Dimensions of Thermal Diffusion Chambers

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1. Introduction

Fitzgerald (1970) and Saxena et al. (1970) discussed the time-history of supersaturation inside a thermal diffusion chamber. In their mathematical treatment both assumed diffusion between two parallel infinite plates. This raises the question, as Fitzgerald mentioned, of just what aspect ratio (ratio of chamber diameter to chamber height) is large enough to permit this assumption.

Twomey (1963) shows that an aspect ratio of 4 was not sufficient to avoid wall effects but does not suggest a suitable value. The purpose of this note is to present some calculations relative to the minimum acceptable aspect ratio.

2. Chamber height

Before discussing the aspect ratio, the actual height of the chamber should be considered. Fitzgerald and Saxena et al., as well as Twomey, have stressed the importance of a shallow chamber and Fitzgerald points out that the time required to reach a steady state is proportional to the square of the chamber depth. The solutions of Saxena et al. provide a useful form for a time constant $\tau$, namely

$$\tau = \pi^2 k H^{-2}, \tag{1}$$

where $k$ is the thermal conductivity of the air and $H$ the height of the chamber. A slightly longer time constant is appropriate for moisture diffusion but the differences are not important here. The number of time constants needed to approach sufficiently close to final conditions depends, of course, on the initial state of the air. For instance, air of very low humidity might take about $8\tau$ to achieve equilibrium, while air introduced at saturation at the lower plate temperature will take only about $4\tau$. Choosing a value of $6\tau$ as being typical and using a value of $k=0.21$ cm$^2$ sec$^{-1}$, the time to equilibrium is about $2.9H^2$ where time is in seconds and $H$ in centimeters. Thus, a chamber 1 cm high will take $\sim 3$ sec to achieve equilibrium whereas a 2-cm chamber will take $\sim 12$ sec. The 3-inch chamber designed by Radke and Hobbs (1969) would take almost 3 min to reach equilibrium. Turbulence inside the chamber will reduce these times by increasing the effective conductivity but this turbulence is to be avoided as Saxena et al. point out. Thus, a chamber height of 1–1.5 cm seems about as high as one can safely go.

3. Aspect ratio

To determine the aspect ratio $A$, assume the chamber to be a cylinder of radius $a$ and height $H$. Assume further that the top ($z=H$) and walls ($r=a$) are held at the temperature $T=T_1$, while the surface $z=0$ is held at $T_b$. The steady-state temperature in such a chamber is given by

$$T(r,z)=T_1-2\Delta T \sum_{n=1}^{\infty} \frac{J_n(\alpha_n r)}{a \alpha_n J_1(\alpha_n a)} \sinh\frac{H-z}{\alpha_n}, \tag{2}$$

where $\Delta T=T_1-T_b$, $J_n(x)$ and $J_1(x)$ are Bessel’s functions of the first kind of order 0 and 1, and $\alpha_n$ are the positive roots of

$$J_n(\alpha_n a)=0.$$

If we confine our attention to the midpoint of the chamber ($r=0$, $z=H/2$) and write $H=2aA^{-1}$, Eq. (2) can be written for the midpoint temperature $T_m$ as

$$\frac{T_1-T_m}{\Delta T} = \sum_{n=1}^{\infty} \left( \frac{a \alpha_n J_1(\alpha_n a)}{A} \cosh\left(\frac{\alpha_n H}{A}\right) \right)^{-1}, \tag{3}$$

and the problem becomes one of finding a value of $A$ such that the right-hand side of Eq. (3) is 0.5 to sufficient accuracy.

To determine how close one should be to 0.5 we consider the case where $T_1=20^\circ C$, $T_b=17^\circ C$, and the vapor pressure is truly linear in the chamber. Here, the design midpoint temperature would be 18.50C and the nominal value of supersaturation (SS) would be 0.38%. If, however, the actual temperature at the center were 18.57C, the chamber would not be saturated at all; and if the center temperature were 18.53C, the SS in the center would only be 0.20%, a reduction of almost one-half from the design value. In fact, if the center temperature were 18.51C, the actual SS would still be about 16% too low. These temperature “errors” correspond to values of $\hat{T}$ of 0.477, 0.487 and 0.497, respectively. To be able to produce a particular SS in the chamber to within 10%, the value of $\hat{T}$ should be 0.500 ±0.002.

To investigate the effect of $A$ on $\hat{T}$, we solve Eq. (3);
the results are given in Table 1. Under the initial conditions given above the chamber would not be saturated for \(A=3\), while \(A=4\) would give SS in the center of 0.25%, about two-thirds of the designed value. Thus, even \(A=4\) is not satisfactory and \(A=5\) just barely so.

Table 2 shows the values of SS in the center for \(A=4\) and \(A=5\) for values of \(\Delta T\) in the range most commonly used. The values were computed assuming the top plate temperature was 20°C and the vapor pressure exactly linear as before. The table shows that \(A=5\) satisfies the 10% criterion for most of the \(\Delta T\)'s commonly used and also that the smaller \(\Delta T\), the more demanding is the criterion.

Since these values apply only at \(r=0\), it remains to be seen whether they can be used at sufficient distances from the center, i.e., the term \(J_0(r\alpha_n)\) has to be included. Values of \(\hat{T}\) as a function of \(r/a\) are shown in Table 3 for \(A=5\) and \(A=7.5\).

Thus, if one wishes to stretch the criterion for \(\hat{T}\) a bit, one can say an aspect ratio of 5 will give substantially the same SS out to \(r/a=0.1\), while one could go to \(r/a=0.3\) or a bit more if \(A=7.5\). In practice this means that one could use about 2.5 mm on either side of the centerline in a 1 cm high chamber where \(A=5\), and about 1 cm on either side if \(A=7.5\).

4. Discussion

The above calculations permit the formulation of design characteristics of a thermal diffusion chamber. The chamber should be between 1 and 1.5 cm high with the smaller value preferred. The aspect ratio should be no less than 5 and preferably 6 or 7 or even larger. It should be noted that one of the wall effects found in these chambers is the formation of a toroidal circulation with rising motion along the walls and sinking motion nearer the middle. A small chamber height would also tend to suppress these motions which can alter the concentration of water drops and hence affect the estimate of nuclei population.

REFERENCES


