Part II: Parcel Model Corroboration

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ABSTRACT

It is common for cloud microphysical models to use a single axis length to characterize ice crystals. These methods use either the diameter of an equivalent sphere or mass–size equations in conjunction with the capacitance model to close the equations for ice vapor diffusion. Single-axis methods unnaturally constrain growth because real crystals evolve along at least two axis directions. Thus, they are unable to reproduce the simultaneous variation in mass mixing ratio, maximum dimension, and mass-weighted fall speeds. While mass–size relations can at times capture the evolution of one of these with relatively low errors, the other properties are generally under- or overpredicted by 20%–40%. Part I of this study describes an adaptive habit method that evolves two axis dimensions, allowing feedbacks between aspect ratio changes and mass mixing-ratio evolution. The adaptive habit method evolves particle habit by prognosing number and mass mixing ratios along with two axis length mixing ratios. Compared with a detailed Lagrangian bin representation of ice habit distribution evolution in a parcel framework, the bulk method reproduces the ice mass mixing ratio, mean axis lengths, and mass-weighted fall speeds generally to within less than 5% relative error for layered and deeper mixed-phase clouds.

1. Introduction

Parameterizing ice processes is complicated because observed ice crystals have a vast array of shapes. At present, it is not possible to evolve ice crystals in a mechanistic sense: Ice crystal growth depends on the manner in which water molecules adhere and then bond to an underlying surface that can be quite complicated (e.g., Libbrecht 2005). Therefore, only approximate methods for growing ice from the vapor phase are possible. Most cloud models use the capacitance method to calculate changes in crystal mass due to vapor diffusion:

$$\frac{dm}{dt} = 4\pi C(c, a) G(T, P) s_{ui}^{\phi},$$

where $C$ is the capacitance, $G$ combines the effects of thermal and vapor diffusion [Lamb and Verlinde 2011, their Eq. (8.41), p. 343], $P$ is the pressure, $T$ is the temperature, and $s_{ui}^{\phi}$ is the ice supersaturation. To first order, crystal shape can be characterized by two axis lengths based on the geometry of a hexagonal prism: The $a$-axis semidimension is the distance from the center of the hexagonal (basal) face to a corner on that face. The $c$-axis semidimension is half of the length of the rectangular (prism) face. The basic shapes of many crystals can then be characterized by the aspect ratio ($\phi = c/a$). Equation (1) is complicated because the mass and the capacitance depend in different ways on the geometry of the crystal. Therefore, some method to close the vapor diffusion equation must be used that relates the capacitance and the particle mass. Nearly all cloud models use either an equivalent volume sphere or mass–size relationships; however, some methods evolve crystals more naturally (e.g., Chen and Lamb 1994; Hashino and Tripoli 2007).

Equivalent spheres are sometimes used to approximate the habits of ice through the use of an effective particle density $\rho_e$ (e.g., Lin et al. 1983; Reisner et al. 1998; Thompson et al. 2004): Because a sphere has
a different overall volume and area than that of a real crystal, the density of the assumed particle must be reduced. Typical methods of reducing the density follow those like Fukuta and Takahashi (1999) in which a ratio is taken of the measured to the modeled crystal volume, a number less than unity, and used to modify the density. While some equivalent spherical methods account for this density variation with size (e.g., Fridlind et al. 2007), the effective particle density is assumed constant in other models (e.g., Reisner et al. 1998; Thompson et al. 2004; Morrison et al. 2009).

More complex methods use mass–size relationships to represent crystal shape (e.g., Mitchell et al. 1990; Harrington et al. 1995; Meyers et al. 1997; Woods et al. 2007; Thompson et al. 2008; Morrison and Grabowski 2008, 2010):

\[ m = a_m L^{10_m}, \]  

where \( L \) is the particle maximum dimension and requisite data are taken from in situ measurements (Locatelli and Hobbs 1974; Mitchell et al. 1990; Heymsfield et al. 2007). Crystal habit is defined through the power-law coefficients, which are constants for a given crystal type with a range of values available in the literature (Westbrook and Heymsfield 2011).

Using either spheres or mass–size relations is necessarily limited and suffers from a number of deficiencies (Sulia and Harrington 2011, hereafter SH11). In comparison to wind tunnel data, the use of equivalent spheres (\( \phi = 1 \)) underestimates ice growth at nearly every temperature except at the transitions between habits (about \(-10^\circ\) and \(-22^\circ\)C) where ice growth is nearly isometric (e.g., Fukuta and Takahashi 1999). Consequently, assuming spherical particles with constant density plays a large role in the overproduction of supercooled liquid in model simulations of mixed-phase clouds (Woods et al. 2007; Thompson et al. 2008; Avramov and Harrington 2010). A primary deficiency with Eq. (2) is that the power-law coefficients are derived from in situ data and therefore contain entire particle growth histories, including variations in temperature, supersaturation, and ventilation. The number of available coefficients leads to a large range of possible predicted water paths in cloud model simulations (e.g., Avramov and Harrington 2010). In addition, such methods are not tied to the mechanisms of growth associated with the two principle crystal axes (\( a \) and \( c \)), and therefore suffer from an inability to allow crystals to evolve in a more natural fashion. Indeed, SH11 and Westbrook and Heymsfield (2011) show that the capacitance model can produce a relatively accurate estimate of mass growth only if the aspect ratio evolves. Alas, using mass–size relations produces capacitances that are too large and growth rates that are too small along with the inability to concomitantly capture ventilation effects (Westbrook and Heymsfield 2011).

In Harrington et al. (2013, hereafter Part I), an approach for bulk ice habit evolution was developed. The method is based in the work of Chen and Lamb (1994, hereafter CL94) with modifications described in SH11. The CL94 method is summarized below as is the adaptive bulk habit method derived in Part I. The adaptive bulk habit approach is then tested in comparison to a detailed bin microphysical treatment (see SH11).

2. Single particle adaptive habit method

The bulk adaptive habit method is similar to the method described in CL94 (termed the “Fickian-distribution method” in SH11) for single particles. In that work, the classical capacitance model for vapor diffusion is combined with a mass distribution hypothesis allowing for aspect ratio evolution. As long as the capacitance and mass are known (spheroids are used here; see Part I), Eq. (1) can be integrated to estimate the mass change of the crystals over a time step. To evolve habit, a way to relate mass and axis length changes is required. CL94 do this by modifying the vapor fluxes onto the \( a \) and \( c \) axes to produce a parameterization for the change in aspect ratio (see Part I) called the mass distribution hypothesis:

\[ \frac{dc}{da} = \delta(T)\phi, \]  

where \( \delta \) is the inherent growth ratio—a ratio of the growth efficiencies (deposition coefficients) for each axis that can be determined from laboratory measurements. In this work \( \delta \) is derived from CL94 and Hashino and Tripoli (2008). Though \( \delta \) values reported in Hashino and Tripoli (2008) were derived for numerical stability in time stepping, the values have two advantages: First, the values are higher for plates and lower for columns than the CL94 best fit, producing slower planar and columnar growth. Since these values are within the range of the data in CL94 (their Fig. 3), using values from both sources provides a plausible range of solutions for the adaptive habit method. Second, the values from Hashino and Tripoli (2008) allow for the possibility of platelike crystals (as laboratory data suggest; Bailey and Hallett 2009) at temperatures below \(-22^\circ\)C unlike the CL94 data, which are for columns. Ice particle density, ventilation, and fall speeds follow SH11 with details provided in Part I.
Comparisons of single particle growth: Critique of current methods

The advantage of predicting two axis lengths is underscored by comparisons made among the Fickian-distribution approach including ventilation, the wind tunnel data summarized in Fukuta and Takahashi (1999), and the use of mass–size relations from the literature. In the comparisons below, crystals are simulated at a single temperature and liquid saturation for 15 min to mimic the conditions experienced by ice crystals growing in the laboratory.

Consistent with earlier studies (CL94; SH11), the Fickian-distribution method compares favorably to laboratory-measured axis lengths, mass, and fall speeds after 15 min of growth at most temperatures (Fig. 1). The model captures the strong growth, greater masses, and reduced fall speeds that occur at both $-15^\circ$C and $-6^\circ$C. Minima in mass occur at transition temperatures ($-10^\circ$ and $-22^\circ$C) where growth is isometric, leading to compact particles with higher fall speeds. While the method captures the overall pattern of growth with temperature exhibited by the laboratory data, there are regions where the errors become relatively large. The $a$-axis length is underpredicted around $-6^\circ$C, leading to aspect ratios that are too large, and the mass is overpredicted near $-12^\circ$C and between $-16^\circ$ and about $-20^\circ$C. In comparisons to data generated from a free-fall chamber, Connolly et al. (2012) also found that the CL94 method inaccurately predicted habit near $-10^\circ$ and below $-20^\circ$C with aspect ratios that are too large. These authors adjusted the inherent growth ratio to produce a better match with the experiments.

For the cases presented here, numerical tests indicate that much of the error in mass and axis length is due to the use of a deposition density $\rho_{dep}$ that CL94 derived from a separate data source (Fukuta 1969) other than the wind tunnel data used for the comparisons (summarized...
in Fukuta and Takahashi 1999). Use of the deposition densities from Fukuta and Takahashi (1999) provides better agreement with the laboratory data, including the sharper peak in mass with temperature around −15°C (not shown). Connolly et al. (2012) also used the CL94 formulation for \( \rho_{\text{dep}} \), so it is conceivable that using density values derived from the experiments of Connolly et al. (2012) might improve agreement with the model. Given the simplicity of formulation, and variation from case to case, the original form of \( \rho_{\text{dep}} \) from CL94 is used here. The ability of the Fickian-distribution method to capture the first-order evolution of mass, axis length, and fall speed using data derived from different sources proves the method flexible, yet robust. Future improvements to the method should include examining the prognosis of \( \rho_{\text{dep}} \).

Using the \( \delta \) data from CL94 as compared to that from Hashino and Tripoli (2008) produces only relatively small differences in the axis lengths, mass, and fall speed for most temperatures (Fig. 1). The largest impacts of varying \( \delta \) occur in the dendritic regime, where the equilibrium vapor pressure difference between liquid and ice maximizes, crystal densities minimize, and therefore differences in \( \delta \) are accentuated. At temperatures below −22°C, large differences in axis length appear because the growth data of CL94 predict columnar growth whereas those from Hashino and Tripoli (2008) are for platelike growth. Though the axis lengths are different for \( T < −22°C \), the mass and fall speed differences are not large, indicating these may have a small dependence on the shape of the crystal at lower temperatures. Despite differing sources of \( \delta \), the method still captures the first-order evolution of crystal habit even though the observed crystal types varied (plates, dendrites, stellar, and columns). This indicates the method might also encompass the growth of more complex ice particles if appropriate \( \delta \) values are determined.

To demonstrate the limitations of current bulk methods, power-law coefficients for Eq. (2) were taken from a number of studies, each labeled on Fig. 1 (Mitchell 1988; Harrington et al. 1995; Walko et al. 1995; Woods et al. 2007). Fall speeds were computed using formulations published for each method, when available; otherwise, the method from Part I is used with appropriate area-size relations from Mitchell (1996). Most methods produce relatively large errors at most temperatures in comparison to both the laboratory data and the Fickian-distribution model, with the largest errors occurring in the dendritic growth regime (−15°C). Maximum semi-axis lengths and mass are overpredicted for dendritic and stellar mass–size relations across the entire platelike growth regime (−9°C to −22°C); however, Woods et al. (2007) and hexagonal plate crystals capture these more accurately. All of the mass–size relations tend to overpredict the fall speeds for dendritic crystals—a result that matches those from Westbrook and Heymsfield (2011). Variation across a given habit regime (e.g., platelike from −10°C to −22°C) is typically not captured because one habit type is usually chosen for a given temperature regime. Abrupt changes occur at boundaries between habits because of the discontinuity in available mass–size coefficients.

At higher temperatures (−10°C), most of the methods produce \( c \)-axis lengths that are too short and masses and fall speeds that are too small. The mass–size relations from Woods et al. (2007) reproduce the fall speed variation above −10°C with relatively high accuracy. However, comparisons of other methods with fall speed data show larger errors at these temperatures. Wind tunnel measurements show that fall speeds minimize at temperatures where dendritic (−15°C) and needle (−6°C) growth occur but maximize where crystal growth is isometric (−10°C)—features that the mass and fall speed size methods do not capture. Though not shown here, spherical methods are unable to predict the variation in mass with temperature even if the effective density varies correctly in time (see SH11). It is worth noting that Connolly et al. (2012) attempted to interpret their laboratory measurements using bulk mass–size methods (e.g., Mitchell 1988) but found that the results were sensitive to the choices of the parameters in the mass–size relations, which is similar to the results shown here.

### 3. Bulk adaptive habit parameterization

The bulk adaptive habit method derived in Part I evolves habit by predicting the mass, number, and \( a \)- and \( c \)-axis mixing ratios (\( q_a, n, A, \) and \( C \), respectively). To avoid using a two-dimensional distribution in \( a \) and \( c \) for ice habit (e.g., Chen and Lamb 1999), the bulk adaptive habit method makes use of a historical factor to relate the \( a \) and \( c \) axes. In Part I, it was shown that

\[
c(t) = \alpha_\delta \delta_\alpha^\delta \delta(t),
\]

where \( \delta_\alpha \) is a weighted time average of the inherent growth ratio, \( \delta \), over the growth history of the particle, and \( \alpha_\delta \) depends on the initial ice particle size at the time of nucleation and on \( \delta_\alpha \). This expression allows the use of a single modified gamma distribution of the \( a \) axis, \( n(a) \), from which the \( c \) axis distribution can be determined. Crystal shape and size are then represented by the distribution characteristic axis lengths, \( a_n \) and \( c_n \). These characteristic lengths are related to the mean (i.e., \( \bar{a} = \bar{va_n} \)) through the distribution shape \( \nu \) that controls breadth. Since \( a_n \) and \( c_n \) are predicted, the value of \( \delta_\alpha \)
can be diagnosed from $a_n$ and $c_n$ at any time. While the characteristic sizes are used to predict the mean crystal habit, axis length mixing ratios are proposed for use in an Eulerian framework (see Part I). Using axis length mixing ratios or characteristic lengths are equivalent in the parcel tests presented here.

The bulk adaptive habit method uses distribution-integrated forms of Eqs. (1) and (3) for mass mixing ratio and the characteristic $a$- and $c$-axis length evolution. In Part I, the differential equation for the ice mass mixing ratio was shown to be

$$\frac{dq_i}{dt} = \frac{N_i}{\rho_a} 4\pi C(a_n, \delta_a) G_{a,j}$$

by integrating the vapor diffusion equation [Eq. (1)] over the $n(a)$. In Eq. (5), $q_i$ is the ice mass mixing ratio, $N_i$ is the ice number density, and $C(a_n, \delta_a)$ is the average capacitance. It is important to note that $\delta_a$ appears because the capacitance depends on both axis lengths. The historical relation between $a$ and $c$ given by Eq. (4) is used to write $C$ in terms of $a$ alone, thus providing the link between aspect ratio evolution and vapor diffusion (see Part I for details). Mean particle density, fall speeds, and ventilation are also appropriately treated and described in Part I.

4. Corroboration of method: Parcel simulations

A useful approach for testing the bulk habit method is to compare it with predictions from a detailed bin model with both methods included in the same Lagrangian parcel model. Certainly, parcel models suffer from a number of deficiencies (see Harrington et al. 2000; Ervens et al. 2011; SH11), and one must be careful in comparing parcel results to actual cloud scenarios. Nevertheless, they are advantageous because the growth of individual ice particles can be tracked avoiding numerical diffusion and other problems that could obfuscate the differences between the bulk and bin approaches. Since the goal here is to determine the relative accuracy of the bulk habit method for capturing mass, aspect ratio, and fall speed variation during vapor growth, the parcel method is ideal.

The bin model of SH11 is used and evolves 200 Lagrangian bins of ice and liquid particles, with the crystal aspect ratios and sizes evolving freely. Mixed-phase clouds are simulated since the Fickian-distribution approach is thought to be most valid at liquid saturation (SH11). Parcels were initiated as in SH11 (see SH11 for details) with a relative humidity of 90%, a cloud condensation nuclei distribution of 100 cm$^{-3}$, and a pressure of 950 hPa. Cloud depth and vertical motion, approximated as a sinusoid, controlled the simulation length. Cloud depth, maximum vertical motion, ice concentration, and initial parcel temperature were varied to create differing environments through which the ice habits evolved. To avoid differences associated with nucleation, ice was formed as spheres only after liquid appears and was nucleated instantaneously conforming to a gamma size spectrum ($r_{\text{average}} = 10 \mu\text{m}$). The only undetermined constant in the bulk habit method is the distribution shape, which is set to $\nu = 8$ for most simulations and was chosen because the bin model produces approximately this distribution shape. It is worth pointing out that when realistic ice nucleation is used in this model, ice spectra take on more complex shapes (see Ervens et al. 2011). Since the goal of this work is the corroboration of the vapor growth method, complicating factors associated with nucleation feedbacks and collection processes are not considered.

4.1. Comparisons for layered mixed-phase clouds

The models were configured for parcel oscillations through conditions typical of a stratiform mixed-phase cloud. Cloud depth was chosen to be 500 m, the maximum vertical motion 0.5 m s$^{-1}$, initial parcel temperature $T_i = -15^\circ\text{C}$, and ice concentration $N_i = 1 \text{ L}^{-1}$. Though particle habit characteristics vary in time ($\delta$ and $\delta_a$), the shallow cloud depth ensures that the temperature remains within the platelike growth regime. During parcel rise, both the liquid and ice water contents (LWC and IWC, respectively) increase in time (Fig. 2). The LWC reaches a maximum at cloud top and then decreases because of ice growth and adiabatic compression as the parcel descends in the downdraft. The LWC oscillations are damped in time as ice growth depletes the vapor through the Bergeron–Findeisen–Wegener process. Because cloud base is ice supersaturated, the IWC, mean axis lengths, mass-mean fall speed, and mean capacitance continually rises during the parcel oscillation. The bulk habit prediction method captures the evolution of the LWC, IWC, mean axis ratios, mass-mean fall speed, and the mean capacitance accurately. Errors relative to the bin model in LWC and IWC are never greater than 2%, and errors in the other quantities are less than 1%.

The excellent agreement between the bin and the bulk habit model is somewhat dependent on the choice of $\nu$ which, to some extent, controls the spread of the distribution. Fixing $\nu$ constrains how the crystals are distributed in size which then affects the overall mass evolution. To illustrate the sensitivity of the bulk habit method to the choice of $\nu$, simulations were conducted where $\nu$ was changed from the primary value used (8) to as low as 4 and as high as 12 (gray shaded region on
The range of results is not insignificant: the maximum differences in IWC are approximately 30% with $n = 4$ defining the lowest IWC and the highest LWC. Since the fall speed depends on the mass, it is not surprising to find a similar range of variability, as well as in the mean axis lengths and capacitance (up to about 15%). It is worth noting that a value of $n = 1$ (Marshall–Palmer distribution), which is used in many cloud models, produces the lowest IWC and the highest LWC. This result is due to the large number of small particles that continually exist in the distribution, which keeps the overall growth rate lower. Constraining distribution shape is a limitation of nearly all bulk models, and so these error ranges occur for any growth method employed. It is also worth pointing out that a value of $n = 8$, chosen based on the bin results, is not necessarily advocated by this work. When feedbacks to nucleation, aggregation, and riming occur, $n$ certainly will be different, and the value chosen should reflect the overall microphysical processes being modeled. Despite an assumed distribution shape, the bulk habit method captures the concomitant first-order evolution of IWC, two mean axis lengths, mass-mean fall speed, and the mean capacitance.

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For comparison with previous methods, two representative mass–size relations were modeled following Harrington et al. (1995). The mass–size relations for dendritic crystals from Walko et al. (1995) and the relationships used in Woods et al. (2007) were used. Other mass–size relations were also modeled, but the results were similar to those presented here. Since it is thought that ice crystals remain isometric until their maximum dimension reaches a threshold size (e.g., Heymsfield et al. 2007), spherical ice with density adjusted so masses match is assumed until the mean maximum dimension of the particles reaches 40 $\mu$m. This is also done because most mass–size relations are representative of larger crystals, although some microphysical models implicitly assume mass–size relations can be extrapolated to smaller
particle sizes [e.g., the Regional Atmospheric Modeling System (RAMS); Walko et al. 1995]. Nevertheless, this extrapolation makes only a small difference in the simulated mass mixing ratios.

The mass–size relation from Walko et al. (1995) produces IWCs and semimaximum dimensions that are too large in comparison to the bin model (Fig. 2), LWCs that are too low, and consequently a cloud that glaciates (removal of liquid due to ice growth) too rapidly. This occurs because the mean maximum semidimension is overpredicted, and the mean capacitance is too large, which leads to growth rates that are too strong. The reason for this physical behavior, as discussed in section 2b, is the fixed particle aspect ratio: Growth is always assumed to be dendritic, resulting in excessive growth rates. The mass-mean fall speed of the ice is similar to that of the bin model because the larger mean size is compensated for by increased drag.

The dendrite mass–size relation from Woods et al. (2007) produces a better match of the IWC, LWC, mean maximum semidimension, and mean capacitance. The mass-mean fall speed is overpredicted, but this may be due to the use of fall speed relations from Woods et al. (2007) that are not coupled to mass- and area-size relations. During the second parcel cycle, when ice sizes are significant and differences in the growth methods begin to manifest, the IWC and LWC diverge from the bin model results, indicating that growth is too slow despite the relatively accurate estimation of the mean capacitance. This result indicates that an accurate capacitance is insufficient for predicting growth: How the gained mass is distributed over the particle is key. When mass–size relations are used, the mass is distributed over the particle in a fashion that is determined only by the maximum semidimension rather than time-dependent crystal growth parameters.

More insight into the divergence of the results is gained from the time derivative of the mean capacitance from the bin, bulk habit method, and mass–size relation from Woods et al. (2007) (Fig. 3). The bulk habit and bin models produce \( \frac{dC}{dt} \) variation that is nearly equivalent, indicating that the average effects of habit evolution are well captured by the new approach. However, the mass–size relation does not produce the sharp maxima in \( \frac{dC}{dt} \) in the updrafts as liquid cloud base is approached (about 10 min and 55 min, where liquid supersaturation \( S_L > 1 \)). These maxima are due to large changes in aspect ratio that occur as rapid ice growth begins near liquid saturation, and at a \( T \) (about \(-16^\circ C\)) that is conducive of dendritic growth. The mass–size relations cannot capture this physical behavior because aspect ratio is implicitly constant in the capacitance.

**b. Comparisons for various temperatures and ice concentrations**

To provide a broader picture of the accuracy of the bulk habit method, a number of simulations were completed for the shallower clouds (500 m deep, 0.5 m s\(^{-1}\) maximum vertical motion). Each simulation used a different initial parcel temperature that varied from \(-5^\circ C\) to \(-20^\circ C\) and ice concentration from 0.1 to 100 L\(^{-1}\) to cover a realistic range. The oscillations over a 500-m depth led to temperature changes of approximately 4°C from cloud base to cloud top. Cloud water contents, mean axis lengths, along with mass-mean fall speed and capacitance (not shown) were averaged over four parcel oscillations for each simulation (Fig. 4). The bulk habit method captures the overall pattern of the simulation-averaged LWC, IWC, and mean axis lengths. While
LWCs are always predicted to within about 5% of the bin model results, the IWC does show some divergence from the bin results in the vicinity of $-10^\circ C$ where relative errors can approach 15%. Axis length prediction is highly accurate (within 1% or 2%) for simulations in the platelike growth regime ($-9^\circ$ to $-21^\circ$C). Ice particles forming near $-5^\circ$ and $-9^\circ$C begin growth isometrically, but are then advected to regions with different growth characteristics (columns or plates, respectively). The changes in aspect ratio that occur as particles transition between different growth regimes is more difficult to capture with a single bulk distribution, and this is the reason for the larger errors around $-5^\circ$ and $-9^\circ$C. Nevertheless, the overall pattern of water contents and particle axis lengths matches that of the bin model.

To illustrate the level of improvement that comes from predicting two axis lengths instead of a single dimension, the simulation set was repeated using the mass–size relations from Woods et al. (2007) (Fig. 5). A single mass–size relation was used based on the initial temperature. The variation of LWC and IWC with temperature and ice concentration compares best to the bin model when ice concentrations are low (<0.1 L$^{-1}$). However, for larger ice concentrations, the LWC and IWC diverge from the bin results by 15%–30%. This is particularly true for ice concentrations $>10$ L$^{-1}$, where ice growth dominates the condensed phase budget. The simulation-averaged maximum semidimension is nearly always overpredicted by more than 20%. The sharp changes are due to the use of different mass–size relations based on initial temperature. Simulations with other mass–size relations produce similar results, with water contents or mean semimaximum dimension being either too larger or too small relative to the bin model by 15%–40%.

c. Comparisons for a deeper cloud system

The bulk habit method is accurate for crystals grown within a particular habit growth regime (i.e., plate- or columnlike). However, atmospheric flows and sedimentation,
though neglected here, move ice particles to temperature regimes that differ from that of their primary habits, altering growth characteristics. To test the performance of the bulk habit method when primary habit regimes change, simulations were undertaken with two initial temperatures of \( T_i = -5^\circC \) and \( -12^\circC \), a cloud depth of 2000 m, a maximum vertical motion of 1 m s\(^{-1}\), and an ice concentration of 25 L\(^{-1}\) for the \(-5^\circC\) case and 5 L\(^{-1}\) for the \(-12^\circC\) case. A single oscillation of the parcel through an updraft and downdraft cycle was simulated.

For \( T_i = -12^\circC \), the parcel oscillation produced a temperature ranging from \(-12^\circC\) to \(-25^\circC\), so ice particles begin growth as platelike particles and are advected into a region where the growth could be columnar or platelike. Inherent growth ratios for columnlike particles for \( T < -20^\circC \) were chosen primarily to examine whether the bulk habit method could capture the change in growth characteristics in comparison to the bin model.

The bin model simulations show (Fig. 6) that the ice concentration is high enough so that the cloud begins to glaciate during the updraft part of the cycle while the IWC increases. The crystals grow as platelike particles throughout the parcel cycle with average aspect ratios of about 0.2. Even though the axis growth rates are changing in time with temperature, the bulk habit method is still able to capture the first-order evolution of the LWC, IWC, the mean axis lengths, the mass-mean fall speeds, and the mean capacitance to a relatively high degree of accuracy (errors < 2%--4%).

Simulations were also conducted with the mass–size relations for dendrites from Walko et al. (1995) and stellar crystals from Woods et al. (2007). Figure 6 shows that using mass–size relations for dendrites overpredicts IWC by over 100%, causing very rapid depletion of the available liquid. This result is not surprising given the strong mass growth for single particles indicated in Fig. 1.

**Fig. 6.** (a) Water content, (b) mean \( a \)– and \( c \)-axis length, (c) mass-mean fall speed, and (d) average capacitance evolution over 2 simulation hours with an initial parcel temperature \(-12^\circC\), ice concentration of 5 L\(^{-1}\), maximum vertical motion of 1 m s\(^{-1}\), and depth of 2000 m. LWC and mean \( a \)-axis length colored black, and IWC and \( c \)-axis length colored red. Lines are as follows: bin model (thick solid), bulk habit model (thick short dash), mass–size for dendrite (Walko et al. 1995, long dash), or stellar (Woods et al. 2007, long–short dash).
Moreover, the fall speeds are largely underpredicted because the maximum semidimension is large for the given mass, hence drag is substantial. This result indicates that in an Eulerian cloud model, the longer in-cloud residence time of these particles would cause very rapid glaciation—a result that matches the cloud resolving simulations of Avramov and Harrington (2010) for dendritic crystals. The slower growth associated with the stellar crystals leads to smaller IWC, larger LWC, and fall speeds that are better approximated than the dendritic crystals. Though the mean capacitance of the stellar crystals is similar to that predicted by the bin model, the distribution of that mass over the particle through the mass–size relation produces slower ice growth and lower IWCs.

Temperatures in the simulation beginning with $-5^\circ C$ decrease to $-18^\circ C$, and so ice particles encounter first columnlike then platelike growth regimes. The results for the bin and bulk habit method (Fig. 7) are qualitatively similar to those for $T_i = -12^\circ C$: glaciation is relatively rapid with the crystals beginning as columns and continuing to grow as columnlike particles throughout the parcel cycle. The aspect ratios of the ice remains relatively modest (around 2) given the cycling through platelike growth temperatures, increasing mass growth along the $a$-axis direction, and producing more isometric particles. The bulk habit method captures the first-order evolution of the LWC, IWC, the mean axis lengths, and the mass-mean fall speeds (not shown). The relative errors between bin and bulk are larger in comparison to the $T_i = -12^\circ C$ case with IWC underpredicted by up to 15% by the new method. This is due to an underprediction of both the mean axis lengths, and analysis shows this occurs near $-10^\circ C$. However, the mean aspect ratio of the particles (not shown) is simulated to a high degree of accuracy (<5%).

Simulations were also conducted with the mass–size relations for needles from Walko et al. (1995) and columns from Woods et al. (2007). Figure 7 shows an under-prediction in IWC and an overprediction in LWC by as much as 40%. Furthermore, the maximum in the IWC is shifted in time. The mean maximum semidimension of the particles is overpredicted in comparison to the bin model, and in the case of needles (Walko et al. 1995), by nearly 600%. Since the power in the mass–size relation is related to $\delta(T)$, it is more consistent with crystal growth theory if the exponent is changed to that of a platelike crystal as the temperature decreases below $-10^\circ C$. This approach was followed using the mass–size relations from Woods et al. (2007) while ensuring conservation of IWC. However, as Fig. 7 shows, the predicted IWC, LWC, and mean semimajor axis length do not change drastically with this modification. The maximum in the IWC is now shifted earlier in time, and the LWC is reduced, but the overall differences relative to the bin method still remain.

5. Summary, remarks, and future development

The vast majority of cloud models predict the growth and evolution of ice particles using only a single axis length. Particle length and mass are typically related to each other by either assuming an equivalent volume sphere or by using mass–size relations (e.g., Reisner et al.
1998; Fridlind et al. 2007; Thompson et al. 2004; Avramov
and Harrington 2010). Neither of these prior methods
allow aspect ratio to explicitly evolve in time, and as
comparisons with wind tunnel data show, this leads to
large errors in the prediction of ice crystal mass, axis
length, and fall speed. Chen and Lamb (1994) provide
a methodology that allows for two axis ratios to evolve in
a manner that compares favorably to laboratory data of
crystal growth at liquid saturation. A bulk adaptive habit
version of this method was developed in Part I consisting
of four prognostic variables (mass, number, and a- and
c-axis mixing ratios) that allow for aspect ratio evolution.
Parcel model simulations show that the bulk habit
method predicts the mean characteristics of habit evo-
lution accurately (typically <5% relative error) in com-
parison to a binned version (SH11). This is in contrast to
the use of mass–size relations, which can, at times, cap-
ture one property relatively accurately but at the ex-
 pense of inaccurate predictions of other properties.

Though the method presented here has numerous
advantages, it has yet to be tested for situations in which
ice crystals are exposed to varying temperatures and lower ice supersaturations. At present, the Fickian
distribution method will continue to evolve crystals
with the shape with which they began growing. This
memory effect is built into the mass distribution rela-
tionship through the aspect ratio, and it is worth pointing
out that other mass distribution relationships do exist
(e.g., Nelson and Baker 1996) that produce no memory
effect. Because of a lack of laboratory data, it is unclear
which, if any, of these mass distribution relationships
works appropriately as temperatures change. Moreover,
it is unclear whether the present method could be used
to model irregular ice that often occurs, though the robust
nature of the parameterizations suggests that this is
possible if appropriate δ values can be determined.

As SH11 point out, the habit prediction method is
capable of reproducing habit evolution at liquid satu-
rations and at T above −22°C in comparison to labora-
tory measurements. At lower ice supersaturations, the
method likely becomes less accurate because of surface
kinetic resistance. This resistance is characterized by
deposition coefficients (growth efficiencies), and is par-
cularly important at low temperatures and ice super-
saturations. Methods to correct for surface effects exist,
but nearly all use a constant deposition coefficient (e.g.,
Harrington et al. 2009) even though it varies with su-
persaturation. Methods to predict the coefficients exist
(e.g., Lamb and Chen 1995), but these have been de-
veloped for spherical particles only. Furthermore, no
detailed tests of the Chen and Lamb (1994) method have
been done at low ice supersaturations, so the accuracy of
the method, and of the capacitance model itself, is an
open question. A unified growth model that captures the
main physical processes at low and high temperatures and
ice supersaturations is needed.

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