large (>200) set of simulations of persistent long-lived contrails have been performed from seconds behind the aircraft until their demise many hours later. Considering a single aircraft (B-767), the effective ice crystal number emission index and many atmospheric conditions were varied over physically interesting ranges. The most persistent, long-lived, and extensive contrails have been emphasized, and, for the first time, the determining factors for contrail lifetime have been assessed with a large simulation set. Contrail lifetimes exceeding 40 h, widths exceeding 100 km, and ice masses exceeding 50 kg per meter of flight path were all encountered in some simulations. Even with the breadth of the simulation set, several important variations have been omitted and will be taken up elsewhere including aircraft type, interaction with
natural cirrus and natural-cirrus nucleation, and effects of horizontal inhomogeneities.

The simulations were analyzed emphasizing leading scaling behaviors and lifetime-integrated significance. The lifetime-integrated ice crystal surface area, $S_2$, was introduced as a simple metric for overall contrail significance. A simple physically based model for $S_2$ was developed and used to organize and interpret the simulation results. It was found to be quantitatively accurate in many limits of the simulation parameter space and to provide a useful framework for organizing the behavior elsewhere. Scaling behaviors for contrail lifetime, width, and maximum total ice mass and surface area encountered were also considered. Several distinct behavior regimes produced by coupled radiative forcing were identified and a simplified model presented for understanding them. The interaction between shear and ice sedimentation was also found to be a key ingredient in the overall dynamics.

The utility of the model given by (5) for $S_2$ is in its simplicity. It provides a means for identifying dynamics and parameters that may be of importance in assessing or perhaps even mitigating contrail impact as well as the needs and uncertainties involved in modeling and understanding them. It suggests that to lowest order the critical dynamics may be organized with respect to five components: 1) the number of crystals that survive the wake-dominated contrail regime, 2) the depth of the supersaturated layer below flight level, 3) vertical motions (either in the ambient or specific to the contrail), 4) late-time ice crystal loss mechanisms other than precipitation, and 5) effects forcing deviations from (5).

Remarkably, in (5) the explicit dependence of $S_2$ on a host of physical (as well as, for the simulations, numerical) variables—RH, $T$, $P$, $du/dz$, $d\theta/dz$, ambient-turbulence level, radiation conditions, aircraft parameters—drops out, even though many are of primary importance to contrail extent, lifetime, or peak properties. Here $S_2$ ultimately still depends on many of them; however, their impact is felt almost entirely through their effects on components 1–5. For example, aircraft parameters have their primary effects through their impact on 1. Variations in RH, $T$, and $d\theta/dz$ have their effects primarily through 1 and, via coupled radiation, 3. Shear and atmospheric diffusion rates have their effect on $S_2$ to lowest order only through 4.

Because of its simplicity, (5) also provides a means for assessing more complex contrail scenarios such as the interaction of multiple contrails. It suggests that a subsequent flight passing through an existing contrail would have its integrated effects reduced substantially because of the much smaller fraction of crystals that would survive the wake-dominated regime as a result. This provides a possible mitigation strategy in that, at least as seen in the simulations, an aged sheared contrail can deplete the moisture above saturation from a large cross-sectional area. In contrast, the later merger of two adjacent contrails through horizontal spread would not reduce the impact to lowest order over that from two separated contrails: the reduction in horizontal extent would be offset by an increase in lifetime.

Both the simulations and the simple model for $S_2$ indicate the potential importance of “cold” contrails despite their generally having small ice content and being subvisible. These contrails may be much longer lived than their more visible counterparts, allowing their lifetime-integrated significance to be comparable or, for some regimes of coupled radiation, greater. The simulations and (5) also highlight the critical importance of ice crystal number on integrated contrail significance and of the effects of vertical motions on in situ crystal-loss mechanisms. Sufficiently lowering ice crystal numbers produced by the aircraft would dramatically reduce contrail significance for essentially all conditions.

Finally, while we have focused on contrails (including contrail cirrus), much of the dynamics considered is clearly of direct relevance to natural cirrus. The cases and models considered here should help our understanding of the latter even though differences and new dynamics enter. Given their simpler localized (in time and space) initialization, contrails may provide a simpler venue for studying some aspects of late-time cirrus evolution.

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APPENDIX A

Simulation Details

Except as noted, the simulation conditions and run procedures are as described in Part I for a B-767. Members of the principal simulation set were each performed in Q3D mode in five segments with different grids (Table A1). When employed, coupled radiation and/or subsidence/uplift were added only after 15-min contrail age since neither are significant enough to compete with wake-induced effects earlier. Only cross-stream oriented vertical wind shear was included with constant value $du/dz$ maintained throughout the supersaturated layer and at least 500 m above flight level; the

\[ S = \frac{1}{2} \int_{0}^{L} dS_{	ext{cr}}(z) dz \]

\[ \frac{dS_{	ext{cr}}}{dz} = \frac{1}{2} \left( \frac{d\theta}{dz} + \frac{du}{dz} \right) \]

\[ S_{2} = \int_{0}^{L} \frac{1}{2} \left( \frac{d\theta}{dz} - \frac{du}{dz} \right) \frac{dS_{	ext{cr}}}{dz} dz \]
shear was reversed above and below to ramp $u$ gradually back to 0, limiting the largest $u$ velocities appearing in the domain and thereby allowing a larger time step for some simulations. All ambient conditions were taken to be uniform in the mean across the horizontal simulation domains. Low-amplitude ambient-turbulence fields were added to the simulations out to 2.0 h (as described in Part I, with $T_{amp} = 0.2$ K, $f_{age} = 600$ s, $t_{add} = 600$ s) but only added to the central 16 km of the domain width. Given the large shears and coarse $x$ resolution, turbulence fields were not added during the final run segment except initially to break the symmetry following the periodic domain doubling downstream. As a sample of numerical tests performed, Fig. A1 compares for one set of conditions multiple turbulent realizations\(^\text{A1}\) at standard grid resolution with simulations at higher resolution and greater downstream extent. From this and similar tests it was judged that the standard $x$ resolution and single turbulence realizations were adequate for present purposes. Total CPU times were typically $\sim 15–40$ h on a single Xeon processor.

When longwave radiation was included a blackbody temperature of downwelling infrared radiation at domain top of 130 K, and relative humidity (with respect to water) of 30% between the ground and the LES domain bottom were assumed. When solar radiation was included nominal conditions assumed were surface albedo of 0.2, latitude 45$^\circ$N, and date 21 June. The diurnal variation in solar angle was included, with an initial fly-through time of 1000 LT. In a preliminary set of 6-h-duration simulations for many conditions and varying fly-through time, the 1000 LT choice tended to maximize solar absorption effects on the contrail over its lifetime.

A set of 16 fully 3D runs out to contrail demise were also included. The basic conditions were as described in Part I and included no mean shear and moisture profiles that did not drop off as steeply as those described above. The basic cases were S10T218m and S10T225m from Part I, each run with $T_{sfc} = 258$, 278, 298 K, or no coupled radiation, plus variations of that set with shallower (500 m) supersaturated layers below flight level.

APPENDIX B

Summary Metrics from Selected Runs Illustrating Radiation Effects

Table B1 provides sample simulation results of the effects of coupled radiation on contrail metrics. These results show that coupled radiation can lead to significant

\[ S(t) \ (m^2\cdot m^{-1}) \]

FIG. A1. Evolution of ice surface area per meter of flight path for case with $z_{dn} = z_{up} = 500$ m, $du/dz = 4$ m s$^{-1}$ km$^{-1}$, $\gamma = 2.5$ K km$^{-1}$, $R_{H1} = 110\%$, $T = 218$ K, $E_{lwceno} = 10^{13}$ kg$^{-1}$, and coupled longwave radiation with $T_{sfc} = 278$ K: four different turbulent realizations at standard resolution (solid lines, minimum $D_x = 200$ m at late times), finer resolution (long dashed, minimum $D_x = 100$ m at late times), and finest resolution (short dashed, minimum $D_x = 50$ m at late times). The downstream domain length was doubled in the latter two cases to 1.28 km.

\(^\text{A1}\) That is, runs differing only by the random perturbations used in initializing the turbulence.
Table B1. Summary metrics [fraction of crystals that precipitate into the subsaturated layer $N_{\text{prec}}/N_{\text{init}}$; maximum contrail depth $D_{\max}$ (km); lifetime-integrated ice surface area per length of flight path $S_{\Sigma}$ ($m^{-2} m^{-1}$); lifetime $t_{\text{live}}$ (h)] for sample contrail simulations illustrating different coupled radiation regimes. The radiation conditions, stratification ($\gamma$: K km$^{-1}$), and depth of the constant RH, supersaturated layer above flight level ($z_{\text{up}}$, m) are indicated for each simulation in the first column of each row; the flight-level temperature (K) in column headings at top. Other conditions held constant are $z_{\text{dn}} = 500$ m, $E_{\text{Iice}} = 10^{15}$ kg$^{-1}$, $w_{\text{up}} = 0$, $du/dz = 4$ m s$^{-1}$ km$^{-1}$, and RH$_i = 110\%$.

<table>
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<th>Radiation</th>
<th>$\gamma$</th>
<th>$z_{\text{up}}$</th>
<th>$N_{\text{prec}}/N_{\text{init}}$</th>
<th>$D_{\max}$</th>
<th>$S_{\Sigma}$</th>
<th>$t_{\text{live}}$</th>
<th>$N_{\text{prec}}/N_{\text{init}}$</th>
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<th>$S_{\Sigma}$</th>
<th>$t_{\text{live}}$</th>
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Increases or decreases in contrail significance with different behavior in different regimes depending on $T$, $T_{\text{sc}}$, and $\gamma$.

REFERENCES


