Multi-sample structural equation models with mean structures, with special emphasis on assessing measurement invariance in cross-national research

Measurement invariance

- measurement invariance:
  “whether or not, under different conditions of observing and studying phenomena, measurement operations yield measures of the same attribute” (Horn and McArdle 1992);
- frequent lack of concern for, or inappropriate examination of, measurement invariance in cross-national research;
- this is problematic because in the absence of measurement invariance cross-national comparisons may be meaningless;
Measurement model

Let $x^g$ be a $p \times 1$ vector of observed variables in country $g$, $\xi^g$ an $m \times 1$ vector of latent variables, $\delta^g$ a $p \times 1$ vector of unique factors (errors of measurement), $\tau^g$ a $p \times 1$ vector of item intercepts, and $\Lambda^g$ a $p \times m$ matrix of factor loadings. Then

$$x^g = \tau^g + \Lambda^g \xi^g + \delta^g$$

The means part of the model is given by

$$\mu^g = \tau^g + \Lambda^g \kappa^g$$

and the covariance part is given by

$$\Sigma^g = \Lambda^g \Phi^g \Lambda^g ' + \Theta^g$$

Model identification

- covariance part:
  - the latent constructs have to be assigned a scale in which they are measured; this is done by choosing a marker item and setting its loading to one;
- means part: two possibilities
  - the intercepts of the marker items are fixed to zero (which equates the means of the latent constructs to the means of their marker variables, $\mu^g = \kappa^g$);
  - the vector of latent means is set to zero in the reference country and one intercept per factor is constrained to be equal across countries;
- although these constraints help with identifying the model, in general they are not sufficient for meaningful cross-national comparisons;
Types of invariance

• **configural invariance**: the pattern of salient and nonsalient loadings is the same across different countries;

• **metric invariance**: the scale metrics are the same across countries;
  \[ \Lambda^1 = \Lambda^2 = \ldots = \Lambda^G \]

• **scalar invariance**: in addition to the scale metrics, the item intercepts are the same across countries;
  \[ \tau^1 = \tau^2 = \ldots = \tau^G \]

Types of invariance (cont’d)

- **factor (co)variance invariance**: factor covariances and factor variances are the same across countries;
  \[ \Phi^1 = \Phi^2 = \ldots = \Phi^G \]

- **error variance invariance**: error variances are the same across countries;
  \[ \Theta^1 = \Theta^2 = \ldots = \Theta^G \]
## Linking the types of invariance required to the research objective

<table>
<thead>
<tr>
<th>Research Objective</th>
<th>Configural invariance</th>
<th>Metric invariance</th>
<th>Scalar invariance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exploring the basic structure of the construct cross-nationally</td>
<td>✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Examining structural relationships with other constructs cross-nationally</td>
<td>✓</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Conducting cross-national comparisons of means</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

### Partial measurement invariance

- for identification purposes, one item per factor has to have invariant loadings and intercepts (marker item); the marker item has to be chosen carefully;
- at least one other invariance constraint on the loadings/intercepts is necessary to ascertain whether the marker item satisfies metric/scalar invariance;
- Cheung and Rensvold (1999) proposed the factor-ratio test in which all possible pairs of items are tested for metric/scalar invariance and sets of invariant items are identified;
- an alternative is to start with the fully invariant model of a given type and relax invariance constraints based on significant modification indices, changes in alternative fit indices, and expected parameter changes;
Assessing metric invariance across different groups

- choose a marker item for each factor (e.g., based on reliability or other considerations; if it later turns out that the marker item does not satisfy metric or scalar invariance, a different marker item may have to be chosen)
- compare the configural with the full metric invariance model;
- use Bonferroni-adjusted modification indices, changes in alternative fit indices, and expected parameter changes to free loadings that are not invariant; the final model should fit as well as the configural model;
- cross-validate the model, if possible;

Assessing scalar invariance across different groups

- compare the final (partial) metric invariance model with the full (or initial) scalar invariance model;
- use Bonferroni-adjusted modification indices, changes in fit indices, and expected parameter changes to free item intercepts that are not invariant; the final model should fit as well as the final (partial) metric invariance model;
- cross-validate the model, if possible;
Figure 1: Proposed Procedure for Assessing Measurement Invariance in Cross-National Consumer Research

{Flowchart description}

Note: If the researcher is not interested in comparing means across countries, tests of scalar invariance can be omitted and the analysis proceeds from assessing metric invariance to investigating factor covariance invariance.
Illustration:

393 Austrian and 1181 U.S. respondents completed the Satisfaction with Life Scale (SWLS; Diener et al. 1985), which is a well-known instrument used to assess the cognitive component of subjective well-being. The scale consists of the following five items:

1. In most ways my life is close to my ideal.
2. The conditions of my life are excellent.
3. I am satisfied with my life.
4. So far I have gotten the important things I want in life.
5. If I could live my life over, I would change almost nothing.

Respondents indicated their agreement or disagreement with these statements using the following five-point scale: 1 = strongly disagree, 2 = disagree, 3 = neither agree nor disagree, 4 = agree, and 5 = strongly agree.

Perform an analysis of measurement invariance on the SWLS and test whether Austrian or American respondents are more satisfied with their lives (if possible).
### Model Comparisons for Life Satisfaction (Austria vs. U.S.)

<table>
<thead>
<tr>
<th></th>
<th>$\chi^2$ value</th>
<th>df</th>
<th>RMSEA</th>
<th>CAIC</th>
<th>CFI</th>
<th>TLI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Configural invariance</td>
<td>56.94</td>
<td>10</td>
<td>.078</td>
<td>308.50</td>
<td>.991</td>
<td>.982</td>
</tr>
<tr>
<td>Full metric invariance</td>
<td>81.38</td>
<td>14</td>
<td>.080</td>
<td>302.40</td>
<td>.987</td>
<td>.982</td>
</tr>
<tr>
<td>Final partial metric invariance</td>
<td>70.22</td>
<td>13</td>
<td>.076</td>
<td>296.99</td>
<td>.989</td>
<td>.984</td>
</tr>
<tr>
<td>Initial partial scalar invariance</td>
<td>71.50</td>
<td>16</td>
<td>.067</td>
<td>273.16</td>
<td>.990</td>
<td>.987</td>
</tr>
<tr>
<td>Mean invariance</td>
<td>240.84</td>
<td>17</td>
<td>.123</td>
<td>411.93</td>
<td>.958</td>
<td>.951</td>
</tr>
</tbody>
</table>

### Results: Final partial scalar invariance model

<table>
<thead>
<tr>
<th></th>
<th>Factor loadings</th>
<th>Item intercepts</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AUT</td>
<td>US</td>
</tr>
<tr>
<td>ls1</td>
<td>.92</td>
<td>.92</td>
</tr>
<tr>
<td>ls2</td>
<td>.90</td>
<td>.90</td>
</tr>
<tr>
<td>ls3</td>
<td>1.00</td>
<td>1.00</td>
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<tr>
<td>ls4</td>
<td>.80</td>
<td>.80</td>
</tr>
<tr>
<td>ls5</td>
<td>1.10</td>
<td>.83</td>
</tr>
<tr>
<td>Latent means</td>
<td>AUT: 3.91</td>
<td>US: 3.26</td>
</tr>
</tbody>
</table>
SIMPLIS specification of final scalar invariance model:

ANALYSIS of LIFE SATISFACTION (FINAL PARTIAL SCALAR INVARINACE)

GROUP: AUSTRIA
Observed Variables: ls1 ls2 ls3 ls4 ls5
Raw data from file ls-aut.dat
Sample Size: 393
Latent Variables: LS
Relationships:
ls1 = CONST + LS
ls2 = CONST + LS
ls3 = 1*LS
ls4 = CONST + LS
ls5 = CONST + LS
LS = CONST

GROUP: USA
Observed Variables: ls1 ls2 ls3 ls4 ls5
Raw data from file ls-usa.dat
Sample Size: 1181
Latent Variables: LS
Relationships:
LS = CONST
Set the path LS -> ls5 free
Set the path CONST -> ls5 free
Set the Error Variance of ls1 free
Set the Error Variance of ls2 free
Set the Error Variance of ls3 free
Set the Error Variance of ls4 free
Set the Error Variance of ls5 free
Set the Variance of LS free
Options mi
End of Problem

GROUP: AUT

Covariance Matrix

<table>
<thead>
<tr>
<th></th>
<th>ls1</th>
<th>ls2</th>
<th>ls3</th>
<th>ls4</th>
<th>ls5</th>
</tr>
</thead>
<tbody>
<tr>
<td>ls1</td>
<td>0.72</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ls2</td>
<td>0.37</td>
<td>0.69</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ls3</td>
<td>0.42</td>
<td>0.38</td>
<td>0.66</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ls4</td>
<td>0.42</td>
<td>0.32</td>
<td>0.41</td>
<td>0.86</td>
<td></td>
</tr>
<tr>
<td>ls5</td>
<td>0.50</td>
<td>0.40</td>
<td>0.46</td>
<td>0.56</td>
<td>1.32</td>
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</tbody>
</table>

Means

<table>
<thead>
<tr>
<th></th>
<th>ls1</th>
<th>ls2</th>
<th>ls3</th>
<th>ls4</th>
<th>ls5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3.57</td>
<td>3.63</td>
<td>3.91</td>
<td>3.81</td>
<td>3.31</td>
</tr>
</tbody>
</table>
GROUP: USA

Covariance Matrix

<table>
<thead>
<tr>
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<th>ls2</th>
<th>ls3</th>
<th>ls4</th>
<th>ls5</th>
</tr>
</thead>
<tbody>
<tr>
<td>ls1</td>
<td>1.06</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>ls2</td>
<td>0.71</td>
<td>0.99</td>
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<td></td>
<td></td>
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<tr>
<td>ls3</td>
<td>0.75</td>
<td>0.78</td>
<td>1.11</td>
<td></td>
<td></td>
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<tr>
<td>ls4</td>
<td>0.59</td>
<td>0.55</td>
<td>0.67</td>
<td>1.06</td>
<td></td>
</tr>
<tr>
<td>ls5</td>
<td>0.65</td>
<td>0.59</td>
<td>0.69</td>
<td>0.60</td>
<td>1.29</td>
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</table>

Means

<table>
<thead>
<tr>
<th></th>
<th>ls1</th>
<th>ls2</th>
<th>ls3</th>
<th>ls4</th>
<th>ls5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.97</td>
<td>3.04</td>
<td>3.26</td>
<td>3.33</td>
<td>2.75</td>
</tr>
</tbody>
</table>

GROUP: AUT

Measurement Equations

\[ \text{ls1} = -0.032 + 0.92 \times \text{LS}, \text{Errorvar.} = 0.30, \text{R}_Y = 0.57 \]
\[ \text{ls2} = 0.12 + 0.90 \times \text{LS}, \text{Errorvar.} = 0.37, \text{R}_Y = 0.50 \]
\[ \text{ls3} = 1.00 \times \text{LS}, \text{Errorvar.} = 0.23, \text{R}_Y = 0.67 \]
\[ \text{ls4} = 0.72 + 0.80 \times \text{LS}, \text{Errorvar.} = 0.48, \text{R}_Y = 0.38 \]
\[ \text{ls5} = -1.00 + 1.10 \times \text{LS}, \text{Errorvar.} = 0.76, \text{R}_Y = 0.43 \]

Variances of Independent Variables

<table>
<thead>
<tr>
<th></th>
<th>LS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.46</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
</tr>
<tr>
<td></td>
<td>11.11</td>
</tr>
</tbody>
</table>
Mean Vector of Independent Variables

\[
\begin{align*}
\text{LS} & \quad \text{--------} \\
3.91 & \quad (0.04) \\
99.44 & 
\end{align*}
\]

Group Goodness of Fit Statistics

Contribution to Chi-Square = 27.44
Percentage Contribution to Chi-Square = 38.37
Root Mean Square Residual (RMR) = 0.060
Standardized RMR = 0.069
Goodness of Fit Index (GFI) = 0.97

Modification Indices and Expected Change

Modification Indices for LAMBDA-X

\[
\begin{align*}
\text{LS} & \quad \text{--------} \\
ls1 & \quad 0.07 \\
ls2 & \quad 0.02 \\
ls3 & \quad 0.00 \\
ls4 & \quad 0.32 \\
ls5 & \quad - - 
\end{align*}
\]

Expected Change for LAMBDA-X

\[
\begin{align*}
\text{LS} & \quad \text{--------} \\
ls1 & \quad 0.00 \\
ls2 & \quad 0.00 \\
ls3 & \quad 0.00 \\
ls4 & \quad 0.00 \\
ls5 & \quad - - 
\end{align*}
\]

No Non-Zero Modification Indices for PHI

The Modification Indices Suggest to Add an Error Covariance Between \(ls5\) and \(ls4\) Decrease in Chi-Square New Estimate

\[13.2 \quad 0.13 \text{ IN GROUP 1}\]

Modification Indices for THETA-DELTA

\[
\begin{array}{cccccc}
\text{ls1} & \text{ls2} & \text{ls3} & \text{ls4} & \text{ls5} \\
\text{--------} & \text{--------} & \text{--------} & \text{--------} & \text{--------} \\
ls1 & - & - & - & - \\
ls2 & 0.09 & - & - & - \\
ls3 & 0.52 & 0.64 & - & - \\
ls4 & 1.14 & 4.46 & 0.16 & - & - \\
ls5 & 0.80 & 1.83 & 5.02 & 13.20 & - & - \\
\end{array}
\]
Expected Change for THETA-DELTA

<table>
<thead>
<tr>
<th>ls1</th>
<th>ls2</th>
<th>ls3</th>
<th>ls4</th>
<th>ls5</th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td>0.01</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>-0.02</td>
<td>0.02</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0.03</td>
<td>-0.05</td>
<td>0.01</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0.03</td>
<td>-0.04</td>
<td>-0.07</td>
<td>0.13</td>
<td>-</td>
</tr>
</tbody>
</table>

Modification Indices for TAU-X

<table>
<thead>
<tr>
<th>ls1</th>
<th>ls2</th>
<th>ls3</th>
<th>ls4</th>
<th>ls5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.20</td>
<td>0.11</td>
<td>1.20</td>
<td>-</td>
</tr>
</tbody>
</table>

Expected Change for TAU-X

<table>
<thead>
<tr>
<th>ls1</th>
<th>ls2</th>
<th>ls3</th>
<th>ls4</th>
<th>ls5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.02</td>
<td>0.01</td>
<td>-0.07</td>
<td>-</td>
</tr>
</tbody>
</table>

No Non-Zero Modification Indices for KAPPA

GROUP: USA

Measurement Equations

\[ ls1 = -0.032 + 0.92*LS, \text{ Errorvar.} = 0.36, \text{ Rý} = 0.66 \]
\[ (0.083) \quad (0.023) \quad (0.019) \]
\[-0.39 \quad 39.53 \quad 18.94 \]

\[ ls2 = 0.12 + 0.90*LS, \text{ Errorvar.} = 0.31, \text{ Rý} = 0.68 \]
\[ (0.079) \quad (0.023) \quad (0.017) \]
\[ 1.55 \quad 39.67 \quad 18.32 \]

\[ ls3 = 1.00*LS, \text{ Errorvar.} = 0.27, \text{ Rý} = 0.76 \]
\[ (0.017) \]
\[ 15.63 \]

\[ ls4 = 0.72 + 0.80*LS, \text{ Errorvar.} = 0.55, \text{ Rý} = 0.49 \]
\[ (0.089) \quad (0.025) \quad (0.025) \]
\[ 8.14 \quad 31.67 \quad 21.70 \]

\[ ls5 = 0.055 + 0.83*LS, \text{ Errorvar.} = 0.72, \text{ Rý} = 0.44 \]
\[ (0.11) \quad (0.032) \quad (0.032) \]
\[ 0.51 \quad 25.84 \quad 22.11 \]
Variances of Independent Variables

\[ \text{LS} \]
\[
\begin{array}{ccc}
\text{LS} & \text{---------} & 0.84 \\
& (0.04) & 18.92 \\
\end{array}
\]

Mean Vector of Independent Variables

\[ \text{LS} \]
\[
\begin{array}{ccc}
\text{LS} & \text{---------} & 3.26 \\
& (0.03) & 108.52 \\
\end{array}
\]

Global Goodness of Fit Statistics

Degrees of Freedom = 16
Minimum Fit Function Chi-Square = 71.50 (P = 0.00)
Normal Theory Weighted Least Squares Chi-Square = 72.49 (P = 0.00)
Estimated Non-centrality Parameter (NCP) = 56.49
90 Percent Confidence Interval for NCP = (33.74 ; 86.77)

Minimum Fit Function Value = 0.045
Population Discrepancy Function Value (F0) = 0.036
90 Percent Confidence Interval for F0 = (0.021 ; 0.055)
Root Mean Square Error of Approximation (RMSEA) = 0.067
90 Percent Confidence Interval for RMSEA = (0.052 ; 0.083)
P-Value for Test of Close Fit (RMSEA < 0.05) = 0.034

Expected Cross-Validation Index (ECVI) = 0.077
90 Percent Confidence Interval for ECVI = (0.056 ; 0.090)
ECVI for Saturated Model = 0.019
ECVI for Independence Model = 3.42

Chi-Square for Independence Model with 20 Degrees of Freedom = 5363.79
Independence AIC = 5383.79
Model AIC = 120.49
Saturated AIC = 60.00
Independence CAIC = 5447.40
Model CAIC = 273.16
Saturated CAIC = 250.84

Normed Fit Index (NFI) = 0.99
Non-Normed Fit Index (NNFI) = 0.99
Parsimony Normed Fit Index (PNFI) = 0.79
Comparative Fit Index (CFI) = 0.99
Incremental Fit Index (IFI) = 0.99
Relative Fit Index (RFI) = 0.98

Critical N (CN) = 704.58
Group Goodness of Fit Statistics

Contribution to Chi-Square = 44.06
Percentage Contribution to Chi-Square = 61.63
Root Mean Square Residual (RMR) = 0.024
Standardized RMR = 0.022
Goodness of Fit Index (GFI) = 0.99

Modification Indices and Expected Change

Modification Indices for LAMBDA-X

<table>
<thead>
<tr>
<th>LS</th>
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</tr>
</thead>
<tbody>
<tr>
<td>ls1</td>
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</tr>
<tr>
<td>ls2</td>
<td>0.02</td>
</tr>
<tr>
<td>ls3</td>
<td>0.00</td>
</tr>
<tr>
<td>ls4</td>
<td>0.32</td>
</tr>
<tr>
<td>ls5</td>
<td>-</td>
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</table>

Expected Change for LAMBDA-X

<table>
<thead>
<tr>
<th>LS</th>
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</tr>
</thead>
<tbody>
<tr>
<td>ls1</td>
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</tr>
<tr>
<td>ls2</td>
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<tr>
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<tr>
<td>ls4</td>
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<tr>
<td>ls5</td>
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</table>

No Non-Zero Modification Indices for PHI

The Modification Indices Suggest to Add an Error Covariance

<table>
<thead>
<tr>
<th>Between</th>
<th>and</th>
<th>Decrease in Chi-Square</th>
<th>New Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>ls2</td>
<td>ls1</td>
<td>9.7</td>
<td>0.05 IN GROUP 2</td>
</tr>
<tr>
<td>ls3</td>
<td>ls1</td>
<td>15.6</td>
<td>-0.07 IN GROUP 2</td>
</tr>
<tr>
<td>ls3</td>
<td>ls2</td>
<td>13.0</td>
<td>0.06 IN GROUP 2</td>
</tr>
<tr>
<td>ls4</td>
<td>ls2</td>
<td>17.6</td>
<td>-0.07 IN GROUP 2</td>
</tr>
<tr>
<td>ls5</td>
<td>ls2</td>
<td>12.6</td>
<td>-0.06 IN GROUP 2</td>
</tr>
<tr>
<td>ls5</td>
<td>ls4</td>
<td>14.3</td>
<td>0.08 IN GROUP 2</td>
</tr>
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</table>

Modification Indices for THETA-DELTA

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<td>12.99</td>
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<td>ls4</td>
<td>0.32</td>
<td>17.64</td>
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<td>1.52</td>
<td>12.64</td>
<td>0.22</td>
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### Expected Change for THETA-DELTA

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</tr>
<tr>
<td>ls1</td>
<td>-</td>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ls2</td>
<td>0.05</td>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ls3</td>
<td>-0.07</td>
<td>0.06</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>ls4</td>
<td>-0.01</td>
<td>-0.07</td>
<td>0.02</td>
<td>-</td>
</tr>
<tr>
<td>ls5</td>
<td>0.02</td>
<td>-0.06</td>
<td>-0.01</td>
<td>0.08</td>
</tr>
</tbody>
</table>

### Modification Indices for TAU-X

<table>
<thead>
<tr>
<th>ls1</th>
<th>ls2</th>
<th>ls3</th>
<th>ls4</th>
<th>ls5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.00</td>
<td>0.20</td>
<td>0.11</td>
<td>1.21</td>
<td>-</td>
</tr>
</tbody>
</table>

### Expected Change for TAU-X

<table>
<thead>
<tr>
<th>ls1</th>
<th>ls2</th>
<th>ls3</th>
<th>ls4</th>
<th>ls5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.00</td>
<td>0.01</td>
<td>-0.01</td>
<td>-0.02</td>
<td>-</td>
</tr>
</tbody>
</table>

No Non-Zero Modification Indices for KAPPA

Max. Mod. Index is 17.64 for Element ( 4, 2) of THETA-DELTA in Group 2
LISREL specification of final scalar invariance model:

group 1: model for ls in AUT (PARTIAL SCALAR INVARIANCE)
DA NI=5 NO=0 MA=CM ng=2
LA
ls1 ls2 ls3 ls4 ls5
RA file=ls-aut.dat re
MO NX=5 NK=1 LX=FI PH=SY,FI TD=DI,FI TX=FI KA=FI
pa lx
111 112 0 114 115
ma lx
1 1 1 1 1
pa ph
121
pa td
131 132 133 134 135
pa tx
141 142 0 144 145
pa ka
151
LK
LS
OU se tv ad=off nd=3 mi all

group 2: model for ls in USA
DA NI=5 NO=0 MA=CM
LA
ls1 ls2 ls3 ls4 ls5
RA file=ls-usa.dat re
MO NX=5 NK=1 LX=FI PH=SY,FI TD=DI,FI TX=FI KA=FI
pa lx
111 112 0 114 215
ma lx
1 1 1 1 1
pa ph
221
pa td
231 232 233 234 235
pa tx
141 142 0 144 245
pa ka
251
LK
LS
OU se tv ad=off nd=3 mi all
group 1: model for ls in AUT (PARTIAL SCALAR INVARINACE)

LAMBDA-X

<table>
<thead>
<tr>
<th>LS</th>
<th>ls1</th>
<th>(0.023)</th>
<th>39.529</th>
<th>ls2</th>
<th>(0.023)</th>
<th>39.668</th>
<th>ls3</th>
<th>1.000</th>
<th>ls4</th>
<th>(0.025)</th>
<th>31.672</th>
<th>ls5</th>
<th>(0.079)</th>
<th>13.960</th>
</tr>
</thead>
</table>

PHI

<table>
<thead>
<tr>
<th>LS</th>
<th>0.462</th>
<th>(0.042)</th>
<th>11.112</th>
</tr>
</thead>
</table>

THETA-DELTA

<table>
<thead>
<tr>
<th>ls1</th>
<th>ls2</th>
<th>ls3</th>
<th>ls4</th>
<th>ls5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.297</td>
<td>(0.027)</td>
<td>0.367</td>
<td>(0.031)</td>
<td>0.231</td>
</tr>
</tbody>
</table>

Squared Multiple Correlations for X - Variables

<table>
<thead>
<tr>
<th>ls1</th>
<th>ls2</th>
<th>ls3</th>
<th>ls4</th>
<th>ls5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.569</td>
<td>0.502</td>
<td>0.667</td>
<td>0.378</td>
<td>0.425</td>
</tr>
</tbody>
</table>

TAU-X

<table>
<thead>
<tr>
<th>ls1</th>
<th>ls2</th>
<th>ls3</th>
<th>ls4</th>
<th>ls5</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.032</td>
<td>(0.083)</td>
<td>0.122</td>
<td>(0.079)</td>
<td>-0.389</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

KAPPA

<table>
<thead>
<tr>
<th>LS</th>
<th>3.906</th>
<th>(0.039)</th>
<th>99.435</th>
</tr>
</thead>
</table>
Group Goodness of Fit Statistics

Contribution to Chi-Square = 27.437
Percentage Contribution to Chi-Square = 38.375
Root Mean Square Residual (RMR) = 0.0600
Standardized RMR = 0.0694
Goodness of Fit Index (GFI) = 0.973

group 2: model for ls in USA

LAMBDA-X

<table>
<thead>
<tr>
<th></th>
<th>ls1</th>
<th>ls2</th>
<th>ls3</th>
<th>ls4</th>
<th>ls5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.922</td>
<td>0.896</td>
<td>1.000</td>
<td>0.797</td>
<td>0.826</td>
</tr>
<tr>
<td></td>
<td>(0.023)</td>
<td>(0.023)</td>
<td>0.000</td>
<td>(0.025)</td>
<td>(0.032)</td>
</tr>
<tr>
<td></td>
<td>39.529</td>
<td>39.668</td>
<td>31.672</td>
<td>31.672</td>
<td>25.837</td>
</tr>
</tbody>
</table>

PHI

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.838</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.044)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>18.925</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

THETA-DELTA

<table>
<thead>
<tr>
<th></th>
<th>ls1</th>
<th>ls2</th>
<th>ls3</th>
<th>ls4</th>
<th>ls5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.364</td>
<td>0.311</td>
<td>0.265</td>
<td>0.548</td>
<td>0.715</td>
</tr>
<tr>
<td></td>
<td>(0.019)</td>
<td>(0.017)</td>
<td>(0.017)</td>
<td>(0.025)</td>
<td>(0.032)</td>
</tr>
</tbody>
</table>

Squared Multiple Correlations for X - Variables

<table>
<thead>
<tr>
<th></th>
<th>ls1</th>
<th>ls2</th>
<th>ls3</th>
<th>ls4</th>
<th>ls5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.662</td>
<td>0.684</td>
<td>0.760</td>
<td>0.492</td>
<td>0.444</td>
</tr>
</tbody>
</table>

TAU-X

<table>
<thead>
<tr>
<th></th>
<th>ls1</th>
<th>ls2</th>
<th>ls3</th>
<th>ls4</th>
<th>ls5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-0.032</td>
<td>0.122</td>
<td></td>
<td>0.724</td>
<td>0.055</td>
</tr>
<tr>
<td></td>
<td>(0.083)</td>
<td>(0.079)</td>
<td>(0.089)</td>
<td>(0.108)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.389</td>
<td>1.545</td>
<td>8.139</td>
<td>0.510</td>
<td></td>
</tr>
</tbody>
</table>
Global Goodness of Fit Statistics

Degrees of Freedom = 16
Minimum Fit Function Chi-Square = 71.497 (P = 0.00)
Normal Theory Weighted Least Squares Chi-Square = 72.487 (P = 0.00)
Estimated Non-centrality Parameter (NCP) = 56.487
90 Percent Confidence Interval for NCP = (33.745 ; 86.771)

Minimum Fit Function Value = 0.0455
Population Discrepancy Function Value (F0) = 0.0359
90 Percent Confidence Interval for F0 = (0.0215 ; 0.0552)
Root Mean Square Error of Approximation (RMSEA) = 0.0670
90 Percent Confidence Interval for RMSEA = (0.0518 ; 0.0831)
P-Value for Test of Close Fit (RMSEA < 0.05) = 0.0337

Expected Cross-Validation Index (ECVI) = 0.0766
90 Percent Confidence Interval for ECVI = (0.0558 ; 0.0895)
ECVI for Saturated Model = 0.0191
ECVI for Independence Model = 3.418

Chi-Square for Independence Model with 20 Degrees of Freedom = 5363.790
  Independence AIC = 5383.790
  Model AIC = 120.487
  Saturated AIC = 60.000
  Independence CAIC = 5447.404
  Model CAIC = 273.160
  Saturated CAIC = 250.841

Normed Fit Index (NFI) = 0.987
Non-Normed Fit Index (NNFI) = 0.987
Parsimony Normed Fit Index (PNFI) = 0.789
Comparative Fit Index (CFI) = 0.990
Incremental Fit Index (IFI) = 0.990
Relative Fit Index (RFI) = 0.983
Critical N (CN) = 704.583

Group Goodness of Fit Statistics

Contribution to Chi-Square = 44.060
Percentage Contribution to Chi-Square = 61.625
Root Mean Square Residual (RMR) = 0.0241
Standardized RMR = 0.0221
Goodness of Fit Index (GFI) = 0.985
What to do when the number of items and/or factors varies across groups?

- **Unequal number of observed variables**
  - introduce imaginary observed variables;
  - specify their intercepts and loadings as zero and error variances as one;
  - adjust degrees of freedom;

- **Unequal number of latent variables**
  - introduce imaginary observed and latent variables;
  - specify latent variables to have zero means, unit variance, and no relationships with other constructs;
  - add imaginary observed variables and adjust degrees of freedom;

\[
y_1 = 3.0 + 1^* (x_1) + \varepsilon_1
\]

<table>
<thead>
<tr>
<th>Var (x_1)</th>
<th>Var (\varepsilon_1)</th>
<th>(b)</th>
<th>(r)</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>9</td>
<td>.989</td>
<td>.687</td>
</tr>
</tbody>
</table>

\[
y_2 = 3.5 + 1^* (x_2 + \delta) + \varepsilon_2
\]

<table>
<thead>
<tr>
<th>Var (x_2)</th>
<th>Var (\delta)</th>
<th>Var (\varepsilon_2)</th>
<th>(b)</th>
<th>(r)</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>0</td>
<td>9</td>
<td>.990</td>
<td>.666</td>
</tr>
<tr>
<td>36</td>
<td>0</td>
<td>9</td>
<td>.991</td>
<td>.883</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
<td>36</td>
<td>1.014</td>
<td>.415</td>
</tr>
<tr>
<td>36</td>
<td>0</td>
<td>36</td>
<td>.998</td>
<td>.686</td>
</tr>
<tr>
<td>9</td>
<td>9</td>
<td>9</td>
<td>.450</td>
<td>.447</td>
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<td>36</td>
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<td>.782</td>
<td>.782</td>
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<td>.446</td>
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<tr>
<td>36</td>
<td>9</td>
<td>36</td>
<td>.782</td>
<td>.603</td>
</tr>
</tbody>
</table>
The full multi-sample LISREL model with mean structures

\[ \eta^g = \alpha^g + B^g \eta^g + \Gamma^g \xi^g + \zeta^g \]
\[ y^g = \tau_y^g + \Lambda_y^g \eta^g + \epsilon^g \]
\[ x^g = \tau_x^g + \Lambda_x^g \xi^g + \delta^g \]

with \( E(\xi^g) = \kappa^g \)

Bagozzi, Baumgartner, and Pieters (1998)

\[ \chi^2(110)=150.51 \]
\[ RMSEA=.030 \]
\[ CFI=.94 \]
\[ TLI=.92 \]
Illustration: A full SIMPLIS model

ANALYSIS of SWB

GROUP: AUSTRIA
Observed Variables: ls1 ls2 ls3 ls4 ls5 pa na
Raw data from file swb-aut.dat
Sample Size: 393
Latent Variables: LS PA NA
Relationships:
ls1 = CONST + LS
ls2 = CONST + LS
ls3 = 1*LS
ls4 = CONST + LS
ls5 = CONST + LS
pa = CONST + 1*PA
na = CONST + 1*NA
LS = CONST + PA + NA
Set the Covariance of PA and NA free
Set the Error Variance of pa to 0
Set the Error Variance of na to 0
Set the path CONST -> PA to 0
Set the path CONST -> NA to 0

GROUP: USA
Observed Variables: ls1 ls2 ls3 ls4 ls5 pa na
Raw data from file swb-usa.dat
Sample Size: 1181
Latent Variables: LS PA NA
Relationships:
pa = CONST + 1*PA
na = CONST + 1*NA
LS = CONST + PA + NA
Set the Variance of PA free
Set the Variance of NA free
Set the Covariance of PA and NA free
Set the path LS -> ls5 free
Set the path CONST -> ls5 free
Set the Error Variance of ls1 free
Set the Error Variance of ls2 free
Set the Error Variance of ls3 free
Set the Error Variance of ls4 free
Set the Error Variance of ls5 free
Set the Error Variance of LS free
Options mi
End of Problem
Illustration: A full LISREL model

group 1: model for swb in AUT
DA NI=7 NO=0 MA=CM ng=2
LA
ls1 ls2 ls3 ls4 ls5 pa na
RA file=swb-aut.dat re
MO NX=2 NY=5 NK=2 NE=1 LX=FI LY=FI TD=DI,FI TE=DI,FR PH=SY,FR PS=SY,FR TX=FR TY=FI KA=FI AL=FR
ma lx
  1 0
  0 1
pa ly
  111 112 0 114 115
ma ly
  1 1 1 1 1
pa ph
  121 122 123
pa te
  131 132 133 134 135
pa ty
  141 142 0 144 145
pa al
  151
LK
PA NA
LE
LS
OU se tv ad=off nd=3 mi

group 2: model for swb in USA
DA NI=7 NO=0 MA=CM
LA
ls1 ls2 ls3 ls4 ls5 pa na
RA file=swb-usa.dat re
MO NX=2 NY=5 NK=2 NE=1 LX=FI LY=FI TD=DI,FI TE=DI,FR PH=SY,FR PS=SY,FR TX=FR TY=FI KA=FI AL=FR
ma lx
  1 0
  0 1
pa ly
  111 112 0 114 215
ma ly
  1 1 1 1 1
pa ph
  221 222 223
pa te
  231 232 233 234 235
pa ty
  141 142 0 144 245
pa al
  251
LK
PA NA
LE
LS
OU se tv ad=off nd=3 mi