Experiment Study and Finite Element Analysis of Single-Axis Acoustic Levitator

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Abstract. A single-axis ultrasonic levitator applied in containerless processing of materials was introduced. It could suspend steel ball with density of 7.9 g/cm\textsuperscript{3} as well as else material samples easily. The finite element model of single-axis ultrasonic levitator was developed to study the incident acoustic fields. Three-dimensional distribution of time-averaged potential is focused and the results are successful to confirm some experimental phenomena such as the movement of sample along concave reflector and the deviation of object near the reflector.

Introduction

Acoustic levitation suspends objects using acoustic radiation force produced by high intensity sound field to balance against gravitational force. It is an important technique for containerless processing \cite{1} and can simulate microgravity environment under ground-based conditions to make the object free from the container \cite{2}. This technique is very suitable for both the manufacture of highly pure materials and the research on undercooling liquids, measuring thermodynamic properties, and studying fluid dynamics of free drops and bubbles. Acoustic levitation has no special restriction on the levitated object such as its electric or magnetic properties, and no coupling with the strong heating effect such as that in electromagnetic levitation. It may find the most attractive applications in containerless processing of nonmetallic substances and low-melting metals and alloys. In the paper, a single-axis ultrasonic levitator and a finite element model for the study of velocity potential distribution are described. Then the time-averaged potential for acoustic radiation force on a small rigid sphere is calculated and to explain the experimental phenomena.

Description of Ultrasonic Levitator

The block diagram of experimental set-up is shown in Fig.1 \cite{3}. Transducer, horn, vibrating plate and reflector are arranged to be coaxial, so as to produce a large lifting force in the vertical direction. The frequency of the ultrasonic generator is adjustable from 18 kHz to 22 kHz. In order to get the larger enlargement factor of amplitude, the diameter of the horn’s carryout terminal must be smaller. But good transmission of wave at the horn tip-fluid interface requires that \(d \geq \lambda_0\), where \(\lambda_0\) is the sound wavelength in the fluid medium and \(d\) is the diameter of the tip \cite{4}. In order to solve the contradictory, a vibrating plate is attached to the tip of the horn. Considering convenience of operation, the acoustic field is taken to be open. The single-axis acoustic levitator works in resonant states in order to obtain an intense acoustic field. Research showed that the set-up could suspend steel sphere with density of 7.9 g/cm\textsuperscript{3} easily. Photograph of levitating samples is shown in Fig.2.

Finite Element Analysis of the Acoustic Field

A method to evaluate levitation force and stability of levitated spheres in various acoustic fields was developed by Barmatz and Collas\cite{5}. Their method is based on Gor’kov’s theory, which gives the time-averaged potential for acoustic radiation force on a small rigid sphere in ideal fluids:
Where \( U \) is the time-averaged potential, \( p'_{in} \) and \( v'_{in} \) are the mean square fluctuations of the incident pressure and velocity at the point where the sample with a radius of \( R_s \) is located, \( \rho_f \) and \( c \) are the density and sound speed of the fluid, respectively.

The acoustic radiation force can be described in terms of the time-averaged potential by

\[
F = -\nabla U
\]  

(2)

Eq. (1) can be transformed into

\[
U = \frac{4}{3} \pi R_s^3 \left[ \frac{2}{3} \left( \frac{p'_{in}}{3 \rho c'} - \rho \frac{v'_{in}}{2} \right) \right] = V_s \cdot U
\]  

(3)

\[
\frac{1}{2} \left( \frac{p'_{in}}{3 \rho c'} - \rho \frac{v'_{in}}{2} \right)
\]  

(4)

\( V_s \) is the volume of the sample. Then the expression of acoustic radiation force can be written as

\[
F = V_s \cdot -\nabla U = V_s \cdot F
\]  

(5)

where

\[
F = -\nabla U
\]  

(6)

The weight of the sample is \( G = V \cdot \rho g \), where \( \rho_s \) is the density of the sample, \( g \) is the gravitational acceleration. The sample can be levitated in the acoustic field demanded \( F \geq G_s \), i.e. \( F \geq \rho_s g \). Therefore it is enough to calculate \( U \) and \( F \).

In this paper, a finite element analysis model for single-axis acoustic levitators is described. The incident acoustic field is obtained by solving the Helmholtz equation through the finite element method.

The finite element model is shown in Fig.3. The incident acoustic field will exist in an infinite space. An additional bound must be given such as \( \Gamma_1 \) at which sound pressure is zero approximately. If the radius of the additional bound is large enough, its effect on acoustic field can be neglected.

The emitter is a cylinder with a height \( H_a \) and a section radius \( R_a \). Its bottom section acts as the emitting surface (denoted by \( \Gamma_e \)), which vibrates in the normal direction with an amplitude \( v_0 \) and an angle frequency \( \omega \) sinusoidally. The other surfaces of the cylinder are stationary. The reflector is a cylinder with height \( H_b \), a section radius \( R_b \), and its upper side was cut out by a spherical surface.
with radius $R$ ($R \geq R_b$). All surfaces of the reflector are stationary. A circular coordinate is applied. The vibrating cylinder and the reflector occupy the same axis of symmetry, namely, the $z$ axis, which is in the antigravity direction. The lowest point of the curved surface is the origin of the circular coordinate. The interval $H$ between the reflector and the vibrating surface is defined as the distance from the origin to the vibrating surface.

Because of the axial symmetry, $\Phi$ is not dependent on the circular coordinate $\phi$ and thus takes the form of $\Phi(\rho, z) \exp(-j\omega t)$. $\Phi$ satisfies the Helmholtz equation with boundary conditions:

$$\frac{\partial \Phi}{\partial n} |_{\Gamma_s} = 0, \quad \frac{\partial \Phi}{\partial n} |_{\Gamma_v} = 0,$$

where $k$ is the wave number, $\Gamma_s$ is the surface of the cylinders with unit outward norm $n$, $\Gamma_v$ is the vibrating surface, and $\Gamma_i$ is the additional bound.

The finite space is divided by triangular mesh. Then the Helmholtz equation can be further written as:

$$\sum_{i=1}^{Np} \int_{\Omega_i} \left[ k^2 \Phi - \nabla \Phi \cdot \nabla \Phi \right] d\rho dz = -\sum_{i=1}^{Np} \int_{\partial \Omega_i} \frac{\partial \Phi}{\partial n} d\Gamma$$

(7)

Where $NE$ is the number of triangular elements, $ei$ is the element of a partition, $\Omega$ is the whole finite region, $r_i = \partial e_i \cap \partial \Omega$, $\{\phi_i\} (i = 1, 2, ..., N_p)$ is the weight functions of element $e_i$, $N_p$ is the number of nodes. The value of $\Phi$ is obtained by solving the equations set. The incident pressure $p$ and $v$ of the incident acoustic field can be derived by the differentiation of $\Phi$ with respect to time and space, respectively. Then the time-averaged potential $U$ can be derived. Based on the distribution of $U$ with respect to space, the acoustic radiation force is determined at last.

**Results and Discussions**

When a levitator is adopted with parameters $Ra=12\text{mm}$, $Ha=10\text{mm}$, $Rb=20\text{mm}$, $Hb=20\text{mm}$, $R=36\text{mm}$, $v_0 = 2m/s$, the three dimensional distribution map of $\overline{U}$ is shown in Fig.4 corresponding to mode 1 and mode 4, respectively. The time-averaged potential minimum denotes the expected position for the levitated sample. In Fig.4 (a), there is one position for the levitated sample. At the lowest point of the curved surface in the reflector, the value of $\overline{U}$ is the maximum. Therefore the sample at the point is very unsteady. When the sample is disturbed and deviates from the point, it will get a radial force which is in the same direction of its movement and depart from the center. This can explain the phenomenon that the sample moved along the concave surface of the reflector.
In Fig. 4(b), there are 4 positions for the levitated sample. The potential well near the reflector deviates from the z axis. This can explain the phenomenon that the sample near reflector deviated from the symmetry axis, which is shown in Fig.5.

By Eq. (6) $F_z$ can be derived. Distributions of $U$ and $F_z$ along z-axis are shown in Fig.6. It can be seen that below a potential well closely, there is a maximum of $F_z$, which determines the maximum levitation force of that potential well.

Conclusions

A single-axis ultrasonic levitator was described. It could suspend steel sphere with density of $7.9 \, \text{g/cm}^3$ easily. To study the complex incident acoustic fields, a finite element analysis model was developed. This model was proved to be a useful tool for analyzing the singe-axis ultrasonic levitator and it explained the movement of sample along the concave reflector and the deviation of object near the reflector successfully.

References
