Assignment 4, Due Feb. 10

Section 3.2: 1, 2*

Extra Problem #1*:
(i) Write the contrapositive of the statement “If $a$ is an element of $E$ and $a \neq 0$, then $a^{-1}$ is an element of $E$”.
(ii) Write the negation of your answer to part (i) of this problem.

Extra Problem #2*:
What is the difference between the two double quantified sentences $\forall x \exists y P(x, y)$ and $\exists y \forall x P(x, y)$ when $P(x, y)$ is the statement:
(i) “The person $x$ is a citizen of the country $y$”;
(ii) $x + y = 2$ where $x$ and $y$ are integers.

Extra Problem #3*:
Determine the truth value of each of the following statements. Explain your answers. Recall that $\mathbb{Z}$ denotes the set of integers and $\mathbb{R}$ denotes the set of real numbers.
(i) $(\forall x \in \mathbb{Z})[x^2 \leq 0]$.
(ii) $(\exists x \in \mathbb{Z})[x^2 \leq 0]$.
(iii) $(\forall x \in \mathbb{Z})[x < 3 \Rightarrow (x^2 < 12) \lor (x = 0)]$.
(iv) $(\forall x \in \mathbb{R})[x > 1 \Rightarrow x^2 > x]$.
(v) $(\forall x \in \mathbb{R})[(x > 0) \land (x < 1) \Rightarrow x^2 < x]$.
(vi) $(\exists x \in \mathbb{R})[(x > 0) \land (3x^3 - 5x^2 + 3x - 5 = 0)]$. 