Coordinate System

The figure below is a sketch of a typical 2-D spectrum of a target galaxy, showing the coordinate system adopted for this calculation. $i$ labels columns (in the spectral direction) and $j$ labels rows (in the spatial direction).

Definitions and Notation

In general, use $f_{ij}$ to denote the flux in a given pixel and $\sigma_{ij}$ to denote the corresponding error. Then indicate the image in parenthesis. These quantities have units of flux density per unit angular size. For the normalized PSF template (after dividing by the peak) we will use $T_{ij}$ and $S_{ij}$ for the value and the error because these are dimensionless. This distinction is useful because it allows us to use the units for to check for errors in the calculations later on.

Here is the notation used for the flux density and its uncertainty in the individual pixels of various images.

<table>
<thead>
<tr>
<th>Description</th>
<th>Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>PSF star (before normalizing peaks)</td>
<td>$f_{ij}$ (psf) and $\sigma_{ij}$ (psf)</td>
</tr>
<tr>
<td>PSF star (after normalizing peaks)</td>
<td>$T_{ij}$ (psf, n) and $S_{ij}$ (psf, n)</td>
</tr>
<tr>
<td>Central-row starlight</td>
<td>$f_{ij}$ (<em>) and $\sigma_{ij}$ (</em>)</td>
</tr>
<tr>
<td>Normalization constant for starlight</td>
<td>$F$</td>
</tr>
<tr>
<td>PSF-normalized starlight</td>
<td>$f_{ij}$ (<em>) (n) and $\sigma_{ij}$ (</em>) (n)</td>
</tr>
<tr>
<td>PSF-normalized and then scaled starlight</td>
<td>$f_{ij}$ (<em>) (s) and $\sigma_{ij}$ (</em>) (s)</td>
</tr>
<tr>
<td>Observed galaxy spectrum</td>
<td>$f_{ij}$ (G) and $\sigma_{ij}$ (G)</td>
</tr>
<tr>
<td>Residual galaxy spectrum after starlight subtraction</td>
<td>$f_{ij}$ (G, r) and $\sigma_{ij}$ (G, r)</td>
</tr>
</tbody>
</table>
Step-by-Step Error Propagation

1. Normalizing and shifting of 2-D PSF star spectrum. Here we divide each row by the peak row. We denote the peak row with \( j = p \), which gives

\[
T_{ij}(\text{psf}, n) = \frac{f_{ij}(\text{psf})}{f_{ip}(\text{psf})}
\]  

(1)

This is a division so we add the fractional errors in quadrature

\[
\left[ \frac{S_{ij}(\text{psf}, n)}{T_{ij}(\text{psf}, n)} \right]^2 = \left[ \frac{\sigma_{ij}(\text{psf})}{f_{ij}(\text{psf})} \right]^2 + \left[ \frac{\sigma_{ip}(\text{psf})}{f_{ip}(\text{psf})} \right]^2
\]  

(2)

We will leave the above equation in that form because what we will need later is \( [S_{ij}(\text{psf}, n)/T_{ij}(\text{psf}, n)]^2 \)

2. Multiplication of the normalized 2-D PSF star image with the central-row starlight spectrum from the galaxy. This produces the “PSF-normalized” starlight image.

\[
f_{ij}(\star, n) = f_{ij}(\star) T_{ij}(\text{psf}, n)
\]

(3)

The fractional error bars on the two quantities on the right-hand side are added in quadrature and the expression is rearranged to give

\[
\sigma_{ij}(\star, n) = f_{ij}(\star, n) \left\{ \left[ \frac{\sigma_{ij}(\star)}{f_{ij}(\star)} \right]^2 + \left[ \frac{S_{ij}(\text{psf}, n)}{T_{ij}(\text{psf}, n)} \right]^2 \right\}^{1/2}
\]

(4)

Now, we can insert the expression for \( [S_{ij}(\text{psf}, n)/T_{ij}(\text{psf}, n)]^2 \) from equation (2) into equation (4).

\[
\sigma_{ij}(\star, n) = f_{ij}(\star, n) \left\{ \left[ \frac{\sigma_{ij}(\star)}{f_{ij}(\star)} \right]^2 + \left[ \frac{\sigma_{ij}(\text{psf})}{f_{ij}(\text{psf})} \right]^2 + \left[ \frac{\sigma_{ip}(\text{psf})}{f_{ip}(\text{psf})} \right]^2 \right\}^{1/2}
\]

(5)

3. Scaling of the “PSF-normalized” starlight image by a factor \( F \).

\[
f_{ij}(\star, s) = F f_{ij}(\star, n)
\]

(6)

The error bar is also scaled by the same factor (since \( F \) is a constant and has no error bar itself).

\[
\sigma_{ij}(\star, s) = F \sigma_{ij}(\star, n)
\]

(7)

and using equation (5) for \( \sigma_{ij}(\star, n) \), we get

\[
\sigma_{ij}(\star, s) = F f_{ij}(\star, n) \left\{ \left[ \frac{\sigma_{ij}(\star)}{f_{ij}(\star)} \right]^2 + \left[ \frac{\sigma_{ij}(\text{psf})}{f_{ij}(\text{psf})} \right]^2 + \left[ \frac{\sigma_{ip}(\text{psf})}{f_{ip}(\text{psf})} \right]^2 \right\}^{1/2}
\]

(8)
4. Subtraction of the scaled starlight image from the observed galaxy image.

\[ f_{ij}(G, r) = f_{ij}(G, r) - f_{ij}(\star, s) \]  

(9)

Since this is a subtraction, we add the *absolute* error bars in quadrature to get

\[ \sigma_{ij}^2(G, r) = \sigma_{ij}^2(G, r) + \sigma_{ij}^2(\star, s) \]  

(10)

or, after inserting equation (8) for \( \sigma_{ij}(\star, s) \),

\[
\sigma_{ij}(G, r) = \left\{ \sigma_{ij}^2(G) + F^2 f_{ij}^2(\star, n) \left\{ \left[ \frac{\sigma_{ij}(\star)}{f_{ij}(\star)} \right]^2 + \left[ \frac{\sigma_{ij}(\text{psf})}{f_{ij}(\text{psf})} \right]^2 + \left[ \frac{\sigma_{ip}(\text{psf})}{f_{ip}(\text{psf})} \right]^2 \right\} \right\}^{1/2}
\]

(11)

**Sanity Check**

We can make the simplifying assumption that the S/N of the PSF star spectrum is very high so that it has no noise at all. If we made such an assumption we would set \( \sigma_{ij}(\text{psf}) = 0 \), which would also imply that \( \sigma_{ip}(\text{psf}) = 0 \). Then equation (8) would take a simpler form:

\[
\sigma_{ij}(\star, s) = F_{ij}(\star, n) \left[ \sigma_{ij}(\star) \right] = F_{T_{ij}(\text{psf}, n)} \sigma_{ij}(\star),
\]

(12)

where in the last step we used equation (3) to substitute for \( f_{ij}(\star, n)/f_{ij}(\star) \).

If we now substitute equation (12) into equation (10) we get

\[
\sigma_{ij}(G, r) = \left[ \sigma_{ij}^2(G) + F^2 T_{ij}^2(\text{psf}, n) \sigma_{ij}^2(\star) \right]^{1/2}
\]

(13)