Study of a five-electrode quadrupole levitation system
Part I: Theoretical aspects
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Abstract: A novel quadrupole levitation system with a five electrode structure is designed and constructed to achieve stable levitation of liquid particles in a 1 g environment. The theoretical aspects relating to the design and operation of the system are given, which include derivation of the three-dimensional potential function in a paraxial form and the stability analysis. Based on the three-dimensional potential expression, the stability of the levitated liquid particle is obtained by solving the equations of motion of the charged particles within the quadrupole for the situations with and without the consideration of a viscous damping force. Numerical calculations are made for different electrode geometries to verify the theoretical analysis and to locate the stability domains. Charge-to-mass ratio of levitated particles can be decided either from numerical results or from the analytical expression. The difference in the charge-to-mass ratios obtained from each method is only about 3%.

List of symbols

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<tr>
<td>a, a, a' etc.</td>
<td>constant in standard Mathieu equation</td>
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<td>β</td>
<td>characteristic number related to the stable Mathieu functions</td>
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<td>C, c</td>
<td>constant, quadrupole spacing parameter</td>
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<td>D(z)</td>
<td>coefficient in potential function with two planes of symmetry</td>
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<td>ε</td>
<td>unit charge</td>
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<td>φ</td>
<td>electric potential</td>
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<td>f</td>
<td>frequency</td>
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<td>φ₀</td>
<td>axial potential along the symmetrical axis (z-axis)</td>
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<td>g</td>
<td>gravitational acceleration</td>
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<td>h</td>
<td>normalised axial co-ordinate</td>
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<td>k</td>
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<td>K</td>
<td>short for kilo</td>
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<td>m, n</td>
<td>random integral number</td>
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<td>p</td>
<td>viscous damping coefficient</td>
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<td>q</td>
<td>characteristic number in the standard Mathieu equations</td>
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<td>R</td>
<td>radius of the quadrupole bar</td>
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<td>r</td>
<td>radial co-ordinate</td>
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<td>R₀</td>
<td>shortest distance from bar to the central axis (z)</td>
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<td>t</td>
<td>time variable</td>
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<td>U</td>
<td>DC component of the bottom electrode</td>
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<td>U₀, V, V₀, Vₐ etc.</td>
<td>DC voltage on the bottom electrode or injecting capillary</td>
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<td>V</td>
<td>amplitude of the AC voltage on bar electrode</td>
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<td>v</td>
<td>volume</td>
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<td>ω</td>
<td>angular frequency</td>
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<td>Ω, Ω₀, Ω₁ etc.</td>
<td>particle frequency</td>
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<td>x, y, z</td>
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1 Introduction

The principle of electrodynamic confinement of charged particles by an AC quadrupole configuration was first discovered by W. Paul et al. in the early 1950s [1]. They designed and constructed a radio frequency quadrupole mass filter that consisted of two hyperbolic end caps and one ring electrode, with a hyperbolic linear surface to confine charged particles such as ions. This device has been later referred to as Paul's ion trap. Paul and his colleagues pioneered the fundamental development of the mass spectroscope. The same principle had also been used in confining macroscopic solid metal particles in a vacuum or in an atmospheric environment [2]. Since then, this technique has been widely used to levitate small particles by many researchers. For example, Johnson and Hendricks [3], at Lawrence Livermore National Laboratory, used it for levitating and transporting laser fusion targets into the fusion chamber; Vedder simulated the hazards of micrometeoroid in space by charging and accelerating microparticles to a hypervelocity in an electrodynamic levitator [4]; Arnold built a fluorimeter based on the same principle to study photoemission from a single levitated microparticle [5]. A recent achievement
of using the electrodynamic levitator to test the Einstein single photon quantum transition has been made by Walther and his associates in the Max-Plank Institute [5]. Applications were also found in the study of particle's rate process [7], the measurement of vapour pressure [8], refractive index [9], mass and charge [10], and infrared absorption of particles in the micron and sub-micron range [11]. However, there have been few changes made to Paul's original scheme. Arnold has once reported a new design of an electrodynamic levitator for confining individual microparticles (normally less than 10 µm in diameter) made from a spherical void [12]. However, the quadrupole field in this spherical void levitator is considerably distorted in the off-axis region, and therefore the levitation is effective only for very small particles. The five-electrode quadrupole levitation scheme presented in this paper has considerable advantages over the conventional ones, for example, the vertical mode of electrode geometry makes observation and illumination easier, particularly for measurement of very fine particles, by employing a laser diffraction method. It is also possible to add auxiliary electrodes to impose ion bombardments to the levitated particles, or induce an electric rotating field to make further study of the electric behaviour of levitated particles. Since the introduction of the fifth electrode, the analysis has to be three-dimensional. The stability analysis based on the two-dimensional potential expression is no longer valid [13] for the five-electrode configuration. In this paper, we present the theoretical aspects of levitation in the quadrupole system, which include deriving the three-dimensional axial potential function for the system, giving the stability of levitated particles based on the potential function, and calculating charge-to-mass ratio that is then compared with the numerical results. It is shown that the stability domains can be made much wider than the conventional electrodynamic levitator by superimposing a DC voltage onto the quadrupole electrodes.

2 3-D potential function for the quadrupole system

2.1 Potential expression for systems with two planes of symmetry

Fig. 1 shows the electrode configurations for the five-electrode quadrupole electrodynamic levitator. Because of its geometrical complexity, an approximation method is used. In general, a three-dimensional potential function \( \phi \), which obeys the Laplace equation, can be expanded into a power series about its symmetric axis (z-axis), i.e.

\[
\phi(x, y, z) = \sum_{m, n = 0} b_m b_n (m + 1) (n + 1) x^m y^n
\]

This is normally true for the system where the separation variable method can be employed. Substituting eqn. 1 into the Laplace equation,

\[
\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0
\]

will yield

\[
\sum_{m, n = 0} \left[ a_m (m - 1)x^{m - 2}y^n + a_n (n - 1)x^m y^{n - 2}
+ a_{m+n} x^m y^n \right] = 0
\]

The general concurrent relationship of the coefficients for the \( x^m y^n \) terms can be obtained from eqn. 3 and is expressed as

\[
ad_m = \frac{(2m + 1) a_m + b_m}{m + 1} \quad \text{and} \quad b_m = \frac{(2n + 1) b_n}{n + 1}
\]

and so forth.

The independent orthogonal co-ordinates \( x \) and \( y \) can be thought of as being symmetrical with respect to the \( z \)-axis. Therefore the general expression of \( a_{m+2, m} \) and \( a_{n+2, n} \) should be the same in form. So, if \( a_{m+2, m} \) is given by

\[
a_{m+2, m} = -\frac{1}{2(m + 2)(m + 1)} a_m + \frac{b_m}{2(m + 2)}
\]

then \( a_{n+2, n} \) must be

\[
a_{n+2, n} = -\frac{1}{2(n + 2)(n + 1)} a_n + \frac{b_n}{2(n + 2)}
\]

where \( b_m(z) \) is a function of uncertainty. The coefficients for the lower power terms can be written or defined as axial potential: \( a_0 = \phi(z) \)

deflection field: \( a_{1n} = E(z) \)
quadrupole field: \( a_{20} = -\frac{1}{4} a_0 \delta + \frac{1}{2} b_0 \)

and so forth.

A general form of three-dimensional electric potential function can then be obtained as follows:

\[
\phi(x, y, z) = \phi(z) - Ex - Fy - \frac{1}{2}(\phi'(z) - D(z))x^2
+ Pxy - \frac{1}{2}(\phi''(z) - D'(z))x^2
+ \frac{1}{2} Fx^2 + Fy^2 + \frac{1}{8}(\phi''''(z)
- 2D'(z) + D(z))x^4 + \cdots
\]

and supplied with computer-controlled DC power supply

\[
+ \frac{1}{4} E^2 y^2 - \frac{1}{2} F^2 x^2 + \frac{1}{2} E'F' x^2 y - E_F x y^2
+ \frac{1}{4} F^2 x^2 + F^2 y^2 - \frac{1}{2} E_F x y^2
- \frac{1}{2} F^2 x^2 + \frac{1}{2} E^2 y^2 - \frac{1}{2} F^2 x^2
+ \frac{1}{8}(\phi''''(z) - 2D'(z))x^4 + \cdots
\]

Fig. 1 Five-electrode quadrupole configuration

where \( D, E, F, P, D_1, E_1, F_1, P_1 \) are all arbitrary functions of the \( z \)-co-ordinate.

### 2.2 Three-dimensional potential function for quadrupole systems

For a system with two planes of symmetry in standard orientation (e.g. quadrupole system, the symmetric axes of the four electrodes either on \( YOZ \) or \( XOZ \) plane), only the terms of even power appear in eqn. 10, i.e.

\[
\phi(x, y, z) = \frac{1}{2} \phi''(z)(x^2 + y^2) + \cdots + \frac{1}{2} \phi''''(z)(x^2 - y^2) + D(z)(x^2 - y^2) - \frac{1}{2} D''(z)(x^4 - y^4) + D_1(z)(x^4 - 6x^2y^2 + y^4) + \cdots +
\]

(11)

Considering only a small displacement off the central axis (\( z \)-axis) and ignoring all the higher terms of \( x, y \), we have

\[
\phi(x, y, z) = \frac{1}{2} \phi''(z)(x^2 + y^2) - \frac{1}{4} \phi''''(z)(x^2 - y^2) \quad (12)
\]

The quadrupole parameter \( D(z) \) is normally a complicated function of the axial co-ordinate \( z \). It is assumed for a long quadrupole that the function \( D(z) \) over the \( z \)-axis is a bell-shaped distribution, which is shown in Fig. 2a, or, alternatively, the rectangular model, as shown in Fig. 2b [13].

![Fig. 2 Quadrupole parameter D(z)]

If the quadrupole is infinitely long, there will be no longitudinal component, and the potential function is only dependent on the \( x \) and \( y \) co-ordinates. In this case, eqn. 12 becomes

\[
\phi(x, y, z) = \frac{1}{2} D_0(x^2 - y^2)
\]

which is just the same as the two-dimensional model used before [13].

For the five-electrode quadrupole configuration, the contribution from the third direction has to be considered. Assuming the equilibrium levitating point is at \((0, 0, z_0)\), we can expand the axial potential function about this equilibrium point into a Taylor series, as follows:

\[
\phi(z) = \phi_0 - E'z + \frac{1}{2} \phi''(z_0)z^2 - \cdots +
\]

(14)

where \( \phi_0 = \phi(0, 0, z_0) \), which is the axial potential at \( z = z_0 \), and

\[
E = -\frac{\partial \phi}{\partial z} \bigg|_{0, 0, z_0} z' = (z - z_0)
\]

(15)

Substituting eqn. 14 into eqn. 12 and assuming that (a) the quadrupole bars are long, and their surfaces are of an ideal hyperbolic shape, which is given by \((x^2 - y^2)/R^2_0 = 1\)

(b) the quadrupole coefficient \( D \) is a constant \((= D_0)\) within the levitation zone.

Considering the voltages on the boundaries of quadrupole bars \((x, y = \pm R_0)\)

\[
\phi(\pm R_0, 0, z) = -\phi(0, \pm R_0, z) = V \cos (\omega t)
\]

(16)

where \( V \) is the amplitude, and \( \omega \) is the angular frequency of the alternating electric voltage on the quadrupole bar electrodes. From eqns. 16 and 12, the potential function of the quadrupole system can be written as

\[
\phi(x, y, z) = \phi_0 - E'z + \frac{2D_0}{R_0^2} x^2 - \frac{\phi_0}{R_0^2} (x^2 + y^2)
\]

\[
+ \frac{V \cos \omega t}{R_0^2} (x^2 - y^2)
\]

(17)

where \( R_0 \) is half of the distance between two opposite quadrupole electrodes and \( E \) is the axial electric field at \( z = z_0 \). Comparing eqn. 17 with eqn. 12, we have

\[
\phi''(z_0) = 4\phi_0/R_0^2
\]

(18a)

\[
D_0 = \frac{4V \cos \omega t}{R_0^2}
\]

(18b)

The physical meaning of \( D_0 \) as a particular parameter of the quadrupole geometry can be clearly seen from eqn. 18b.

From eqn. 17, one can immediately conclude that, if and only if the axial potential \( \phi_0(z) \), which is the axial potential at the levitating point, is known, the potential in three-dimensions in the neighbourhood of the axis can be solely decided. As \( D_0 \) represents the dynamic confining field, we can find that for a dynamic stabilisation, it is required that \( V > \phi_0 \). This can be seen from the partial derivatives of eqn. 17:

\[
\frac{\partial \phi}{\partial x} = \frac{\partial \phi_0 - V \cos \omega t}{R_0^2} x
\]

(19a)

\[
\frac{\partial \phi}{\partial y} = \frac{\partial \phi_0 + V \cos \omega t}{R_0^2} y
\]

(19b)

The confinement force in the \( x \) direction is equal to \( qE_x \). The contribution from the DC component (electric field force from the bottom electrode) is always to expel particles away from the axis. If \( V < \phi_0 \), there will be no alternating centring force, and this consequently results in the loss of the particle. The same situation will occur in the \( y \) direction, as \( E_y \) is only 180° out of phase with \( E_x \). In this case, the potential shape on the transverse cut plane passing the levitating point \((0, 0, z_0)\) is no longer a hyperbolic one, and the equipotential lines become a set of elliptical contours.

### 2.3 Numerical verification of the analytical expressions

With the aid of the CRAY X-MP, the supercomputer at Rutherford Appleton Laboratory (RAL), and the three-dimensional finite element package TOSCA, \( \phi_0 \) can be calculated numerically [14].

To verify the validity of eqn. 17, we check eqn. 18a. This expression is directly obtained by comparing eqn. 17 with eqn. 18. If eqn. 18a can be proved to be right, then the validity of eqn. 17 is also proved.

Rewriting eqn. 18a as

\[
\phi''(z_0) - 4\phi_0/R_0^2 = 0
\]

(20)

we have the solution to eqn. 20, which is in the form of

\[
\phi''(z) = A \exp(-2z/R_0) + B \exp(2z/R_0)
\]

(21)

It must be convergent as \( z \) approaches infinity, so we discard the exponential term with positive index. According to the boundary condition at \( z = R_0 \), \( R_0 \) is the radius
of the bottom electrode), the voltage $\phi_d(z) = V_0$, which is 
The DC voltage supplied to the bottom electrode:

$$\phi_d(z) = V_0 \exp \left(2R_b/R_0 \exp \left(-2z/R_0 \right) \right)$$

(22)

The normalised form for eqn. 22 is written as

$$\eta(z) = \frac{\phi_d(z)}{V_0} = \exp \left[c - 2z/R_0 \right]$$

(23)

or

$$\eta(h) = \frac{\phi_d(h)}{V_0} = \exp \left(c - h \right)$$

(24)

where $c = 2R_b/R_0$, $h = z/R_b$, which is normally larger than 1.0 and $\eta(h)$ is the normalised axial potential.

Eqn. 24 can be easily plotted against the normalised axial co-ordinate $h = z/R_b$. Fig. 3 shows the comparison of the numerical axial potentials with the analytical expression $\eta(h)$ for different spacing parameters $R_b/R$ ($R$ is the radius of the quadrupole bar electrodes).

Figs. 4 and 5 are the axial electric field and the second derivatives of the axial potential separately for the current model, with the bar spacing parameter $R_b/R = 1.16$, which can give a better approximation of the cylindrical bar to the hyperbolic field.

Eqn. 24 is very important in understanding the influence of $R_b$.

From eqn. 24, we can derive that the electric field along the $z$-axis is

$$E_z = \frac{\partial \phi}{\partial z} = -\frac{d}{dz} \left[ V_0 \exp \left(1 - z/R_b \right) \right] = cE_0 \exp \left(1 - z/R_b \right)$$

(25)

where $E_0 = V_0/R_b$.

Hence, the normalised electric field in $z$-axis will be

$$E_z = (E_z/E_0) = c \exp \left(1 - z/R_b \right) = -\frac{\partial \phi(h)}{\partial h}$$

(26)

Since $z/R_b$ is always larger than 1, and $c = 2R_b/R_0$, it can be clearly seen that increasing $R_b$ will result in increasing the electric field sharply.

Therefore the three-dimensional potential function, as described in eqn. 17, can be written, according to eqn. 24, as

$$\phi(x, y, z) = V_0 \exp \left(1 - z/R_b \right) \left[1 - (x^2 + y^2)/R_b^2\right]$$

$$+ \frac{V \cos \cot \alpha}{R_b^2} \left(x^2 - y^2\right)$$

(27)

Eqn. 27 was obtained for the first time to express the three-dimensional potential function in the central axial zone within the five-electrode quadrupole levitation system. Compared to the old two-dimensional model.
it not only introduces the bar spacing parameter \( R_0 \) as a very important factor in deciding the levitation field and confinement force, but also shows that the amplitudes of the confinement forces from both transverse directions (x and y) are not the same as was anticipated before. This can be examined by writing the potential function on the transverse plane passing the point (0, 0, \( z_0 \)) as follows:

\[ \phi(x', y') = 1 + c_1 x'^2 - c_2 y'^2 \]  

where

\[ x' = x/R_0 \]  
\[ y' = y/R_0 \]  
\[ c_1 = (V_{\text{ac}}/\phi_0 - 1) \]  
\[ c_2 = (V_{\text{ac}}/\phi_0 + 1) \]

and \( V_{\text{ac}} = V \cos(\omega t) \).

Notice that \( \phi_0 \) is the axial potential at the (0, 0, \( z_0 \)), which can be considered as a constant. Eqn. 28 represents a saddle surface with the saddle point at (0, 0, \( z_0 \)) (Fig. 6), where \( \phi(x', y') = 1 \), and \( x = \pm(c_2/c_1)y \), which are two lines crossing at the point (0, 0, \( z_0 \)). For \( x' > 1.0 \) or \( x' < 1.0 \), the potential function is governed by a set of hyperbolic functions \( c_1 x^2 - c_2 y^2 = C \) and \( c_2 y^2 - c_1 x^2 = C \), which are conjugate to each other (C is positive). The principle of electrodynamic confinement potential; as soon as the particle starts to roll down the unstable hillside, the potential saddle is reversed. As long as the AC frequency is high enough, the particle will be stably confined at the centre.

From the levitation point of view, the lateral confinement force is given by the quadrupole AC voltage \( V_{\text{ac}} = V \cos(\omega t) \). If \( V < \phi_0 \), then \( c_1 \) is always less than 0; the potential shape is elliptical, and the electric field is divergent, which will give no confinement at all. Hence, \( V > \phi_0 \) is a necessary condition for the particle's confinement at its balancing point (saddle point).

The potential contours in the \( x \) direction and \( y \) direction when \( c_1 - c_2 \) have almost the same curvatures. Otherwise, if \( c_1 \neq c_2 \), the amplitude of the electric forces in \( x \) and \( y \) direction, acting on the levitating particle, have a difference of \( (\phi_0 - 1)/V_{\text{ac}} \phi_0 + 1) \).

The cross-over effect observed from the experimental programme [13] can be explained by eqn. 27. As \( 1 - z/R_0 \) is always negative, increasing \( R_0 \) will lead \( E_z \) to increase. However, this is only valid for a small range of \( R_0 \). If \( R_0 > 1 \), it is clear from Fig. 3 that eqn. 27 is no longer a good approximation for the axial potential function.

3 Stability analysis

3.1 Motion equations

According to eqn. 17, the equation of motions of a levitated charged particle in the quadrupole levitation system, in its component forms, can be expressed as:

\[ m \frac{d^2 x}{dt^2} = Q E_x = -Q \frac{\partial \phi}{\partial x} \]

\[ = Q(\phi_0 - V \cos(\omega t)x/R_0^2) \]  

(32a)

\[ m \frac{d^2 y}{dt^2} = Q E_y = -Q \frac{\partial \phi}{\partial y} \]

\[ = Q(\phi_0 + V \cos(\omega t)y/R_0^2) \]  

(32b)

\[ m \frac{d^2 z}{dt^2} = Q E_z - mg = Q E_z - mg - \frac{4 \phi_0 Q}{R_0^3} z' = 0 \]  

(32c)
Hence, the above equations become
\[\frac{d^2\phi}{dt^2} = \frac{2Qx}{mR_0^2} \cos \omega t - \frac{2\phi}{mR_0^2} \cos \omega t = 0\]  
(33a)
\[\frac{d^2\phi}{dt^2} = \frac{2Qx}{mR_0^2} \cos \omega t + \frac{2\phi}{mR_0^2} \cos \omega t = 0\]  
(33b)
\[\frac{d^3\phi}{dt^2} = \frac{4\phi}{mR_0^2} = 0\]  
(33c)

\(Q/m\) is the specific charge of the levitated particle. A stable levitation of the particle at the point \(z_0\) implies that eqns. 33a-c must have stable solutions simultaneously. From eqn. 33c, one can easily find out that it has stable solutions only when
\[\frac{4\phi_0 Q}{mR_0^2} > 0 \quad \text{or} \quad \phi_0 Q > 0\]  
(34)

The motion in \(z\) direction is a harmonic oscillation with frequency \(\Omega = \sqrt{(4\phi_0 Q/mR_0^2)}\). As \(z'\) is normally very small, and there is an air viscous force that reduces the harmonic oscillation, so the resultant motion in \(z\) direction is a damped oscillation until it becomes stable. Eqn. 34 clearly indicates that the levitation force against gravity is the Coulomb repulsive force. The radial stability is decided by eqns. 33a and 33b, which can be written into the canonical form of the Mathieu equation [14]:
\[\frac{d^2u}{ds^2} + (a - 2q \cos 2s)u = 0\]  
(35)

Where
\[\xi = \frac{\omega t}{2}\]  
(36a)
\[a = -a_e - a_s - \frac{8\phi_0 Q}{m o^2 R_0^2}\]  
(36b)
\[q = q_e = -q_s = \frac{4VQ}{m o^2 R_0^2}\]  
(36c)

\(u\) represents \(x\) and \(y\) co-ordinates, and constants \(a\) and \(q\) are the characteristic numbers of the Mathieu equation, which are irrelevant to the initial conditions.

### 3.2 Stability characteristics of the quadrupole system

The solution to eqn. 35 can be put into the form [16]
\[u(\xi) = u_{1l} e^{i\zeta} \sum \limits_{s = -\infty}^{\infty} C_{2s} e^{-2is\xi} + u_{1l} e^{-i\zeta} \sum \limits_{s = -\infty}^{\infty} C_{2s} e^{-2is\xi}\]  
(37)

We are only interested in solutions denoted 'stable', where \(u\) remains finite as \(\xi \rightarrow \infty\). The constant \(\mu\) determines the type of solution to be stable or unstable, and it depends on the value of constants \(a\) and \(q\). The stable solution requires \(\mu = \beta i\), which is imaginary, and \(\beta\) is not a whole number. The stable solution can be written as
\[u = a_{1l} \sum \limits_{s = -\infty}^{\infty} C_{2s} \cos (2s + \beta)x + a_{1l} \sum \limits_{s = -\infty}^{\infty} C_{2s} \sin (2s + \beta)x\]  
(38)

The integration constants \(a_l\) and \(a_{1l}\) are dependent on the initial conditions \(u_0, \partial u_0/\partial t\) and the initial phase of the AC field \(\cos \omega t\). The integral constants \(C_{2s}\) and \(\beta\) are functions of \(a\) and \(q\). If \(q = 0\), there is no AC component, and the equation must be unstable because it cannot hold \(a > 0\) unless eqn. 33c is also satisfied. The AC component brings a dynamic equilibrium to the particle movement. It has been shown that eqn. 35 has a stable solution only when \(a\) and \(q\) satisfy certain relationships [16]. This can be plotted in the \(a-q\) plane, which is shown in Fig. 7. The characteristic data are obtained from

![Stability diagram formed by first four characteristic numbers](image)

Mathieu function tables [18], and \(a_j, b_j\) are the characteristic numbers for \(ce_j(\xi, q)\) and \(se_j(\xi, q)\), which are two independent solutions to eqn. 38. The particle will be stably bounded when the values \(a\) and \(q\) are within the region bounded by the curve of \(a_0\) and \(b_0\). The first three can be written as
\[a_0 = -\frac{1}{2} q^2 + \frac{q^2}{12} q^4 - \frac{29}{23} q^6 + O(q^8)\]  
(39a)
(corresponding to \(\beta = 0\))
\[b_1 = 1 - q - \frac{1}{2} q^2 + \frac{3 q^2}{12} q^4 - \frac{15 q^4}{4} q^6 - \frac{3 q^6}{6} q^8 + O(q^{10})\]  
(39b)
\[a_1 = 1 - q - \frac{1}{2} q^2 + \frac{3 q^2}{12} q^4 - \frac{15 q^4}{4} q^6 + \frac{3 q^6}{6} q^8 + O(q^{10})\]  
(39c)
(corresponding to \(\beta = 1\))

To maintain the stability in \(z\) direction, according to eqn. 36, \(a\) has to be negative, which refers to the area below the \(q\) axis. Hence, the basic stable area for \(x-y\) directions is formed by \(a_0\) and \(b_1\), with \(q\) axis, which is shown in Fig. 8. Notice that \(q_0\) and \(q_1\) are different by just a sign; as the stability figure is symmetrical to the \(a\)-axis, the stability in \(x, y\) directions are satisfied simultaneously.

From the stability chart Fig. 8, we can clearly predict that once the AC and DC voltages are fixed, the driving frequency \(\omega\) will select particles with given \(Q\)-to-\(m\) to be stably levitated. It only selects particles with given \(Q\)-to-\(m\) ratio but does not include any restrictions on the particles' size and initial conditions. We can reasonably expect to have a range of different sized particles, but the system will automatically select charged particles
with specific Q-to-m ratio. For a given Q-to-m ratio, there exist an upper and a lower limit either of frequency, if voltage remains unchanged, or a range of AC voltage

\[ I_I \]

\[ = \text{unstable area} \]

\[ \text{Fig. 8 Basic stability area of the quadrupole system} \]

\[ Q = 8\phi(\frac{Q}{m})_0(\omega^2 R_0^2)^{-1} \]

\[ \text{The stable area is bounded by } a_0 \text{ and } b_1 \]

amplitudes while frequency is fixed. For example, experimental results show that for a levitated charged particle with diameter of 550 \( \mu \text{m} \) and Q-to-m ratio of 5.4 \( \times 10^{-4} \text{ Ckg}^{-1} \), the tuning range of AC voltage (amplitude) is from 1.5 to 5.3 kV, while the frequency is fixed at 50 Hz. If we calculate \( a \) and \( q \) from eqn. 36, the range of corresponding \( q \) numbers are \( q_{\text{min}} = 0.27 \) and \( q_{\text{lim}} = 0.96 \) separately, which just refer to the border of the stability chart (Fig. 8) at \( a = -0.03 \). For the same reason, if \( V \) is equal to 3.1 kV, the corresponding frequency range for the same particle will be from 40 to 100 Hz. This result can be used to decide the range of Q-to-m ratios of the particle by tuning the AC voltage or frequency.

So, for a working frequency of \( f = 50 \text{ Hz} \), \( V = 3.0 \text{ kV} \) (\( \omega = 100\pi \)), and the Q-to-m ratio is between \( 1.4 \times 10^{-4} \) and \( 8.46 \times 10^{-4} \text{ Ckg}^{-1} \).

The operation of the quadrupole system enables us to make particle levitations for as wide Q-to-m ratio ranges as possible. An alternative method for the quadrupole system is to apply a DC component \( U \) to the quadrupole bars as well as the AC voltages \( V \cos \omega t \). That is, to define for eqn. 18b, that

\[ D_0 = 4(U + V \cos \omega t)/R_0^2 \]

Thus, the characteristic numbers become

\[ a_s = 8(U - \phi(\frac{Q}{m})_0(\omega^2 R_0^2)^{-1} \]

\[ a_y = -8(U + \phi(\frac{Q}{m})_0(\omega^2 R_0^2)^{-1} \]

and

\[ q = q_s = q_y = \frac{4VQ}{m\omega^2 R_0^2} \]

The stability diagram for this situation has been changed since \( a_y \neq a_s \). Let us consider some cases:

Case 1: \( U = \phi_0 \). Then

\[ a_s = 0 \quad (44a) \]

\[ a_y = -8(2\phi(\frac{Q}{m})_0(\omega^2 R_0^2)^{-1} \]

Because \( (a_s)/new = \frac{1}{2}(a_s)/old \), if all settings are the same, the \( (Q/m) \) ratio will be half of that obtained before applying DC voltage \( U \); or in other words, for the same particle to be levitated in the system where a DC voltage \( (U = \phi_0) \) is superimposed on the bar electrodes, the AC frequency \( \omega \) will be changed by \( 1/\sqrt{2} \). This is just a straightforward prediction. Actually, the stable area is reduced, so that the \( Q/m \) ratio range has reduced.

The stable area for \( x \) direction is along the \( q \) axis, so the whole stable range for both \( x \) and \( y \) directions is the shaded area, as shown in Fig. 9. A particular point is at \( q = 0.906 \), \( a = -0.501 \), because at this point, only the particles with the same \( (Q/m) = 0.227(\omega^2 R_0^2)/V \) can be levitated. In this situation, the levitator works as a mass filter or a mass spectrometer [17]. This effect can also be used to measure the axial potential by adjusting the DC voltage \( U \) on the quadrupole. The result will be very accurate in vacuum as there is no influence from viscous medium in the surrounding.

\[ \text{Fig. 9 Stability area when } U = \phi_0 \]

\[ a_s = 0, a = a_y = 8(2\phi(\frac{Q}{m})_0(\omega^2 R_0^2)^{-1} \]

\[ \begin{align*}
0.906, -0.501 \text{ is a particular point at which the levitator works as a mass filter.} \\
0.906, -0.501 \end{align*} \]

Case 2: \( U \gg \phi_0 \). This is very easy to achieve, as \( \phi_0 \) is usually only about a few hundred volts and \( U \) could be up to more than 10 kV. In this case, we can assume that

\[ a = a_s \approx -a_s \approx 8U(\frac{Q}{m})_0(\omega^2 R_0^2)^{-1} \]

\[ a/q = 2U/V \]

The stable area, as shown in Fig. 10, will be the same as that used in the mass filter and mass spectrometers [16]. The quadrupole can then work either as a mass filter, which only allows very restricted \( Q/m \) particles to be levitated (very high \( U \)), or as a general levitator (less high \( U \)). The stable range is decreased when \( U \) (or parameter \( a \)) increases. The maximum \( a \), which refers to the apex on a-q stable diagram in Fig. 10, is \( a_{\text{lim}} = 0.23699 \), and the corresponding \( q_{\text{lim}} = 0.706 \).

\[ \text{IEE PROCEEDINGS-A, Vol. 138, No. 6, NOVEMBER 1991} \]
Case 3: \( U \geq \phi_0 \). Consider \( U = 5\phi_0/3 \), then
\[
a = a_x = -1/(4a_\eta) = (2\phi_0/3)\left(\frac{Q}{m}\right)(a^2R^2)\frac{1}{b}
\]
(46)

The operation is similar to the three-dimensional quadrupole used by many researchers (see, for example: Arnolds, 1986 [5], Wuerker et al. 1959 [2], and Philip, 1983 [10]).

The stability diagram is shown in Fig. 11: the stable area can be changed according to the relationship between \( a_x \) and \( a_\eta \).

4.5 Stability with consideration of viscous medium

The experimental programme of levitation was carried out in air. It is believed that the oscillation was actually damped by the air viscous force. Hence, in considering the stability problem of levitation, the influence of air or other viscous medium cannot be excluded.

According to Stokes law, the equation of motion of a charged particle in the quadrupole field, with the consideration of a kind of damping force, can be written as
\[
\frac{d^2u}{d\xi^2} + 2\eta \frac{du}{d\xi} + (a - 2q \cos 2\xi)u = 0
\]
(47)
where \( \eta = 6\pi n r_p \rho \omega \), \( r_p \) is the radius of the particle; and \( \eta \) is the air viscosity.

Assume that the solution to eqn. 47 takes the form
\[
u(\xi) = \xi^\mu e^{-\eta \xi}
\]
(48)
Putting eqn. 48 into eqn. 47, we have
\[
\frac{d^2\xi^\mu e^{-\eta \xi}}{d\xi^2} + \left(b - 2q \cos 2\xi\right)\xi^\mu e^{-\eta \xi} = 0
\]
(49)
where \( b = a - \eta^2 \). Eqn. 49 is still a canonical Mathieu equation, so we can write the solution to eqn. 47 as
\[
\xi(\xi) = a^\mu e^{-\eta \xi} \sum_{s=-\infty}^{\infty} C_s e^{2s\xi}
\]
(50)
As mentioned in the previous section, only \( \mu = i\beta \), which is imaginary, will give a stable solution. However, if \( \eta > \mu \), from eqn. 50, the solution to eqn. 47 will converge, according to exp \( \left[-(\eta - \mu)\xi\right] \). Therefore it can be said that the stable regions for a particle's motion in a viscous medium are extended beyond the critical curves corresponding to \( \mu = \eta \). The stability chart with consideration of the air viscous force is obtained with the aid of a computer and is shown in Fig. 12.

4 Conclusions

The three-dimensional potential function for the five-electrode quadrupole levitation system has been given, and numerical verification shows that the axial potential function is valid for a range of spacing parameters \( R_0/R \) from 1.0 to 1.5. The current model chooses \( R_0/R = 1.16 \), which can give a very good approximation to the hyperbolic field.

The three-dimensional potential function in the levitation area is a saddle shape, with the saddle point as the equilibrium point. The confining field force from the quadrupole is not equivalent in two orthogonal directions (x and y directions).

One of the necessary conditions for a stable confinement is that the amplitude of AC component \( V \) should be larger than the DC component, at the levitation point contributed from the bottom electrode (\( \phi_0 \)). If \( V \gg \phi_0 \), the balancing fields from x and y directions are nearly equal.

The determination of \( Q/m \) using the exponential dependence of the axial field is overestimated for all settings of the quadrupole spacing parameter. For \( R_0/R = 2 \), the maximum error is overestimated by approximately 40%. For the current model \( R_0/R = 1.16 \), the difference between the numerical estimates and the calculated value from eqn. 17 is less than 5% for most of the levitating zones.

The quadrupole levitation system can provide a three-dimensional trap to the particle. The conditions for stable levitation in the transverse plane are governed by the standard Mathieu equation.
The stable levitated $Q/m$ particles can be determined from the stability diagram, for a given voltage and frequency. At a working frequency of 50 Hz, with $V_{ac} = 3.0$ kV, the $Q$-to-$m$ ratio is about $10^{-4}$ (Ckg$^{-1}$). The higher the frequency, the larger the $Q$-to-$m$ ratios being levitated.

Changing the stability by giving the quadrupole bars a certain DC bias will make the levitation system work in different application areas, such as a mass spectrometer, or a levitator suitable for levitating particles with a wide range of $Q$-to-$m$ ratio.

The effect of air viscous force on the quadrupole levitator is actually extending the stability domain for a factor dependent on the viscosity. The stability diagram of the quadrupole levitator in viscous medium shows that the stable domain will follow the $iso-\mu$ lines rather than $iso-f$ curves.

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Study of a five-electrode quadrupole levitation system
Part II: Experimental results

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Indexing terms: Electrostatic levitation, Dynamic confinement, Numerical modelling

Abstract: Stable levitation of nonconductive liquid particles up to 1 mm is demonstrated on a microcomputer-controlled five-electrode quadrupole levitation system. Charged liquid particles for the levitation use are obtained by an electrostatic spraying process from the tip of a capillary inside the central bottom electrode. A CCTV camera is used to record the illuminated particles to give a picture of the particles on the monitor screen for the purpose of viewing, recording and digitising. Working liquids can be doped with a solvent of varying concentrations to give various conductivities with little change to their viscosity. The experimental results agree well with the theoretical and numerical values. Design consideration for a further complex system are discussed.

List of symbols and abbreviations

- $r_p$: particle's radius
- $R_0$: shortest distance from bar to the central axis (z)
- $R_b$: radius of the bottom electrode ($=1/2R$ for most of the cases)
- $t$: time variable
- $U$: DC component of the bar electrodes
- $V$: amplitude of the AC voltage on bar electrode
- $V_{dc}$: DC voltage on the bottom electrode or injecting capillary
- $\omega$: angular frequency
- $x, y, z$: Cartesian co-ordinates
- $AIQ$: automated injection quadrupole levitation system
- $VSQ$: variable spacing quadrupole levitation system

1 Introduction

The principle of electrodynamic levitation (EDL) [1] has been adopted by many researchers to build levitation devices, for example, for the handling of laser fusion targets [2], the measurement of optical properties of small liquid particles [3–6], the absolute measurement of mass [7] and charge [8] of very small charged particles and in the study of aerocolloidal phenomena [9–10]. The unique advantage of EDL, using an AC quadrupole electrode structure, lies in its simplicity and effectiveness in handling very small particles in the micron and sub-micron range. The dynamic confinement feature of an AC quadrupole configuration is the principal contribution to the development of mass spectrometers and ion filters [11, 12]. The theoretical aspects of the novel five-electrode quadrupole levitation system, which was designed based on a standard EDL, have been given in the previous paper [13]. In this paper, we present the operation and experimental data on the five-electrode quadrupole levitation system. It consists of three systems: the automatic DC/AC power control system, the liquid injection system and the viewing and measuring system. The experimental levitation results are given together with comparison of the theoretical and numerical analyses. Discussions of the experimental results and conclusions are helpful for the design and construction of the new levitation system. Although the study is purely basic and fundamental, it could be used in the study of cell morphology, plant pathology or some other biochemical process.
2 Description of the experimental device

Fig. 1 shows the structure of the five-electrode levitation system. It consists of four quadrupole bar electrodes mounted in the vertical z-direction and a bottom electrode located on the centre line at the base. The quadrupole electrodes are supplied with an AC voltage, and the bottom electrode is supplied with a DC high voltage. The quadrupole electrodes are cylindrical, with their lengths much larger than the radius \( L \gg R \). The bottom electrode has a hemispherical end and an opening from the bottom hemisphere is \( R_b = 4.5 \, \text{mm} \). The whole system is inside a transparent plexiglas chamber.

There are two levitation systems used in the experimental programme for different purposes:

(a) variable spacing quadrupole levitation system (VSQ): There is a sharp needle with the end radius about \( 0.01 \, \text{mm} \), slightly protruding from the central injecting capillary which is inside and in contact with the bottom electrode. It can be protruded when the liquid injection is needed. The capillary tube is made from a normal medical syringe with outside diameter of \( 1 \, \text{mm} \). The capillary will obtain charges through induction. As soon as the charges have been formed at the tip of the cone, the liquid is extended into a filament at the free end. Fig. 3 is a set of photographs of the launching process, taken from the screen of the TV camera. For a highly conductive liquid like water, the charging process takes very little time for charges to accumulate, whereas, for a liquid of very low conductivity (e.g. silicon oil), the charging and launching process will take a longer time to form a fully charged particle.

(b) automated injection quadrupole levitation system (AIQ): The quadrupole bar radius \( R = 9.0 \, \text{mm} \), and length \( L = 210 \, \text{mm} \); the bar spacing parameter \( R_w/R \) is fixed at 1.16. It consists of a computer-controlled stepper motor that automatically drives a microlitre syringe (Hamilton 7001-N type 1.0 \( \mu \)l syringe) for the liquid injection. A BBC computer program has been developed to control the amount and the speed of injection. This is the optimised version based on the original system (VSQ).

The levitated particle is illuminated by two fibre-optic strobos with adjustable intensity and is viewed on a TV monitor through a CCTV camera that takes pictures through a side window of the plexiglas chamber and is connected to a video recorder or a video digitiser for recording or line measurement.

The stepper motor controlled liquid injector designed for the AIQ uses a 1.0 \( \mu \)l syringe and the diameter of the capillary is \( 460 \, \mu \text{m} \) with no sharp needle inside it.

A block schematic figure of the experimental set-up is shown in Fig. 2.

3 Experimental process

3.1 Generation and launching of liquid particles
The charging and launching of a levitated particle is through the capillary tube of outside radius \( 1 \, \text{mm} \), with a sharp needle inside the capillary for the VSQ, or a hypodermic needle of diameter \( 460 \, \mu \text{m} \) for AIQ without a sharp needle. The capillaries for both systems are inside the bottom electrode and can be protruded to produce an electric-field enhancement and hence particle injection. Liquid is injected by the syringe to form a meniscus at the tip of the capillary when DC voltage is not applied. The launching process is by means of electrostatic atomisation from a filament of liquid formed by the electrostatic field. Charges are built up when the liquid is forming at the tip of the capillary. The needle is electrically connected to the bottom electrode, which is supplied by a high DC voltage. The liquid in contact with the capillary will obtain charges through induction. As soon as the charges have been formed at the tip of the cone, the liquid is extended into a filament at the free end. Fig. 3 is a set of photographs of the launching process, taken from the screen of the TV camera. For a highly conductive liquid like water, the charging process takes very little time for charges to accumulate, whereas, for a liquid of very low conductivity (e.g. silicon oil), the charging and launching process will take a longer time to form a fully charged particle.

3.2 Particle diameter measurement
Levitated particles are viewed on a TV monitor through a Hitachi-CCTV camera. The video picture of the illuminated particle is then processed by a video image digitiser to produce a digitised picture that can be magnified and stored on a floppy disc. The aspect ratio of the image is calibrated either by viewing a 1 mm steel ball located on the bottom electrode or with a standard resolution glass plate for the calibration of microscope or video camera. The focus and the position of the camera are then fixed. The particle's diameter can be therefore determined directly from the size of the digitised picture.

Another method to determine the particle's diameter is through a video processing card [14]. The video image processing card counts the number of video scan lines making up the particle on screen and a conversion factor
from number of scan lines to actual diameter is achieved by software which can be read on the monitor. The circuit diagram of the video processing card is shown in the Appendix (Fig. 10).

3.3 Height and voltage measurement

An increase in the DC voltage applied to the bottom electrode causes the levitated particle to rise in height above the electrode. By mounting the quadrupole on a moveable platform and keeping the TV camera fixed on the bench, it is possible to determine the levitating position of the particle using a height gauge. An alternative scheme is to use the video digitiser and count the scanned lines from the top of the screen to the particle and then decide the reference height with respect to the bottom electrode. This method is typically used to measure very small height changes in accordance with the computer-controlled DC power supply.

The voltage variations on the electrode have been automated using an interface from the BBC microcomputer to the Brandenburg Model 2707 Alpha Series power supply. Software has been developed that sets the voltage to 500 V, and it is increased in steps of 100 V until the voltage reaches a maximum of 2.5 kV.

4 Experimental results

4.1 Effect of different interelectrode spacing (R_o)

The quadrupole spacing parameter R_o, as we can see from previous paper [13], plays a very important role in the electric levitation field. Experimental testing of the effect of R_o on the electric field was carried out in the VSQ levitation system. While a particle is levitated, the bar spacing is changed, and a record is made of the change of levitating height while all the other parameters remain unchanged. The VSQ system can make a change in R_o/R continuously from 1.57 to 2.33. Fig. 4 shows the experimental data of V_o against levitating height, for a particle with diameter about 465 μm, under different bar spacing parameters. The AC voltage is 3.1 kV (peak), with frequency of 50 Hz. The estimated charge-to-mass ratio from the Coulomb model is about 2.4 × 10^{-4} C/kg.

It was intended to keep one particle levitated all the time while making such measurement. However, during the manual operation of changing the bar spacing, it was possible to encounter an instability, and the particle would be lost.

Fig. 4 Dependence of levitating height against DC voltage on the bottom electrode under different bar spacings

Particle size for R_o/R = 2.33 curve is 340 μm, and for the rest is 465 μm; AC voltage amplitude = 3.1 kV, frequency = 50 Hz; numerical calculated Q/m ratio is $5.4 \times 10^{-4}$ C/kg for the particles.

Fig. 5 shows the same experiment, but with the spacing parameter plotted in the horizontal axis against the vertical normalised height, with respect to changes of DC voltages on the bottom electrode. Although it can be made possible to change the spacing over a wider range, the experiment showed that outside the range of 1.67–2.33, the levitated particle was no longer within the central zone.

Fig. 5 Levitating height against bar spacing under different DC voltage of the bottom electrode

Particle size 465 μm; AC frequency = 50 Hz; and amplitude = 3.1 kV

4.2 Dependence of levitating height on V_o

Levitation height and the applied DC voltage on the bottom electrode are directly measurable parameters. The relationship between them can be used to test the validity of the theoretical analysis and also reflects the charging efficiency of the liquid.

Fig. 6 shows the change of the levitating height against the applied DC voltage, for different particle diameters (different Q/m ratios), under fixed spacing parameter R_o/R = 1.16 in the AIQ levitation system. Variable AC
parameters, such as frequency and amplitude, are chosen for levitation of particles with different charge-to-mass ratios. The solid curves are obtained by drawing the function 1.16 in $V_{dc} - K$, and the $K$ values are chosen for best-curve fitting.

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**Fig. 6** Dependence of levitation height on the DC voltage of bottom electrode

Bar spacing $R_0/R = 1.16$; liquid is the DC-200 and $I_1$ compound liquid doped with iodide sodium; $K$ value is related to the geometry of the quadrupole and charge-to-mass ratio of the particle, but obtained here by curve fitting.

- Diameter 138 μm; $K = 3.089$; DC300/1000 cs (0.1 NaI)
- Diameter 184 μm; $K = 1.693$; DC200/1000 cs (0.01 g)
- Diameter 240 μm; $K = 4.32$; Glyc (0.05 NaI)
- Diameter 313 μm; $K = 4.74$; DC1000 cs (0.1 g)
- Diameter 626 μm; $K = 5.22$; DC1000 cs (0.1 g)

---

The small changes of levitation height with respect to the small changes in $V_{dc}$ are shown in Tables 1 and 2. In Table 1, the change of $V_{dc}$ will cause a change of levitation height $dH$; the relationship $dV_{dc}/dH$ can be used to determine the logarithm index from the analytical expression $z/R_0 = z \ln V_{dc} - K$, where $z = H + R_0$ and $R_0 = 4.5$ mm. The average index $z = 1.1432$. The AC parameters are 54 Hz/3.1 kV. The height data are obtained from a height gauge with accuracy of 0.001" (~0.25 mm). A small increase of the input of $V_{dc}$ can be made, as small as a 10 V step. The relation of $\Delta V_{dc}$ and $\Delta H$ can be used to approximate $dV_{dc}$ against $dH$, which can be derived as $dz = x R_0 V_{dc}^{-1} \frac{dV_{dc}}{dH}$ (see Section 5). For some occasions, the change of the levitation height corresponding to the change of $V_{dc}$ is so small that even the TV camera cannot detect the change.

4.3 Particle size limits

The generation and charging of a liquid particle are through an electrostatic atomisation process. A limit to the particle radius is imposed for certain charges as there is an equilibrium between the liquid surface tension that holds the particle together, whereas the surface charges repel one another, which tends to break up the liquid. The maximum limiting charge is well known as the Rayleigh limiting charge, which is only dependent on the liquid surface tension $\gamma$ and the particle radius $r_p$. Another limiting expression for $Q/m$, referred to as the Maxwell value, arises from the maximum induction charge onto a conducting particle resting on a flat plane in a uniform electric field [15]. Fig. 7 is the experimental data.

---

**Table 1:** The levitating height against small changes of $V_{dc}$ for a 500 μm particle with charge to mass ratio $8.4 \times 10^{-6} \text{C/kg (approximately)}$

<table>
<thead>
<tr>
<th>$V_{dc}$ (V)</th>
<th>$\Delta V_{dc}$</th>
<th>$H_2$</th>
<th>$dH$ (mm)</th>
<th>$z$</th>
</tr>
</thead>
<tbody>
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<td>100 605</td>
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<td>0.96</td>
<td></td>
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<tr>
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<td>1.13</td>
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<td></td>
</tr>
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</tr>
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</tr>
<tr>
<td>2000 712</td>
<td>50 720</td>
<td>0.203</td>
<td>1.806</td>
<td></td>
</tr>
</tbody>
</table>

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**Table 2:** Levitating height against small changes of $V_{dc}$ for a 620 μm particle with charge to mass ratio $5.4 \times 10^{-6} \text{C/kg (approximately)}$

<table>
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<th>$V_{dc}$ (V)</th>
<th>$\Delta V_{dc}$</th>
<th>$H_2$</th>
<th>$dH$ (mm)</th>
<th>$z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2200 606</td>
<td>10 624</td>
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<td>1.016</td>
<td></td>
</tr>
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<td>2.5 608</td>
<td>0.152</td>
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<td></td>
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<tr>
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<td>1 606</td>
<td>0.050</td>
<td>1.13</td>
<td></td>
</tr>
<tr>
<td>1500 536</td>
<td>10 555</td>
<td>0.431</td>
<td>0.96</td>
<td></td>
</tr>
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<td>5 546</td>
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<td>1.13</td>
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</tr>
<tr>
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<td>1.41</td>
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</tr>
<tr>
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<td>10 476</td>
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<td></td>
</tr>
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<td>1.35</td>
<td></td>
</tr>
<tr>
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<td>0.96</td>
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</tr>
<tr>
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<td></td>
</tr>
<tr>
<td>500 318</td>
<td>2 322</td>
<td>0.101</td>
<td>1.13</td>
<td></td>
</tr>
<tr>
<td>500 318</td>
<td>1 No change</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The average index $z$ is calculated to be 1.121: AC voltage setting 56 Hz/3.1 kV

The liquids are of DC-200 silicon oil and glycerol; $Q/m$ ratios are calculated from numerical results.

---

**Fig. 7** $Q/m$ ratios of levitated particles with different diameters and liquids

The liquids are of DC-200 silicon oil and glycerol; $Q/m$ ratios are calculated from numerical results.

---

The liquids are of DC-200 silicon oil and glycerol; $Q/m$ ratios are calculated from numerical results.
4.4 Exploring the stable zone

The stable levitation zone can be explored by an experimental method. In the previous paper [13], the stability of the quadrupole levitation system was given by the stability of the Mathieu functions, which are drawn in an \( a-q \) plane. When a particle is levitated within the levitation zone, the stability of the particle is found to be dependent on the electric parameters, such as the frequency and the amplitude of the AC voltage on the bar electrodes. Suppose a particle is stably levitated, changing the frequency or the amplitude is equivalent to changing \( q \) parameter in the \( a-q \) plane. This will result in changing the particle from a stable to an unstable area.

Fig. 8 shows some typical experimental \( a-q \) parameters for a stable levitation in the \( a-q \) plane. This is a general consideration, as the formulas for calculating the parameter \( a = \frac{qQ}{\rho_0 m^2 R_0^2} \) include the axial potential \( \phi_0 \), which was obtained from numerical calculated values.

The stability of a levitated particle is influenced by air viscous forces. This can be seen from Table 3, where \( p \), the viscous damping coefficient, is calculated from the Stokes formula as follows:

\[
p = 6 \frac{\eta \rho}{\mu \rho_0} \omega
\]

or

\[
p = 4.5 \eta / \rho_0^2 \omega
\]

where the air viscosity \( \eta = 18.1 \times 10^{-6} \) (kg m⁻¹ s), \( \rho_0 \) and \( m \) are the radius and mass of the particle, respectively, and \( \omega \) is the angular frequency \( (=2\pi f) \).

Unless the frequency is very low (say about 20 Hz), or the particle is very small and light in weight, the air viscous influence can be ignored. A typical levitated particle is of DC-200 silicon oil, with diameter 300 \( \mu \)m for a frequency of 60 Hz. The \( p \) factor for this situation is only about 0.01.

Levitation of air bubbles and some light fly ash were also observed. However, because of the difficulty in determining the particle's mass and charge individually (not the mass-to-charge ratio) in the micro range, the stable areas for these kinds of particles are not considered.

5 Discussions of the results

From Fig. 4, we can see that when the bar spacing becomes wider, the same levitated particle will rise to a higher equilibrium position. Experimental observation of this phenomenon is clear. When \( R_p/R \) becomes even larger, the particle is likely to be unstable and eventually lost. The change in height, when the bar spacing is changed under constant DC voltage level, is almost linear. A conclusion can be immediately drawn that the increase of the bar spacing \( R_p/R = 1.67-2.33 \) will result in linear increase of axial electric field within a small range.

The charged particles were obtained through an electrostatic spraying from a filament of liquid formed by an electrostatic field. Conductivity will have an obvious effect on the spray frequency and particle size. Normally, for high conductive liquids, the sprayed particles are small and high in \( Q \)-to-\( m \) ratio, whereas for liquids with low conductivity, the spray frequency is lower, and the particle size is comparatively bigger (low \( Q \)-to-\( m \) ratio). In our experiments, we only need a particle to be charged so that it can be expelled by the electrostatic force to overcome the gravitational force, hence the conductivity is not so important. However, for any liquid to be sprayed by an electrostatic field, as long as it stays in contact with the electrode for a long enough time, charges can be accumulated and uniformly distributed on the surface, so the particle can be treated as a perfect conductor. Only an individual particle is needed for a levitation experiment each time. It is therefore possible to allow the particle to remain fixed to the injection needle for a considerable time to ensure it is fully charged. The ejection and levitation processes are then obtained by a slight increase of DC voltage.

Fig. 9 shows the experimental data of levitation height of a 313 \( \mu \)m diameter particle against the \( V_e \) in comparison with the Coulomb model which assumes the particle height is inversely proportional to the square root of \( V_e \) [16]. The broken line is drawn according to an analytical function \( 1.16 \ln(V_e) - 4.743 \). Actually, the levitating height of different particles under different bar spacing quadrupole systems against the DC voltage can all be
expressed by a simple relationship, as \( z/R_s = (1/c) \ln ((V_{dc}) - K) \), shown in Fig. 6. \( K \) is a geometrical constant depending on the quadrupole geometry and particle's \( Q/to-m \) ratio. From the derivation of the previous paper.

\[ \frac{Q}{m} = \frac{1}{2} \frac{1}{R_s^2} \ln \left( \frac{V_{dc}}{e} \right) - K \]

where \( K \) is a constant, and \( R_s \) is the radius of the sphere.

Fig. 9 Experimental data of the levitating height against \( V_{dc} \) for size 313 \( \mu \)m DC-200 200 est. liquid particle, in comparison with the theoretical curves.

The levitation balance in the \( z \)-axis implies that

\[ E_z = -\frac{\partial \phi}{\partial z} + eE_0 \exp \left[ \ln \left( 1 - z/R_s \right) \right] \]

Putting this into eqn. 3, we can derive that

\[ z/R_s = a \ln \left( V_{dc} \right) - K \]

where

\[ K = a \ln \left[ a (Q/m)^{-1} (R_s/c) \right] - 1 \quad (a = 1/c) \]

Table 4: Comparison of the constant \( K \) values

<table>
<thead>
<tr>
<th>Diameters (( \mu )m)</th>
<th>240</th>
<th>184</th>
<th>626</th>
<th>313</th>
<th>138</th>
</tr>
</thead>
<tbody>
<tr>
<td>Best fitted ( K )</td>
<td>4.32</td>
<td>3.693</td>
<td>5.223</td>
<td>4.743</td>
<td>3.089</td>
</tr>
<tr>
<td>Calculated ( K ) (eqn. 6)</td>
<td>3.767</td>
<td>3.26</td>
<td>4.78</td>
<td>4.28</td>
<td>2.76</td>
</tr>
<tr>
<td>( Q/to-m ) estimated from ( K )-fit</td>
<td>( 5.2 \times 10^{-4} )</td>
<td>( 8.95 \times 10^{-4} )</td>
<td>( 2.4 \times 10^{-4} )</td>
<td>( 3.62 \times 10^{-4} )</td>
<td>( 1.5 \times 10^{-3} )</td>
</tr>
<tr>
<td>Numerical ( Q/to-m ) (C kg(^{-1}))</td>
<td>( 8.4 \times 10^{-4} )</td>
<td>( 1.3 \times 10^{-4} )</td>
<td>( 3.5 \times 10^{-4} )</td>
<td>( 5.4 \times 10^{-4} )</td>
<td>( 2 \times 10^{-4} )</td>
</tr>
</tbody>
</table>

6 Conclusions

Levitations in AC quadrupole systems are very stable in an atmospheric environment, for liquid particles from a few microns up to 626 \( \mu \)m in diameter, with a \( Q/to-m \) ratio less than the Rayleigh limiting charge.

The effect of varying the bar spacing \( R_0 \) is to change the axial electric field. This occurs through the image charges of the bottom electrode and the bar electrodes. For a small range of separation parameter \( R_0/R \), from 1.67 to 2.33, the normalised levitation height has a linear relationship with the bar spacing parameter of the quadrupole.

The exponential index \( a \) (which is also the spacing parameter \( R_0/R \)) in the axial potential function of quadrupole system \( A1Q \) is experimentally determined to be 1.13, which is very close to the analytically derived value (1.16).

The \( Q/to-m \) ratios of levitated particles calculated from the best fitted \( K \) values of the experimental data are all underestimated by about 30%, compared to those obtained from the numerical value. The calculated \( K \) values are all underestimated by about 13% relative to the fitted \( K \) values.

The charging and generation of a charged levitated particle is through an electrostatic spray process from an extended filament formed by the electrostatic field. The charging time will depend on the relaxation time, which
can be expressed as $\tau = \varepsilon/\sigma$. For low conductivity (e.g. $10^{-13}$ Sm$^{-1}$), it needs more time to become a fully charged particle, whereas a liquid of conductivity of $10^{-8}$ Sm$^{-1}$ will establish an equilibrium charge in just $10^{-8}$ s.

The Coulomb method of estimating the $Q/m$ ratio and predicting the relationship of height and DC voltage is valid only for a very small range, where $V_e$ is low (<1 kV). The dependence of $V_e$ on the levitation height is a kind of exponential relationship.

The stability of the levitated particle in the quadrupole system can be expressed by the $\alpha-q$ number in the $\alpha-q$ plane, which separates the stable and unstable areas. For some particles with very small radius, or extremely lightweight particle or air bubbles, it is necessary to consider the air viscous force, which in fact extends the stable area according to the damping factor $\rho$.

The experimental data of stable levitation in the quadrupole system have all stayed in the basic stable area of the $\alpha-q$ plane. A typical levitated particle with diameter of 300 $\mu$m has a damping coefficient $\rho = 0.017$, which can be ignored, and the stability characteristics for such kinds of particle are nearly equivalent to the stability characteristics in vacuum.

7 Acknowledgments

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Component list for Fig. 10:

Integrated circuits:
IC₁, TL074CN  IC₆, ZN426  IC₉, HCT4040  
IC₂, LM1881  IC₈, TLO72CP  IC₁₀, HCT574  
IC₃, HCT4040  IC₇, LM311N  IC₁₁, HCT04  
IC₄, HCT574  IC₈, HCT4316  IC₁₂, HCT02

Resistors:
R₁, 75  R₂, 75  R₃, 75  R₄, 1.5 K  R₅, 4.7 K  R₆, 1 K  
R₇, 100  R₈, 1 M  R₉, 4.7 K  R₁₀, 4.7 K  R₁₁, 4.7 K  R₁₂, 560  
R₁₃, 4.7 K  R₁₄, 4.7 K  R₁₅, 560  R₁₆, 1 K  R₁₇, 10 K  R₁₈, 10 K  
R₁₉, 1 K  R₂₀, 680 K  R₂₁, 75  R₂₂, 75  R₂₃, 75  R₂₄, 75  
R₂₅, 75  R₂₆, 75  R₂₇, 75  R₂₈, 75  R₂₉, 75  R₃₀, 75  
R₃₁, 75  R₃₂, 75  R₃₃, 75

Capacitors:
C₁, 0.1 μF  C₂, 10 pF  C₃, 0.1 μF  
C₄, 0.1 μF  C₅, 470 pF  C₆, 0.1 μF  
C₇, 0.1 μF

Fig. 10  Circuit diagram of the video processing card