1. Prove that a simple lattice polygon with at least 4 vertices always has an interior diagonal, i.e. a diagonal that is entirely inside the polygon. (A diagonal is any line connecting two non-adjacent vertices.)

2. We have seen in the previous problem that any simple polygon with at least 4 vertices has at least one interior diagonal. What is the least number of internal diagonals a simple $n$–gon may have? Experiment to find the answer, and prove your guess by mathematical induction.

3. Assume that $ABC$ is a lattice triangle, such that there are no lattice points on its sides (except for its vertices), and exactly one lattice point $O$ is inside. Prove that $O$ is the centroid of the triangle $ABC$. (The centroid of a triangle is the intersection of its medians.)

4. Prove that the sum of the internal angles of any simple, not necessarily convex, $n$–gon (a polygon with $n$ vertices) is equal to $\pi(n - 2)$. 