# Smoothing, Persistence, and Hedge Fund Performance Evaluation

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**Abstract**

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**Keywords**: Hedge funds, return smoothing, performance persistence, Bayesian analysis, time-varying betas.

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**Abstract**

Hedge funds often hold illiquid assets whose true value is slowly reflected in reported returns. As a result, reported returns can become a smoothed version of true realized returns and, thus, bias the evaluation of hedge fund performance. To address this problem, we provide a Bayesian framework for the performance evaluation of managed portfolios that accounts for: return smoothing; time-variation in performance and factor loadings; and the often short-lived nature of such portfolios. Using simulated data, we estimate several restricted versions of our model and find that smoothing affects performance evaluation more than time-variation or the fact that many of these funds are short-lived. We apply the model to equity hedge funds and find that, even for these relatively liquid strategies, smoothing causes an upward bias in excess performance measures, e.g. the fund's $\alpha$, and a downward bias in risk measures. In particular, we show that a moderate level of smoothing can cause the standard OLS-$\alpha$ to over-estimate equity funds' abnormal performance by more than 1% annually.

# 1 Introduction

Hedge funds' illiquid strategies can turn true risk into fake performance by preventing economic returns–that reflect all available information–from being immediately impounded into reported returns. The manifestation of this phenomenon is often referred to as return smoothing, which may, or may not, be intentional.[1] In addition to concerns regarding the liquidity of their assets, hedge funds have time-varying risk exposures,[2] and often exist for only a relatively short time, resulting in a large set of partially overlapping returns with only a short time duration. Existing performance evaluation methods recognize the importance and interdependencies of these features, but so far do not deal with them in a comprehensive modeling framework. To address these features of hedge fund return data, and their interactions, we propose a dynamic linear model for the performance evaluation of hedge funds that simultaneously accounts for return smoothing, time-variation in factor loadings, and the funds' relatively short lived histories. The model allows us to assess which one of these features has a bigger impact on performance evaluation of hedge funds.

We apply the model to a sample of equity hedge funds from the CISDM database covering the period 1994-2005. We find that return smoothing has an economically and statistically significant effect on performance evaluation by biasing abnormal performance upward, and factor loadings downward. In particular, we show that the bias in $\alpha$ for equity hedge funds, which represent a best case scenario given their relatively higher liquidity, can be as high as 1.21% on an annualized basis. Simulation evidence shows that modeling shrinkage and dynamics contribute marginally, as opposed to smoothing, to a correct assessment of hedge funds' risks and rewards.

Our paper is closely related to Getmansky, Lo, and Makarov (2004) and, more generally, to the literature focusing on the effect of liquidity on the performance appraisal of hedge fund returns.[3] Getmansky, Lo, and Makarov (2004) show that funds in investment categories characterized by scarce liquidity are more likely to display return smoothing. Furthermore,

[1] See Asness, Krail, and Liew (2001) and Getmansky, Lo, and Makarov (2004) for illiquidity smoothing and Bollen and Pool (2006) and Pool and Bollen (2007) for fraudulent smoothing.

[2] See Mamaysky, Spiegel, and Zhang (2007) in the context of mutual funds.

[3] See also Asness, Krail, and Liew (2001), Weisman (2002), Bollen and Pool (2006) and Bollen and Whaley (2007). In particular, Bollen and Pool (2006) and Bollen and Whaley (2007) argue that smoothing can be fraudulent and propose a scheme to detect it.

they show that smoothed returns tend to load on lagged risk factors, which may enhance traditional performance measures. This implies that a regression on current risk factors alone results in downward biased betas and upward biased alphas. This intuition is also present in Asness, Krail, and Liew (2001), who estimate betas by regressing hedge fund returns on current and lagged factors, and adding up the slope coefficients. However, while addressing smoothing, this approach cannot assess the extent of it because the smoothing parameter is not identified. On the other hand, while Getmansky, Lo, and Makarov (2004) focus on smoothing by estimating a moving average model for fund returns, they do not use this information to estimate unbiased betas.

We contribute to the literature by estimating for the first time both betas and smoothing parameters simultaneously by modeling observed returns as a linear combination of current and past returns, in which the coefficients are a partition of unity. Our approach allows for current observed returns to load on current and lagged risk factor, and at the same time can handle the moving average (MA) structure of the error terms.

Most Bayesian performance evaluation for hedge funds has been conducted in an unconditional framework, i.e. assuming constant model coefficients. However, this approach is likely to yield unreliable results for two reasons. First, if expected returns and risks are time-varying, then measured abnormal performance might just reflect time-varying risk premia (Ferson and Harvey (1991) and Ferson and Schadt (1996)). Second, both alpha and betas are likely to be time-varying because hedge funds, as well as mutual funds, are dynamic trading strategies (Mamaysky, Spiegel, and Zhang (2007)). This is true even if the assets in the portfolio have constant risk factor loadings.[4] We fill this gap in the literature by modeling the alpha and the betas of the portfolio returns as stationary auto-regressive (AR) processes.

Finally, we observe that our work complements those studies that use Bayesian methods to evaluate short-lived portfolios. In the context of mutual funds, Pastor and Stambaugh (2002) improve on standard performance evaluation measures using seemingly unrelated assets. They take advantage of information contained in long lived non-benchmark assets to evaluate the performance of short lived portfolios. Building on this methodology, Busse

[4]However, the portfolio manager could pursue a constant risk profile for her portfolio by trading in securities characterized by time-varying factor loadings.

and Irvine (2006) propose a factor model with time-varying betas to study mutual fund persistence. Kosowski, Naik, and Teo (2007) apply the seemingly unrelated approach to hedge fund returns. They use as non-benchmark assets the returns on the indexes of hedge funds' strategies. As noted by Pastor and Stambaugh (2002), the use of seemingly unrelated assets must be done with care as these assets might actually lead to a deterioration of the efficiency of the estimate of $\alpha$ if they are not sufficiently related with the fund's returns. We avoid this problem by using shrinkage in order to exploit the information contained in the cross section. In particular, we let similar funds, e.g. funds in the same investment category, share similar intercept and slope coefficients.

The remainder of the paper is organized as follows. Section 2 introduces the proposed model of hedge returns and the methodology to estimate the model parameters. Section 3 presents a simulation study to compare the performance of several restricted models. Section 4 describes the data used in the empirical analysis. Section 5 reports the empirical results. Section 6 concludes the study.

# 2 Methodology

## 2.1 Model Specification

We fix the notation first. Hedge funds are indexed by $i = 1, \ldots, I$, fund categories (strategies) are indexed by $j = 1, \ldots, J$, and observations are indexed by $t = 1, \ldots, T$. The variables of interest are given by:

**$y_{it}$:** true economic returns for fund $i$;

**$x_t$:** $(K+1) \times 1$ vector of factors; note that the first element is equal to one and the remaining $K$ elements represent risky factors (typically benchmark returns);

**$\theta_{i0}, \theta_{i1}, \ldots, \theta_{iL}$:** vector of return smoothers for fund $i$; they are between zero and one and sum to one;

**$\beta_{i1}, \ldots, \beta_{iT}$:** true, time varying $\beta s$;

**$y_{it}^0$:** observed returns; weighted average of present and past returns, where the weights are given by the thetas.

Returns are generated by the following dynamic linear model:

$$y_{it} = \beta_{it}{'}x_t + \varepsilon_{it}, \quad i = 1, \ldots, I, \ t = 1, \ldots, T, \tag{1}$$

where the error terms are given by $\varepsilon_{it} \underset{iid}{\sim} N(0, \sigma^2_{\varepsilon_i})$. We can write (1) in matrix notation as follows:

$$\begin{bmatrix} y_{i1} \\ \cdot \\ \cdot \\ y_{iT} \end{bmatrix} = \begin{bmatrix} x_1' & 0 & 0 & 0 \\ 0 & x_2' & 0 & 0 \\ 0 & 0 & \ldots & 0 \\ 0 & 0 & 0 & x_T' \end{bmatrix} \begin{bmatrix} \beta_{i1} \\ \cdot \\ \cdot \\ \beta_{iT} \end{bmatrix} + \begin{bmatrix} \varepsilon_{i1} \\ \cdot \\ \cdot \\ \varepsilon_{iT} \end{bmatrix}, \tag{2}$$

or, more compactly, as

$$Y_i = X\beta_i + \varepsilon_i, \quad \varepsilon_i \sim N(0, \sigma^2_{\varepsilon_i} I_T), \tag{3}$$

where $Y_i$ is a $T \times 1$ vector, $X$ is a $T \times T(K+1)$ matrix, $\beta_i$ is a $T(K+1) \times 1$ vector, and $\varepsilon_i$ is a $T \times 1$ vector. The contribution of each fund to the total likelihood function takes a simple form:

$$p(Y_i|X, \beta_i, \sigma^2_{\varepsilon_i}) \propto \sigma^{-T}_{\varepsilon_i} exp\left\{-\frac{(Y_i - X\beta_i)'(Y_i - X\beta_i)}{2\sigma^2_{\varepsilon_i}}\right\}. \tag{4}$$

## 2.2 Return Smoothing

Information can only be impounded in a security price after the security is traded. Getmansky, Lo, and Makarov (2004) argue that, due to illiquidity, observed returns are a smoothed function of true economic returns, i.e. the returns that reflect all the available information. Ignoring the subscript $i$ indicating each fund, observed returns are given by

$$y^o_t = \theta_0 y_t + \theta_1 y_{t-1} + \cdots + \theta_L y_{t-L}, \tag{5}$$

where

$$\theta_l \in [0, 1], \quad l = 0, 1, \ldots, L \tag{6}$$

$$\sum_{i=0}^{L} \theta_i = 1 \tag{7}$$

It is clear from (5) that a Sharpe ratio (or other performance measures) calculated with smoothed returns is going to be upward biased. While the mean of $y^o_t$ is an unbiased estimate of the mean of $y_t$, the variance of $y_t^o$ is going to underestimate the variance of $y_t$. In the remainder of the paper, we follow Asness, Krail, and Liew (2001) and consider the case $L = 1$, i.e. reported returns are a weighted average of current and lagged realized returns. However, our framework can easily incorporate more complex weighting scheme. For every fund, we have

$$\begin{aligned} y_t^o &= \theta_0 \beta_t \prime x_t + \theta_1 \beta_{t-1} \prime x_{t-1} + \theta_0 \varepsilon_t + \theta_1 \varepsilon_{t-1} \\ &= \begin{bmatrix} \theta_0 & \theta_1 \end{bmatrix} \begin{bmatrix} x'_{t-1} & 0 \\ 0 & x'_t \end{bmatrix} \begin{bmatrix} \beta_t \\ \beta_{t-1} \end{bmatrix} + \varepsilon_t^o, \quad t = 2, \ldots, T \end{aligned} \tag{8}$$

where $\varepsilon_t^0 = \theta_0 \varepsilon_t + \theta_1 \varepsilon_{t-1}$. Using matrix notation, we have

$$\begin{bmatrix} y_2^o \\ \cdot \\ \cdot \\ y_T^o \end{bmatrix} = \begin{bmatrix} \theta_0 & \theta_1 & 0 & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \theta_0 & \theta_1 \end{bmatrix} \begin{bmatrix} x'_1 & 0 & 0 & 0 \\ 0 & x'_2 & 0 & 0 \\ 0 & 0 & \ldots & 0 \\ 0 & 0 & 0 & x'_T \end{bmatrix} \begin{bmatrix} \beta_1 \\ \cdot \\ \cdot \\ \beta_T \end{bmatrix} + \begin{bmatrix} \varepsilon_1^o \\ \cdot \\ \cdot \\ \varepsilon_T^o \end{bmatrix}.$$

For every fund $i = 1, \ldots, I$, we can write the above expression more compactly as

$$\begin{aligned} Y_i^o &= \Theta_i X \beta_i + \varepsilon_i^o \\ &= X_i^0 \beta_i + \varepsilon_i^o \quad \varepsilon_i \sim N(0, \sigma_{\varepsilon_i}^2 \Omega_i), \end{aligned} \tag{9}$$

where

$$[\Omega_i]_{vp} = [\Theta_i \Theta'_i]_{vp} \begin{cases} \theta_{i0}^2 + \theta_{i1}^2 & \text{if } v = p \\ \theta_{i0}\theta_{i1} & \text{if } |v - p| = 1 \\ 0 & \textit{otherwise} \end{cases}$$

As can be seen, the disturbances in the model induced by smoothing are no longer spherical. The likelihood function is now given by

$$p(Y_i^o | X_i^o, \beta_i, \sigma_{\varepsilon_i}^2, \theta_i) \propto \sigma_{\varepsilon_i}^{-T^o} |\Omega_i|^{-1/2} \exp\left\{ -\frac{(Y_i^o - X_i^o \beta_i)' \Omega_i^{-1} (Y_i^o - X_i^o \beta_i)}{2\sigma_{\varepsilon_i}^2} \right\}, \tag{10}$$

where $T^0 = T - 1$. (In general $T^0 = T - L$, where L represents the lags used in smoothing returns.)

## 2.3 Prior Distributions

Hedge funds are typically classified into categories (e.g. equity long/short) that are supposed to reflect the type of risk on which fund managers load on. We postulate that individual funds' $\alpha$ and $\beta$s are made of two components: a time-varying component, which reflects individual characteristics such as skill, or private information; and a long-run strategy component, common to all funds in a given category. Formally, the evolution of $\beta_{it}$ is given by:

$$
\begin{aligned}
\beta_{it} &= (I-\rho_i)\bar{\beta}_i + \rho_i \beta_{i,t-1} + \eta_{it}, \quad \eta_{it} \underset{iid}{\sim} N(0, \Delta_i) && (11)\\
\bar{\beta}_i &\underset{iid}{\sim} N(\beta^j_{Str}, \Sigma^j), && (12)\\
diag(\rho_i) &\underset{iid}{\sim} TN_{(-1,1)}(0, \Sigma_\rho), && (13)\\
diag(\Delta) &\underset{iid}{\sim} IG(sh, sc) && (14)\\
diag(\Sigma) &\underset{iid}{\sim} IG(sh, sc) && (15)
\end{aligned}
$$

where the index $j$ refers to one of the available $J$ investment strategies to which fund $i$ belongs. We derive the hyper-parameter $\beta^j_{Str}$ empirically with the following three-step procedure:

1. for each fund in strategy $j$, we regress fund returns on current and lagged risk factors depending on whether we are accounting for smoothing or not;

2. we add the slopes relative to the current and lagged regressor of any risk factor to obtain a factor beta;

3. we finally obtain $\beta^j_{Str}$ as the average of the funds betas.

The covariance matrix of the truncated normal distribution for the prior on $\rho$ is diagonal. Since we are mostly interested in the persistence of performance, we assign a large prior variance to the autoregressive coefficient of $\alpha$ and a small prior variance to the the autoregressive coefficient of the $\beta s$. The shape and scale hyper parameters of the Inverse Gamma distributions are chosen to convey very little information about the variances.

To complete the model we need two more prior distributions. Conditional on $\beta$, the variances of the error term in the observation equation 1 come from a inverse gamma distribution:

$$\sigma^2_{\varepsilon_i} \underset{iid}{\sim} IG(sh, sc), \tag{16}$$

where the shape and scale parameters are chosen to impound as little information as possible into the posterior distributions. The prior distribution for the smoothing parameter $\theta$ is given by

$$p(\theta) \sim Beta(a, b), \tag{17}$$

where the hyper-parameters $a$ and $b$ can be chosen to reflect information on smoothing practices available from previous studies, e.g. that of Getmansky, Lo, and Makarov (2004). In practice, simulation evidence shows that setting both parameters of the beta pdf equal to one (i.e. using a uniform) is enough to recover the true theta.

For the rest of the paper, it is convenient to write (11) in matrix notation by stacking the $\beta_{it}$s into a bigger vector. Imposing the initial condition that $\beta_{i0} = \bar{\beta}_i$, we have

$$\beta = A_1\bar{\beta} + A_2\eta, \tag{18}$$

where

$$A_1 = \begin{bmatrix} I_k \\ I_k \\ \vdots \\ I_k \end{bmatrix} \quad A_2 = \begin{bmatrix} I_k & 0 & 0 & 0 \\ \rho & I & 0 & 0 \\ \cdot & \cdot & \cdot & 0 \\ \rho^{T-1} & \cdot & \rho & I_k \end{bmatrix}.$$

It follows that the prior for the stacked vector $\beta$ is given by

$$p(\beta|\bar{\beta}, \Delta, \rho) \sim N(A_1\bar{\beta} + A_2(I_T \otimes \Delta)A_2'). \tag{19}$$

Ignoring subscripts, we summarize the prior distributions below:

$$\begin{aligned}
p(\beta|\bar{\beta}, \rho, \Delta) &\sim N(A_1\bar{\beta}, A_2(I_T \otimes \Delta)A_2'), \\
p(\bar{\beta}|\beta_{Str}, \Sigma) &\sim N(\beta_{Str}, \Sigma), \\
p(\Delta|sh, sc) &\sim IG(sh, sc), \\
p(\Sigma|sh, sc) &\sim IG(sh, sc), \\
p(\sigma_\varepsilon^2|sh, sc) &\sim IG(sh, sc), \\
p(\rho|\Sigma_\rho) &\sim TN_{(-1,1)}(0, \Sigma_\rho), \\
p(\theta|a, b) &\sim Beta(a, b).
\end{aligned}$$

## 2.4 Posterior Distributions

The posterior distributions can be derived under the hypothesis that there is not smoothing ($\theta = 1$), and under the general hypothesis of smoothing ($\theta \in [0, 1]$). In this section we report posterior distribution relative to the general case and leave the derivation of both cases in the Appendix.

Indicating the relevant data with $D_T$, the posterior distributions are given by

$$\begin{aligned}
p(\beta|\bar{\beta}, \sigma_\varepsilon^2, \Delta, \rho, \theta, D_T) &\sim N(B, V), \\
p(\bar{\beta}|\beta, \Sigma, \Delta, \rho, \theta, D_T) &\sim N(\bar{B}, \bar{V}), \\
p(\sigma_\varepsilon^2|\beta, \theta, D_T) &\sim IG(T^o/2 + sh, SSR/2 + sc), \\
p(\sigma_k^2|\{\bar{\beta}\}_i \in strat_j) &\sim IG(n_j + sh, SSR_{\sigma_k}/2 + sc), \quad k = 1, \ldots, K, \\
p(\delta_k^2|\bar{\beta}_k, \beta_k, \rho_k) &\sim IG(T/2, SSR_{\delta_k}/2), \quad k = 1, \ldots, K, \\
p(\rho_k|\beta, \bar{\beta}, \Delta) &\sim TN_{(-1,1)}(\hat{\rho}_k, \hat{\sigma}_{\rho_k}), \quad k = 1, \ldots, K, \\
p(\theta|\beta, \sigma_\varepsilon^2, D_T) &\sim Metropolis - Hastings.
\end{aligned}$$

With the exception of $\sigma_k^2$, it is implicit that all the distributions should be indexed by $i$. The posterior parameters and distributions are derived in the appendix.

Note that the smoothing parameters $\theta$ does not affect the distributions of those parameters that are up in the hierarchy. These distributions are therefore identical to those of the non-smoothing case. This happens because smoothing only affects the likelihood and not the underlying generating process.

# 3 Simulation Study

In this section we perform a controlled experiment to isolate three aspects of performance evaluation, namely smoothing, time-variation in coefficients, and short-livedness. To simplify matters, we only consider 100 funds (I=100) drawn from one category (J=1). We assume a one factor (K=1) model with time-varying coefficients (dynamic CAPM) as the data generating process. Table A.3 summarizes the characteristics of the generated data. The posterior probabilities and empirical priors are obtained as in section 2. The simplest model considered imposes three restrictions: $\theta = 1$ to ignore smoothing; $\Sigma \to \infty$ to ignore shrinkage;[5] and $\Delta = \rho = 0$ to eliminate the dynamics of the intercept and slopes. By imposing just one, two or none of these restrictions, we obtain the other 7 versions of the model for a total of 8 ($2^3$) possible combinations.

Table 2 presents our simulation results. The first four columns identify the eight estimated models by setting fields equal to one whenever a particular feature of the model is retained, e.g. model 1 is the most complete model, while model 8 is the most simplified one. For both $\alpha$ and $\beta$, we report the mean bias ($Bias$), the mean absolute error ($MAD$), and the root mean squared error ($RMSE$). Finally, the last column of the table reports the posterior probability of the models ($Post.\ Pr.$), which is estimated using a Reversible Jump MCMC algorithm following the strategy outlined in Lopes and West (2004), where a fixed number of samples are generated from the posterior density of each model using the standard MCMC algorithm and then a Reversible Jump Metropolis Hastings algorithm over the 'super model', which contains all three of the set of models of interest, is conducted by using samples from the posterior density which were previously generated. Lopes and West (2004) demonstrate that this approach is better at recovering the underlying true model, from the set of competing models, when compared with a range of standard numerical approaches for estimating the Bayes Factor (log marginal probability or posterior probability of the model) such as the Harmonic Mean estimator or Newton Raftery Estimator, see Newton and Raftery (1994), both of which tend to favor models with unnecessary complexity.

Model 1 ranks first in terms of the root mean squared error with regard to both $\alpha$ and $\beta$, although models that ignore tine-variation, or shrinkage, alone follow closely. As can be seen from the last column of Table 2, Model 1 also dominates the other models in terms of relative posterior probabilities. The most striking feature of Table 2 is the superior fit of all the models that account for smoothing, which clearly shows how smoothing can lead to under-estimating the true risks taken by hedge funds. Next, assuming a constant $\alpha$ and $\beta$ also results in a loss of fit. Finally, ignoring shrinkage only results in a minimal loss of fit. However, this is to be expected because shrinkage is more likely to improve the scale of the posterior distribution of the parameters (i.e. the posterior variance) rather than the location (i.e. posterior mean).[6]

[5] In fact we set the diagonal elements of the matrix equal to 1,000.

[6] Note the similarity with seemingly unrelated approach of Pastor and Stambaugh (2002) which is designed

# 4 Data

For our empirical application, we use data from the Center for International Securities and Derivatives Markets (CISDM) hedge fund database (formerly MAR Database). This database is divided into a performance and a fund information file which collect quantitative and qualitative information on living and defunct hedge funds, funds-of-funds, and CTAs. We use the fund information file to select those funds that are denominated as hedge funds, and to assign each fund to an investment strategy. We use the performance file to obtain monthly returns.

Hedge fund databases are characterized by several biases (see Liang (2000) and Fung and Hsieh (2000)). First, to address the incubation (or backfill) bias, we drop the first 12 observations of each fund. Next, in order to mitigate the survivorship bias, we use only data reported after December 1993, since CISDM started reporting information on defunct funds in 1994.[7] To ensure some degree of statistical accuracy we include in our analysis only funds that have at least 30 consecutive monthly observations.

From inspection of the data, we see that several funds report the same returns for many consecutive months. We thus eliminate funds that report the same return for at least three consecutive months.[8] This leaves us with a total of 2,000 hedge funds distributed across 21 categories, which we report in Table 3. The third column of the table reports the number of funds in each category, which ranges from 3 to 834. The last three columns report the minimum, median, and maximum length of the funds' return series in a given category. As can be seen, most categories have at least a fund that spans the twelve years for which we have data. Figure 2 provides further information on the time dimension of the data. The most salient feature of the bar chart is that more than 50% of the funds included in the final sample have data for less than 5 years (60 monthly observations). Only 88 funds, accounting for 4.40% of the total, have data between 11.5 and 12 years.

Table 4 presents descriptive statics on fund returns across strategies, which are codified according to Table 3. The second and third columns report the minimum and maximum return, while the last for columns report the average mean, standard deviation, skewness,

to improve the efficiency of the estimator of a fund's $\alpha$.

[7]Note that the filters are imposed exactly in this order.

[8]Precisely, we define reported returns as equal if they are within 5 basis points of each other.

and kurtosis of fund returns in the 21 categories considered. As can be seen from the second and third columns, the range of attainable returns is wide, with negative returns being more extreme than positive ones. This a clear sign that, even after the imposed filters, the final sample still suffers from the presence of aberrant observations. While returns of -100% are consistent with bankruptcy, returns in excess of 100% (on a monthly basis!) are hard to conceive. Aware of this problem, we also implement our analysis excluding funds with extreme returns.

Table 5 reports the factors used in our dynamic linear model.[9] They include four traditional buy-and-hold strategies, and three trend-following strategies constructed from option prices. The traditional factors are excess returns on the S&P 500, the monthly return on the Russell 2000 index minus the monthly return on the S&P500, the change in the 10-year treasury constant maturity yield, and the change in the Moody's Baa yield less 10-year treasury constant maturity yield. The other factors are the returns on bond, currency, and commodity trend-following strategies.

# 5 Empirical Application: Equity Funds

## 5.1 Variable Selection

To reduce the computational size of the problem, the empirical application focuses on equity hedge funds (codes 7,8, and 9), which account for approximately 50% (937/2000) of our sample. This filtering allows for yet another simplification by reducing the number of required factors to explain returns. To decide which of the seven factors to include in the model specification, we employ a fitting ($R^2$) criterion. Panel A of Table 6 reports the number of times each factor, in addition to the intercept, is selected by the best regression model. The numbering of the $\beta s$ reflects the ordering of the factors in Table 5. The first column of each panel indicates the number of factors included in the regressions. For instance, the first row says that when only one regressor is allowed in the regression, the market factor ($\beta_1$) is selected 539 times out of 937. As can be seen, the equity market factor is (uniformly) the most selected factor, followed by the size and credit spread factors respectively.

[9]These factors were originally proposed by Fung and Hsieh (2004). Details on their construction are available at `http://faculty.fuqua.duke.edu/~dah7/DataLibrary/TF-FAC.xls`.

Panel B of Table 6 reports average estimates of the betas associated with the selected factors as well as average $R^2s$. The fit ranges between 25% and 40%, which is an indication that traditional factor models for hedge fund returns are misspecified, either because more factors are needed, or probably because smoothing and time variation in the alpha and betas are not accounted for. That smoothing might be an important explanation of the poor fit of the models presented in Table 6 is evident in regressions that include lagged factors. In unreported results, we find that the $R^2$ increases by 30-50% when lagged factors are included.

## 5.2 Estimation Results

### 5.2.1 Model Comparison

In this subsection we give a graphical representation of the estimates of the most complete model (Figure 3)and compare the smoothing, persistence, and shrinkage parameter estimates of the eight possible models (Table 7). In Figure 3 we present the funds' posterior means of the smoothing and performance persistence parameters. As can be seen, most of the estimates of $\theta$ lie between 0.5 and 1 across all 3 strategies. Theoretically, if smoothing is intentional, a fund manager that wants to maximize his reported Sharpe ratio would not smooth beyond one half the fund's current realized returns since the observed returns variance is minimized at $\theta = 0.5$. The location of the smoothing parameters is between 0.8 and 0.9 and is highest for the long-only strategy, and lowest for the market-neutral strategy. This finding is consistent with the belief that the latter strategy is less liquid than the former since it might make a greater use of derivatives, which might be characterized by stale prices.

With regard to abnormal performance, we find that it displays various levels of persistence across the three investment categories. Figure 3 shows that most of the performance persistence parameters range from approximately -0.1 to +0.5 for the long-only strategy, and between -0.5 and +0.5 for the long-short and market-neutral strategies. This finding offers new insights on performance persistence studies which are typically implemented by estimating a model with constant $\alpha$ over different consecutive samples. We improve on this paradoxical approach by making persistence part of the estimation, and find that for many funds positive performance is more likely to be followed by negative performance. Notice that our framework lends itself to be used as a model of expected abnormal performance which could be used to implement fund-of-funds trading strategies, but this is beyond the scope of the present study which focuses on performance evaluation. We leave this important issue for future research.

The density plots shown so far only give information on the means of posterior distributions. In Table 7 we provide information on both the location and the scale of the posterior distributions of $\theta$ and $\rho_\alpha$ for the eight estimated models (the mdoels are ordered as in Table 2). Notice that for models that do not account for a specific feature of observed retruns (e.g. smoothing), the posterior parameters is set equal to the corresponding implicit restriction (e.g. equal to one for smoothing). As can be seen, the posterior mean (and median)of the smoothing parameter $\theta$ is consistent across the 8 models and is slightly decreasing from the long-only strategy going to the market-neutral strategy. Most importantly, the column $Pct > 0$ indicates that our estimates are also significant. This column reports the proportion of funds whose smoothing estimates is at least two standard deviation away from the value of one, which represents the null hypothesis of no-smoothing. The table therefore shows (considering model1) that more than 60% of the 937 equity funds are characterized by statistically significant smoothing. We will show that this smoothing is also economically relevant.

The correct null hypothesis for $\rho_\alpha$ is that it is equal to zero (lack performance persistence/autocorrelation). As can be seen, the proportion of significant $\rho_\alpha$s is far smaller than that obtained for $\theta$, and is decreasing over fund strategies (going from Panel A to Panel B, considering model 1). Interestingly, this proportion increases for model 6, reaching approximately 40%, which only accounts for funds dynamics. The lower significance of the persistence parameter is probably due to the lack of information, since a lot of time series observations are likely needed to appropriately asses the dynamics of a single fund.

Finally the last three columns of Table 7 show a moderate amount of shrinkage of the individual funds' $\alpha$ and $\beta's$ toward their strategy counterparts. The shrinkage can be better appreciated by looking at Figure 4, which reports a density plot of the funds' long-run alpha and betas ($\bar{\alpha}$ and $\bar{\beta}'s$) across the three equity strategies considered in the paper. Figure 4 also reveals the essentially unimodal nature of the distribution, and a lower shrinkage for the long-only strategy. With regard to the actual location of the distribution, we see that hedge funds do deliver $\alpha$ in the long run, but they also bear more risk than what is usually perceived. Even for the market-neutral equity strategy, we see that the distribution of the posterior means of the market beta is skewed to the right indicating a sensitivity to market movements.

#### 5.2.2 Impact of Smoothing on Measured Risk and Performance

In this section we report several statistics on the regression parameters estimated with both OLS and Bayesian methods. Table 8 compares OLS and Bayesian estimates under the assumption of no smoothing ($\theta = 1$). For each strategy, we report the sample mean, standard deviation, and several percentiles of the distribution of the estimated coefficients. The results are categorized by investment strategy. In addition to the $\alpha$ and $\beta$ coefficients, for completeness, the last column also reports the estimated Bayesian standard deviation of the shocks. As can be seen, udner the assumption of no smoothing, the OLS estimates and posterior means of the Bayesian estimates are virtually identical. This similarity in the estimates given by the two methods is to be expected, given the diffuseness of the priors. This similarity also indicates that our estimates under the assumption of smoothing are not driven by the use of Byesian methods.

Table 9 reports OLS and Bayesian estimates under the assumption of smoothing ($\theta \in [0, 1]$). In the first three columns the OLS estimates are obtained by regressing the hedge funds' returns on current and lagged factors and summing the two betas relative to each factor. This OLS approach, proposed by Asness, Krail, and Liew (2001), provides an approximate way of dealing with smoothing. The remaining columns of the tables report average (by strategy) of the posterior means of the alphas, betas, standard deviations, and smoothing parameters. Both the OLS and Bayesian approach give substantially higher betas than those obtained under the assumption no-smoothing indicating that smoothing induces a downward bias in the estimated risk exposures.[10] The estimated alphas are also typically lower when we account for smoothing, but the difference is in the scale of basis points and less easy to detect from the table.

In addition to the numerical evidence reported in the tables, we provide graphical evidence on the extent of smoothing and its impact on performance evaluation. Figure 5 presents a box plot with the distribution of the estimated parameter $\theta$ for the three equity strategy

[10]Note that if the true loading are negative, then smoothing actually induces an upward bias in the betas, and a downward bias in the alphas.

considered in this subsection (model5). The boxplots, and Table 9, indicate that the median smoothing is between 0.90, 0.86, and 0.75 for the Long Only, Long/Short, and Market Neutral equity groups respectively. We postulate that equity hedge funds are among those funds less affected by smoothing, and, therefore, 0.90 can be considered an upper bound on the median $\theta$ that we can expect to find in other strategy groups. Hence, we are inclined to state that smoothing is pervasive phenomenon in the hedge fund industry.

Next, we asses the impact of smoothing on performance evaluation. In Figure 6, we present scatter plots of the difference between the standard OLS (from model 1) and the Bayesian estimates of $\alpha$ and the $\beta$s (from model 5) against smoothing ($\theta$). In the construction of the scatter plots (and the fitted line), we drop funds with an estimated $\theta$ in excess of the $20^{th}$ percentile of the strategy group to which it belongs. Furthermore, to reduce the influence of outliers, the fitted lines are fit to the data after eliminating observations with a studentized residual in excess of 3. With regard to the effect on abnormal performance, the plots show that the decrease in alpha going from $\theta = 1$ to the lowest $\theta$ in the range is approximately 10 basis point, which is equivalent to an annualized abnormal performance difference of 1.21%. More precisely, a standard deviation change in $\theta$ generates a bias of approximately 5, 2, and 1 basis points for the first, second, and third strategy respectively. This shows that smoothing has an economic impact as well as statistical.[11]

Like abnormal performance, a correct assessment of the funds' risk exposures is an important aspect of the investment decision process. With regard to the loading on the market factor, Figure 6 shows that accounting for smoothing yields substantially higher estimates of the factor loading, with the discrepancy in beta going from approximately zero up to 0.20 as $\theta$ decreases. The same pattern emerges for the loading on the size spread factor. Probably, the effect of smoothing is less strong for this factor, reflecting the fact that the size spread factor accounts for less variation in hedge funds' returns than does the market factor.

# 6 Conclusion

When the underlying real hedge fund returns are smoothed, due either to illiquidity or outright fraudulent behavior, this smoothing can lead to biases in performance evaluation studies

[11]The economic significance is obtained by multiplying the standard deviation of $\theta$ by the slope coefficient.

which are both statistically and economically significant. In the cases that we considered, i.e. equity hedge funds, we find that, even for these relatively liquid strategies, smoothing causes an upward bias in excess performance measures, e.g. the fund's $\alpha$, and a downward bias in risk measures. In particular, we show that a moderate level of smoothing can cause the standard OLS $\alpha$ to over-estimate equity funds' abnormal performance by more than 1% annually. In addition, we find that ignoring the dynamic aspect of hedge funds' trading strategies, and the similarity of hedge funds in a given category, is potentially important. However, these effects are dwarfed by the effect of ignoring return smoothing. Finally, we anticipate that the findings from this paper represent a lower bound of the impact of smoothing on hedge fund performance evaluation for the rest of the hedge fund industry as we expect the effect of smoothing to be more pronounced for more illiquid investment strategies because of the discretion and difficulty that managers have in marking to market their positions.

# A Appendix

## A.1 Simulation example: unconditional CAPM

This example is based on the unconditional CAPM model and shows what can go wrong if we ignore return smoothing. Consider what happens when reported returns are a weighted average of current and past true economic returns. Excess returns for fund $i$ at time $t$ are generated by the model

$$y_{it} = \alpha_i + \beta_i' x_t + \varepsilon_{it},$$

where $y_{it}$ is the fund's excess return and $x_t$ is the excess return on the market. The parameters $\beta_i$ and $\alpha_i$ represent systematic risk and abnormal performance respectively. We generate returns for 2000 funds and set $T = 60$ for all of them. We set $\alpha_i = 0$ for all funds and draw the $\beta_i$ from a uniform distribution with support equal to $[0, 2]$. For each fund we draw the return smoother $\theta$ from a uniform distribution so that some funds smooth returns more than other. We regress $y_t^o = \theta_i y_t + (1 - \theta_i) y_{t-1}$ on a constant and $x_t$ using OLS. We report the bias for both estimated parameters as a function of $\theta$.

As shown by Figure 1 return smoothing inflates estimated measures of risk-adjusted performance ($\alpha$) while deflating estimated measures of risk ($\beta$).

## A.2 Posterior Distributions: base case ($\theta = 1$)

$(i)$ $p(\beta|\bar{\beta}, \sigma^2, \rho\Delta, D_T)$: using the likelihood in (4) and the prior in (18), we have

$$\begin{aligned} p(\beta|\bar{\beta}, \sigma_\varepsilon^2, \rho, \Delta, D_T) &\sim N(X\beta, \sigma_\varepsilon^2 I_T) \times N(A_1\bar{\beta}, A_2(I_T \otimes \Delta)A_2') \\ &\sim N(B, V), \end{aligned} \tag{20}$$

where,

$$\begin{aligned} B &= V \times \left(X'Y/\sigma_\varepsilon^2 + A_2(I_T \otimes \Delta)A_2')^{-1} A_1 \bar{\beta}\right) \\ V &= \left(X'X/\sigma_\varepsilon^2 + A_2(I_T \otimes \Delta)A_2')^{-1}\right)^{-1}. \end{aligned}$$

$(ii)$ $p(\bar{\beta}|\beta, \Sigma, \Delta, \rho, D_T)$: plug expression (18) into (3) to get

$$\begin{aligned} Y &= XA_1\bar{\beta} + XA_1\eta + \varepsilon \\ Y &= X^*\bar{\beta} + \varepsilon^*, \quad \varepsilon^* \sim N(0, Q), \end{aligned}$$

where $Q = [\sigma_\varepsilon^2 I_T + X A_2 (I_T \otimes \Delta) A_2' X']$. Using the prior for $\bar{\beta}$, we have

$$\begin{aligned} p(\bar{\beta}|\beta, \Sigma, \Delta, \rho, D_T) &\propto exp\left\{ -\frac{1}{2}(\bar{\beta} - \beta_{Str})'\Sigma^{-1}(\bar{\beta} - \beta_{Str}) \right\} \\ &\quad \times exp\left\{ -\frac{1}{2}(Y - X^*\bar{\beta})' Q^{-1} (Y - X^*\bar{\beta}) \right\} \\ &\propto N(\bar{B}, \hat{V}), \end{aligned} \tag{21}$$

where, $\bar{B} = \bar{V} \times (\Sigma^{-1}\beta_{Str} + X^{*\prime} Q^{-1} Y)$ and $\bar{V} = (\Sigma^{-1} + X^{*\prime} Q^{-1} X^*)^{-1}$.

(*iii*) $p(\sigma_\varepsilon^2|\beta, D_T)$: using the IG prior for $\sigma_\varepsilon^2$, we have

$$\begin{aligned} p(\sigma_\varepsilon^2|\beta, D_T) &\sim p(\sigma_\varepsilon^2|\beta) \times p(Y|X, \beta, \sigma_\varepsilon^2) \\ &\propto \left(\frac{1}{\sigma_\varepsilon^2}\right)^{sh+1+\frac{T}{2}} exp\left\{ -sc/\sigma_\varepsilon^2 - (Y - X\beta)'(Y - X\beta)/2\sigma_\varepsilon^2 \right\} \\ &\sim IG(T/2 + sc, SSR/2 + sc), \end{aligned} \tag{22}$$

where $SSR \equiv (Y - X\beta)'(Y - X\beta)$.

(*iv*) $p(\Sigma|\{\bar{\beta}\}_i \in strat_j)$: using the IG prior for $\Sigma$, we have

$$\begin{aligned} p(\Sigma|\{\bar{\beta}\}_i \in strat_j) &\sim p(\Sigma|sh, sc) \times \prod_{i \in strat_j} p(\bar{\beta}_i|\beta_{Str}, \Sigma) \\ &\propto \prod_{k=1}^{K} \left(\frac{1}{\sigma_k^2}\right)^{sh+1+\frac{n_j}{2}} \exp\left\{ -\frac{sc}{\sigma_k^2} - \sum_{i \in strat_j} \frac{(\bar{\beta}_k - \beta_{Str,k})^2}{2\sigma_k^2} \right\}, \end{aligned} \tag{23}$$

where $\bar{\beta}_k$ and $\beta_{Str,k}$ are the $k^{th}$ elements of the vectors $\bar{\beta}$ and $\beta_{Str}$ respectively. The kernel in (23) factors in several independent kernels for each $\delta_k$:

$$p(\sigma_k^2|\{\bar{\beta}\}_i \in strat_j) \sim IG(n_j/2 + sh, SSR_{\sigma_k}/2 + sc), \quad k = 1, \ldots, K, \tag{24}$$

where the definition of $SSR_{\delta_k}$ is obvious.

(*v*) $p(\Delta|\bar{\beta}, \beta_1, \ldots, \beta_T, \rho)$: the information for this posterior density is contained in (11) and (14). Therefore, denoting the demeaned $\beta$ with $\beta^*$,

$$\begin{aligned} p(\Delta|\bar{\beta}, \beta) &\propto p(\Delta) \times p(\beta_1, \ldots, \beta_T|\Delta) \\ &\propto \prod_{k=1}^{K} \left(\frac{1}{\delta_k^2}\right)^{sh+1+\frac{T}{2}} exp\left\{ -sc/\delta_k^2 - \sum_{t=1}^{T} (\beta_{kt}^* - \rho_k \beta_{kt-1}^*)^2 / 2\delta_k^2 \right\}, \end{aligned} \tag{25}$$

where $\beta^*_{kt}$ refers the $k^{th}$ element of the vector $\beta^*_t$. The kernel in (25) factors in several independent kernels for each $\delta_k$:

$$p(\delta_k^2|\bar{\beta}_k, \beta_k, \rho_k) \sim IG(T/2 + sh, SSR_{\delta_k}/2 + sc), \quad k = 1, \ldots, K, \tag{26}$$

where the definition of $SSR_{\delta_k}$ is obvious.

$(vi)$ $p(\rho_k|\beta, \bar{\beta}, \Delta)$: Using independence of the elements of $\rho$, we have

$$\begin{aligned} p(\rho_k|\beta, \bar{\beta}, \Delta) &\propto TN_{(-1,1)}(0, \sigma^2_{\rho,k}) \times \prod_{t=1}^{T} N(\rho\beta^*_{k,t-1}, \delta_k^2) \\ &\propto TN_{(-1,1)}(\hat{\rho}_k, \hat{\sigma}_{\rho_k}), \quad k = 1, \ldots, K, \end{aligned} \tag{27}$$

where $\hat{\rho}_k = \hat{\sigma}_{\rho_k} \times \sum_{t=1}^{T} \beta^*_{k,t}\beta^*_{k,t-1}$ and $\hat{\sigma}_{\rho_k} = 1/(1/\sigma_k^2 + \sum_{t=1}^{T} \beta^{*2}_{k,t-1}/\delta_k^2)$.

## A.3 Posterior Distributions: general case ($\theta \in [0, 1]$)

$(i)$ $p(\beta|\bar{\beta}, \sigma^2_\varepsilon, \Delta, \rho, \theta, D_T)$: using the likelihood in (10) and the prior in (18), we have

$$p(\beta|\bar{\beta}, \sigma^2_\varepsilon \Delta, \theta, D_T) \sim N(B, V), \tag{28}$$

where,

$$\begin{aligned} B &= V \times \left(X^{o\prime}\Omega^{-1}Y^o/\sigma^2_\varepsilon + (A_2(I_T \otimes \Delta)A_2')^{-1}A_1\bar{\beta})\right) \\ V &= \left(X^{o\prime}\Omega^{-1}X^o/\sigma^2_\varepsilon + (A_2(I_T \otimes \Delta)A_2')^{-1}\right)^{-1}. \end{aligned}$$

$(ii)$ $p(\bar{\beta}|\sigma^2_\varepsilon, \Delta, \rho, \theta, D_T, )$: the only difference is in the matrix Q and the sample size (shorter by one observation). Plug expression (18) into (9) to get

$$\begin{aligned} Y^o &= X^o A_1 \bar{\beta} + X^o A_2 \eta + \varepsilon \\ &= X_1^* \bar{\beta} + \varepsilon^*, \quad \varepsilon^* \sim N(0, Q), \end{aligned}$$

where now we have $Q = [\sigma^2_\varepsilon \Omega + X^o A_2 (I_T \otimes \Delta) A_2' X^{o\prime}]$. The rest of the derivation is just like the base case.

$(iii)$ $p(\sigma^2_\varepsilon|\beta, \theta, D_T)$: with a prior for $\sigma^2_\varepsilon$ given by (16), we have

$$\begin{aligned} p(\sigma^2_\varepsilon|\beta, \theta, D_T) &\sim p(\sigma^2_\varepsilon|\beta) \times p(Y^o|X^o, \beta, \sigma^2_\varepsilon) \\ &\propto \left(\frac{1}{\sigma^2_\varepsilon}\right)^{sh+1+\frac{T}{2}} exp\left\{-sc/\sigma^2_\varepsilon - (Y^o - X^o\beta)'\Omega^{-1}(Y^o - X^o\beta)/2\sigma^2_\varepsilon\right\} \\ &\sim IG(T^o/2 + sh, SSR/2 + sc), \end{aligned} \tag{29}$$

where now $SSR \equiv (Y^o - X^o\beta)'\Omega^{-1}(Y^o - X^o\beta)$.

$(iv)$ $p(\Sigma|\{\bar{\beta}\}_i \in strat_j)$: same as base case.

$(v)$ $p(\Delta|\beta_1, \ldots, \beta_T)$: same as base case.

$(vi)$ $p(\rho_k|\beta, \bar{\beta}, \Delta)$: same as base case.

$(vii)$ $p(\theta|\beta, \sigma_\varepsilon^2, D_T)$: Combining the likelihood in (10) with the prior for $\theta$, we have

$$\begin{aligned} p(\theta|\beta, \sigma_\varepsilon^2, D_T) &\sim Beta(a, b) \times N(X^o\beta, \sigma_\varepsilon^2\Sigma) \\ &\propto \frac{\theta^{a-1}(1-\theta)^{b-1}}{|\Omega|^{1/2}} exp\left\{-\frac{(Y^o - X^o\beta)'\Omega^{-1}(Y^o - X^o\beta)}{2\sigma_\varepsilon^2}\right\} \end{aligned} \quad (30)$$

This posterior distribution is not a standard one and a Metropolis-Hastings algorithm is required to sample from it.

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Table 1: Simulation: Data Generating Process

This table reports the parameters used in the data generating process in the simulation study.

| | | |
|---|---|---|
| *Parameters:* | | |
| $\beta_{OLS} = [0, 1]$ | $\sigma_{\varepsilon} = 0.01^2$ | $\theta \sim Uni[0, 1]$ |
| $\Delta = [0.01^2 \ 0.1^2]$ | $\Sigma = [0.01^2 \ 0.1^2]$ | $\Sigma_{\rho} = [1 \ 0.1]$ |
| $x_t \sim N(0.01, 0.15)$ | | |
| *Simulation Set Up*: | | |
| 1 strategy ($J = 1$) | 100 funds ($I = 100$) | |
| 1 factor ($K = 1$) | $T_i \sim Uni[30, 144]$ | |

Table 2: Simulation: Model Comparison

This table presents statistics on the fitting of funds' estimates of $\alpha$ and $\beta$. The first four column identify the model under investigations. A one indicate that the smoothing, dynamic, or shrinkage feature is retained and a zero indicates otherwise. $Bias(\cdot)$ is the average difference between the estimated and true parameters for all funds. $MAD(\cdot)$ is the mean absolute deviation. $RMSE(\cdot)$ is the root mean squared error of the estimates. The last column reports the posterior probability, given the priors and the likelihood, of the estimated models.

| | Sm | Dy | Sr | $Bias(\hat{\alpha})$ | $MAD(\hat{\alpha})$ | $RMSE(\hat{\alpha})$ | $Bias(\hat{\beta})$ | $MAD(\hat{\beta})$ | $RMSE(\hat{\beta})$ | Post. Pr. |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 0.000 | 0.008 | 0.011 | -0.000 | 0.062 | 0.081 | 1.000 |
| 2 | 1 | 1 | 0 | 0.000 | 0.009 | 0.011 | -0.000 | 0.063 | 0.082 | 0.000 |
| 3 | 1 | 0 | 1 | 0.000 | 0.014 | 0.018 | -0.005 | 0.092 | 0.116 | 0.000 |
| 4 | 0 | 1 | 1 | 0.000 | 0.031 | 0.047 | -0.488 | 0.494 | 0.595 | 0.000 |
| 5 | 1 | 0 | 0 | 0.000 | 0.014 | 0.018 | -0.005 | 0.092 | 0.117 | 0.000 |
| 6 | 0 | 1 | 0 | 0.000 | 0.046 | 0.067 | -0.521 | 0.527 | 0.639 | 0.000 |
| 7 | 0 | 0 | 1 | 0.000 | 0.015 | 0.019 | -0.477 | 0.484 | 0.574 | 0.000 |
| 8 | 0 | 0 | 0 | 0.000 | 0.015 | 0.019 | -0.486 | 0.493 | 0.586 | 0.000 |

Table 3: Hedge Fund Strategies

This table presents the fund categories (strategies) considered in the analysis. Each strategy is assigned a code which will be used in the remainder of the paper. The third column reports the number of funds in each category. The remaining three columns report the sample size of the fund with the lowest, median, and maximum number of observations within each category.

| | | | Observations | | |
|---|---|---|---|---|---|
| Code | Category | N. Funds | Min | Median | Max |
| 1 | Capital Structure Arbitrage | 4 | 46 | 53.5 | 84 |
| 2 | Convertible Arbitrage | 102 | 30 | 58 | 144 |
| 3 | Distressed Securities | 77 | 31 | 56 | 144 |
| 4 | Emerging Markets | 183 | 30 | 63 | 144 |
| 5 | Equity Long Only | 40 | 30 | 60 | 144 |
| 6 | Equity Long/Short | 834 | 30 | 60 | 144 |
| 7 | Equity Market Neutral | 96 | 30 | 54 | 144 |
| 8 | Event Driven Multi Strategy | 90 | 30 | 74.5 | 144 |
| 9 | Fixed Income | 21 | 34 | 58 | 144 |
| 10 | Fixed Income - MBS | 33 | 32 | 74 | 112 |
| 11 | Fixed Income Arbitrage | 56 | 31 | 45 | 102 |
| 12 | Global Macro | 101 | 30 | 60 | 144 |
| 13 | Market Timing | 5 | 75 | 81 | 105 |
| 14 | Merger Arbitrage | 66 | 33 | 63 | 144 |
| 15 | Multi Strategy | 28 | 34 | 64 | 144 |
| 16 | Option Arbitrage | 7 | 37 | 47 | 108 |
| 17 | Other Relative Value | 3 | 91 | 111 | 120 |
| 18 | Regulation D | 4 | 80 | 86 | 115 |
| 19 | Relative Value Multi Strategy | 40 | 30 | 66.5 | 142 |
| 20 | Sector | 179 | 30 | 60 | 144 |
| 21 | Short Bias | 31 | 33 | 64 | 144 |
| | All | 2000 | 30 | 60 | 144 |

Table 4: Hedge Fund Descriptive Statistics

This table presents descriptive statistics on monthly returns reported by hedge funds in the CISDM data base. The first column reports the codes associated to each strategy (see Table 1). The second and third column report the smallest and largest monthly reported return in each strategy. The last four columns report the strategy average of the mean, standard deviation, skewness, and kurtosis of hedge fund returns.

| Code | Min | Max | Mean | SD | SK | KUR |
|---|---|---|---|---|---|---|
| 1 | -0.039 | 0.162 | 0.015 | 0.021 | 1.581 | 5.378 |
| 2 | -0.410 | 0.520 | 0.008 | 0.025 | -0.192 | 2.905 |
| 3 | -0.583 | 0.610 | 0.011 | 0.034 | 0.088 | 3.664 |
| 4 | -1.000 | 2.257 | 0.012 | 0.069 | -0.290 | 4.567 |
| 5 | -0.549 | 0.835 | 0.009 | 0.063 | -0.233 | 1.235 |
| 6 | -1.000 | 1.225 | 0.010 | 0.051 | 0.178 | 2.679 |
| 7 | -0.820 | 0.362 | 0.006 | 0.025 | -0.153 | 2.788 |
| 8 | -0.543 | 0.885 | 0.011 | 0.033 | -0.152 | 3.530 |
| 9 | -0.201 | 0.212 | 0.006 | 0.025 | -0.760 | 3.515 |
| 10 | -0.357 | 0.322 | 0.009 | 0.025 | -2.059 | 17.175 |
| 11 | -0.528 | 0.249 | 0.004 | 0.024 | -1.020 | 7.266 |
| 12 | -0.518 | 0.742 | 0.008 | 0.048 | 0.282 | 3.478 |
| 13 | -0.144 | 0.262 | 0.010 | 0.033 | 0.524 | 2.383 |
| 14 | -0.320 | 1.842 | 0.006 | 0.021 | -0.310 | 4.178 |
| 15 | -0.215 | 0.408 | 0.008 | 0.027 | 0.250 | 2.796 |
| 16 | -0.120 | 0.238 | 0.005 | 0.032 | 0.988 | 3.773 |
| 17 | -0.068 | 0.145 | 0.009 | 0.025 | 0.671 | 3.982 |
| 18 | -0.108 | 0.214 | 0.008 | 0.020 | 0.993 | 5.735 |
| 19 | -0.192 | 0.617 | 0.009 | 0.017 | -0.017 | 6.149 |
| 20 | -0.483 | 0.909 | 0.012 | 0.067 | 0.363 | 3.423 |
| 21 | -0.574 | 0.660 | 0.003 | 0.070 | 0.183 | 2.143 |
| All | -1.000 | 2.257 | 0.010 | 0.047 | 0.005 | 3.528 |

Table 5: Seven Factors Descriptive Statistics

This table presents descriptive statistic on the seven factors. The first three factors are bond, currency, and commodity trend-following risk factors respectively. They are constructed by Fung and Hsieh (2004) and are available at `http://faculty.fuqua.duke.edu/~dah7/DataLibrary/TF-FAC.xls`. The equity market factor is the Standard & Poors 500 index monthly total return. The size spread factors is the monthly return on the Russell 2000 index minus the monthly return on the Standard & Poors 500. The bond market factor is the monthly change in the 10-year treasury constant maturity yield. The Credit Spread Factor is the monthly change in the Moody's Baa yield minus the 10-year treasury constant maturity yield.

| Factors | Min | Max | Mean | SD | SK | KUR | $\rho$ |
|---|---|---|---|---|---|---|---|
| Equity Market | -0.147 | 0.094 | 0.006 | 0.043 | -0.577 | 3.549 | -0.015 |
| Size Spread | -0.164 | 0.183 | -0.000 | 0.038 | 0.258 | 7.421 | -0.133 |
| Bond Market | -0.005 | 0.006 | -0.000 | 0.002 | 0.389 | 2.727 | 0.249 |
| Credit Spread | -0.003 | 0.005 | -0.000 | 0.001 | 0.918 | 5.022 | 0.372 |
| Bond Trend | -0.254 | 0.689 | -0.005 | 0.152 | 1.507 | 6.153 | 0.094 |
| Currency Trend | -0.301 | 0.903 | -0.002 | 0.191 | 1.339 | 6.029 | 0.010 |
| Commodity Trend | -0.229 | 0.648 | -0.008 | 0.130 | 1.466 | 7.141 | -0.142 |

Table 6: Variable Selection and Unconditional Models' Fit

This table presents selection and estimation results of the variable selection criterion based on $R^2$. For each of the 937 equity funds, we estimate $2^7$ regression models using all the possible combinations available with seven factors. We then divide these regressions into seven groups depending on the number of included factors, and rank them according to their $R^2$. Finally, we retain, for every fund and every group, the regression with the highest $R^2$. Panel A reports the number of times each factor is selected by our procedure. Panel B reports average estimates of $\alpha$ and $\beta$, and average $R^2s$.

| Factors | $\alpha$ | $\beta_1$ | $\beta_2$ | $\beta_3$ | $\beta_4$ | $\beta_5$ | $\beta_6$ | $\beta_7$ | $R^2$ |
|---|---|---|---|---|---|---|---|---|---|
| *PANEL A: selected factors* | | | | | | | | | |
| 1 | 937 | 539 | 216 | 38 | 49 | 23 | 31 | 41 | |
| 2 | 937 | 704 | 547 | 143 | 149 | 119 | 88 | 124 | |
| 3 | 937 | 767 | 651 | 293 | 329 | 295 | 191 | 285 | |
| 4 | 937 | 821 | 731 | 468 | 504 | 449 | 335 | 440 | |
| 5 | 937 | 865 | 810 | 617 | 666 | 618 | 516 | 593 | |
| 6 | 937 | 906 | 871 | 781 | 807 | 770 | 703 | 784 | |
| 7 | 937 | 937 | 937 | 937 | 937 | 937 | 937 | 937 | |
| *PANEL B: estimates and fit* | | | | | | | | | |
| 1 | 0.005 | 0.573 | 0.471 | -1.357 | -12.024 | -0.005 | 0.027 | 0.048 | 0.249 |
| 2 | 0.004 | 0.526 | 0.431 | -1.017 | -7.101 | -0.006 | 0.029 | 0.037 | 0.334 |
| 3 | 0.004 | 0.498 | 0.383 | -1.223 | -5.666 | -0.006 | 0.027 | 0.034 | 0.364 |
| 4 | 0.004 | 0.467 | 0.352 | -1.000 | -4.185 | -0.006 | 0.019 | 0.029 | 0.379 |
| 5 | 0.004 | 0.444 | 0.322 | -0.890 | -3.311 | -0.006 | 0.014 | 0.021 | 0.387 |
| 6 | 0.004 | 0.423 | 0.302 | -0.792 | -2.838 | -0.006 | 0.011 | 0.016 | 0.391 |
| 7 | 0.004 | 0.409 | 0.280 | -0.666 | -2.432 | -0.006 | 0.008 | 0.014 | 0.392 |

Table 7: Model Comparison

This table presents the average and the median of the funds'posterior means of $\theta$ and $\rho_\alpha$ and, the average of the posterior mean of the shrinkage estimates. The column $Pct < 1$ reports the proportion of the smoothing estimates that are at least two standard deviations away from the value of one. The column $Pct \gtrless 0$ reports the proportion of the persistence estimates that are at least two standard deviations away from the value of zero. The results are reported for each of the 8 models considered in the study, and are categorized by investment strategy. Fields corresponding to model restrictions report values implied by the restriction. For instance, the fields for model8 are not estimates. These fields simply indicate the restrictions we use to estimate a model with no smoothing, no dynamics, and no shrinkage.

| | $\theta$ | | | $\rho_\alpha$ | | | $\Sigma$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | *Mean* | *Median* | $Pct < 1$ | *Mean* | *Median* | $Pct \gtrless 0$ | $\alpha$ | $\beta_1$ | $\beta_2$ |
| | | | | *PANEL A: Equity Long Only* | | | | | |
| 1 | 0.862 | 0.886 | 0.538 | 0.177 | 0.233 | 0.077 | 0.005 | 0.514 | 0.242 |
| 2 | 0.846 | 0.863 | 0.590 | 0.250 | 0.311 | 0.128 | 1000 | 1000 | 1000 |
| 3 | 0.889 | 0.921 | 0.359 | 0.000 | 0.000 | 0.000 | 0.005 | 0.588 | 0.282 |
| 4 | 1.000 | 1.000 | 0.000 | 0.317 | 0.339 | 0.103 | 0.005 | 0.442 | 0.193 |
| 5 | 0.898 | 0.916 | 0.333 | 0.000 | 0.000 | 0.000 | 1000 | 1000 | 1000 |
| 6 | 1.000 | 1.000 | 0.000 | 0.407 | 0.432 | 0.282 | 1000 | 1000 | 1000 |
| 7 | 1.000 | 1.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.006 | 0.506 | 0.220 |
| 8 | 1.000 | 1.000 | 0.000 | 0.000 | 0.000 | 0.000 | 1000 | 1000 | 1000 |
| | | | | *PANEL B: Equity Long/Short* | | | | | |
| 1 | 0.824 | 0.839 | 0.654 | 0.090 | 0.107 | 0.108 | 0.003 | 0.506 | 0.309 |
| 2 | 0.811 | 0.821 | 0.738 | 0.252 | 0.294 | 0.151 | 1000 | 1000 | 1000 |
| 3 | 0.799 | 0.879 | 0.397 | 0.000 | 0.000 | 0.000 | 0.003 | 0.528 | 0.366 |
| 4 | 1.000 | 1.000 | 0.000 | 0.338 | 0.330 | 0.204 | 0.003 | 0.390 | 0.255 |
| 5 | 0.806 | 0.874 | 0.421 | 0.000 | 0.000 | 0.000 | 1000 | 1000 | 1000 |
| 6 | 1.000 | 1.000 | 0.000 | 0.436 | 0.442 | 0.300 | 1000 | 1000 | 1000 |
| 7 | 1.000 | 1.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.004 | 0.410 | 0.294 |
| 8 | 1.000 | 1.000 | 0.000 | 0.000 | 0.000 | 0.000 | 1000 | 1000 | 1000 |
| | | | | *PANEL C: Equity Market Neutral* | | | | | |
| 1 | 0.815 | 0.827 | 0.621 | 0.009 | -0.009 | 0.074 | 0.003 | 0.230 | 0.064 |
| 2 | 0.779 | 0.778 | 0.768 | 0.192 | 0.220 | 0.042 | 1000 | 1000 | 1000 |
| 3 | 0.745 | 0.860 | 0.253 | 0.000 | 0.000 | 0.000 | 0.003 | 0.256 | 0.107 |
| 4 | 1.000 | 1.000 | 0.000 | 0.300 | 0.299 | 0.147 | 0.003 | 0.184 | 0.084 |
| 5 | 0.742 | 0.845 | 0.305 | 0.000 | 0.000 | 0.000 | 1000 | 1000 | 1000 |
| 6 | 1.000 | 1.000 | 0.000 | 0.400 | 0.405 | 0.211 | 1000 | 1000 | 1000 |
| 7 | 1.000 | 1.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.004 | 0.229 | 0.115 |
| 8 | 1.000 | 1.000 | 0.000 | 0.000 | 0.000 | 0.000 | 1000 | 1000 | 1000 |

## Table 8: Individual Estimates: No Smoothing (model8)

This table presents the sample mean ($Mean$), standard deviation( $SD$) and percentiles ($5^{th}$ $pct$, and $95^{th}$ $pct$ )of the estimated OLS and bayesian parameters. The results are categorized by investment strategy. In addition to the $\alpha$ and $\beta$ coefficients, the last column also report the estimated Bayesian standard deviation of the shocks.

| | OLS | | | BAYES | | | |
|---|---|---|---|---|---|---|---|
| | $\alpha$ | $\beta_1$ | $\beta_2$ | $\alpha$ | $\beta_1$ | $\beta_2$ | $\sigma_\varepsilon$ |
| | | | *PANEL A: Equity Long Only* | | | | |
| Mean | 0.004 | 0.713 | 0.292 | 0.004 | 0.714 | 0.293 | 0.042 |
| St. Dev | 0.009 | 0.565 | 0.322 | 0.009 | 0.564 | 0.326 | 0.027 |
| $Min$ | -0.019 | -0.585 | -0.432 | -0.019 | -0.586 | -0.421 | 0.011 |
| $5^{th}$ | -0.007 | -0.446 | -0.190 | -0.007 | -0.443 | -0.191 | 0.011 |
| $50^{th}$ $pct$ | 0.003 | 0.762 | 0.266 | 0.003 | 0.762 | 0.267 | 0.033 |
| $95^{th}$ $pct$ | 0.022 | 1.769 | 0.882 | 0.022 | 1.758 | 0.878 | 0.103 |
| $Max$ | 0.028 | 2.008 | 1.068 | 0.028 | 2.007 | 1.118 | 0.110 |
| | | | *PANEL B: Equity Long/Short* | | | | |
| Mean | 0.004 | 0.428 | 0.322 | 0.004 | 0.427 | 0.322 | 0.038 |
| St. Dev | 0.008 | 0.485 | 0.386 | 0.008 | 0.485 | 0.386 | 0.022 |
| $Min$ | -0.040 | -2.534 | -0.928 | -0.038 | -2.550 | -0.934 | 0.007 |
| $5^{th}$ $pct$ | -0.009 | -0.183 | -0.125 | -0.009 | -0.183 | -0.123 | 0.012 |
| $50^{th}$ $pct$ | 0.005 | 0.351 | 0.246 | 0.005 | 0.352 | 0.247 | 0.033 |
| $95^{th}$ $pct$ | 0.016 | 1.371 | 1.050 | 0.016 | 1.369 | 1.033 | 0.078 |
| $Max$ | 0.035 | 2.957 | 3.288 | 0.035 | 2.943 | 3.288 | 0.167 |
| | | | *PANEL C: Equity Market Neutral* | | | | |
| Mean | 0.003 | 0.064 | 0.070 | 0.003 | 0.063 | 0.069 | 0.023 |
| St. Dev | 0.005 | 0.306 | 0.177 | 0.005 | 0.306 | 0.177 | 0.014 |
| $Min$ | -0.005 | -0.927 | -0.286 | -0.005 | -0.934 | -0.286 | 0.007 |
| $5^{th}t$ | -0.003 | -0.275 | -0.173 | -0.003 | -0.274 | -0.172 | 0.008 |
| $50^{th}$ $pct$ | 0.002 | 0.036 | 0.030 | 0.002 | 0.037 | 0.031 | 0.020 |
| $95^{th}$ $pct$ | 0.014 | 0.492 | 0.406 | 0.014 | 0.492 | 0.407 | 0.056 |
| $Max$ | 0.025 | 1.777 | 0.570 | 0.025 | 1.780 | 0.570 | 0.079 |

Table 9: Individual Estimates: Smoothing (model5)

This table presents the sample mean ($Mean$), standard deviation( $SD$) and percentiles ($5^{th}$ $pct$, and $95^{th}$ $pct$ )of the estimated OLS and bayesian parameters. The results are categorized by investment strategy. In addition to the $\alpha$ and $\beta$ coefficients, the last two columns also report the Bayesian estimates of the standard deviation of the shocks and of the smoothing parameter.

| | OLS | | | BAYES | | | | |
|---|---|---|---|---|---|---|---|---|
| | $\alpha$ | $\beta_1$ | $\beta_2$ | $\alpha$ | $\beta_1$ | $\beta_2$ | $\sigma_\varepsilon$ | $\theta$ |
| *PANEL A: Equity Long Only* | | | | | | | | |
| Mean | 0.003 | 1.005 | 0.075 | 0.003 | 0.802 | 0.293 | 0.048 | 0.898 |
| St. Dev | 0.009 | 0.780 | 0.273 | 0.008 | 0.621 | 0.337 | 0.032 | 0.059 |
| $Min\ pct$ | -0.032 | -0.564 | -0.468 | -0.016 | -0.611 | -0.500 | 0.013 | 0.713 |
| $5^{th}$ | -0.007 | -0.416 | -0.369 | -0.007 | -0.454 | -0.198 | 0.013 | 0.778 |
| $50^{th}$ $pct$ | 0.003 | 0.980 | 0.059 | 0.003 | 0.892 | 0.265 | 0.036 | 0.916 |
| $95^{th}$ $pct$ | 0.018 | 2.398 | 0.578 | 0.019 | 1.923 | 0.920 | 0.117 | 0.960 |
| $Max$ | 0.023 | 3.095 | 1.019 | 0.026 | 2.368 | 1.065 | 0.140 | 0.968 |
| *PANEL B: Equity Long/Short* | | | | | | | | |
| Mean | 0.004 | 0.743 | 0.158 | 0.004 | 0.498 | 0.361 | 0.043 | 0.806 |
| St. Dev | 0.009 | 0.770 | 0.287 | 0.009 | 0.545 | 0.441 | 0.025 | 0.193 |
| $Min$ | -0.053 | -3.172 | -0.914 | -0.049 | -2.877 | -1.020 | 0.007 | 0.072 |
| $5^{th}$ $pct$ | -0.009 | -0.155 | -0.180 | -0.010 | -0.183 | -0.173 | 0.015 | 0.280 |
| $50^{th}$ $pct$ | 0.004 | 0.618 | 0.128 | 0.004 | 0.421 | 0.281 | 0.037 | 0.874 |
| $95^{th}$ $pct$ | 0.015 | 2.215 | 0.587 | 0.016 | 1.513 | 1.210 | 0.090 | 0.955 |
| $Max$ | 0.039 | 5.868 | 2.414 | 0.035 | 3.516 | 3.569 | 0.206 | 0.977 |
| *PANEL C: Equity Market Neutral* | | | | | | | | |
| Mean | 0.003 | 0.127 | 0.033 | 0.003 | 0.089 | 0.072 | 0.026 | 0.742 |
| St. Dev | 0.005 | 0.367 | 0.145 | 0.005 | 0.345 | 0.188 | 0.016 | 0.249 |
| $Min$ | -0.006 | -0.689 | -0.355 | -0.005 | -1.009 | -0.353 | 0.008 | 0.080 |
| $5^{th}$ $pct$ | -0.003 | -0.234 | -0.185 | -0.003 | -0.287 | -0.175 | 0.010 | 0.127 |
| $50^{th}$ $pct$ | 0.002 | 0.042 | 0.016 | 0.002 | 0.048 | 0.037 | 0.023 | 0.845 |
| $95^{th}$ $pct$ | 0.012 | 0.808 | 0.324 | 0.013 | 0.608 | 0.408 | 0.062 | 0.956 |
| $Max$ | 0.029 | 2.038 | 0.609 | 0.026 | 1.860 | 0.613 | 0.087 | 0.986 |

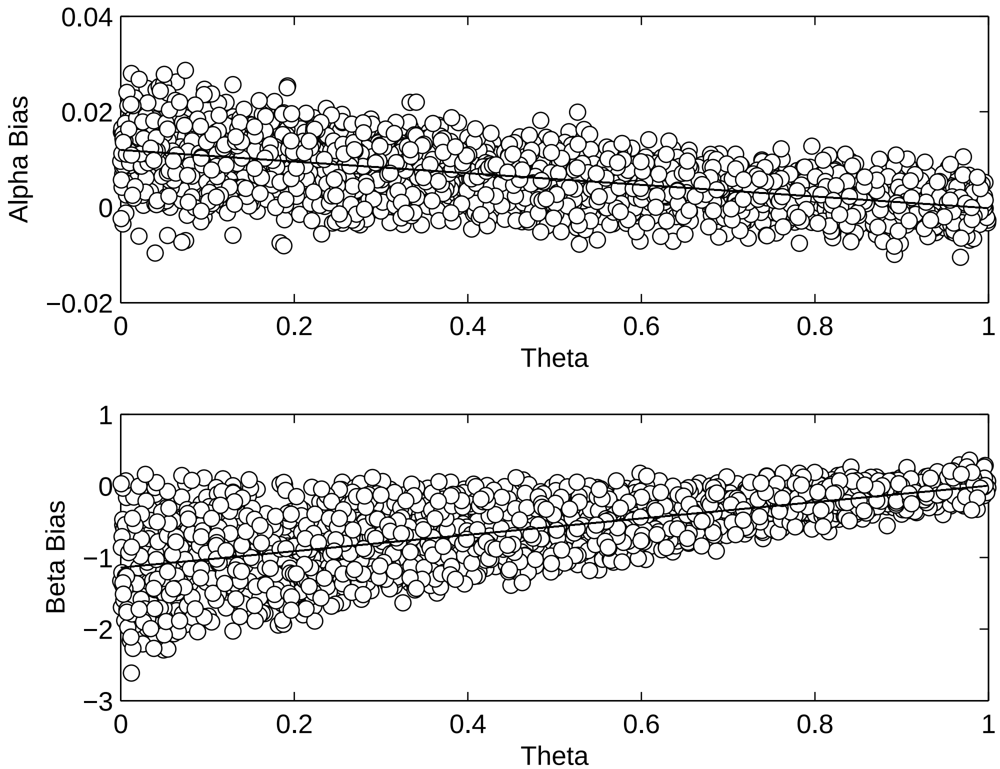


Figure 1: Smoothing Bias

Excess returns for fund $i$ at time $t$ are generated by the model $y_{it} = \alpha_i + \beta_i\prime x_t + \varepsilon_{it}$, where $y_{it}$ is the fund's excess return and $x_t$ is the excess return on the market. The parameters $\beta_i$ and $\alpha_i$ represent systematic risk and abnormal performance respectively. We generate returns for 2000 funds and set $T = 60$ for all of them. We set $\alpha_i = 0$ for all funds and draw the $\beta_i$ from a uniform distribution with support equal to $[0, 2]$. For each fund we draw the return smoother $\theta$ from a uniform distribution so that some funds smooth returns more than other. We regress $y_t^o = \theta_i y_t + (1 - \theta_i)y_{t-1}$ on a constant and $x_t$ using OLS. We report the bias for both estimated parameters as a function of $\theta$.

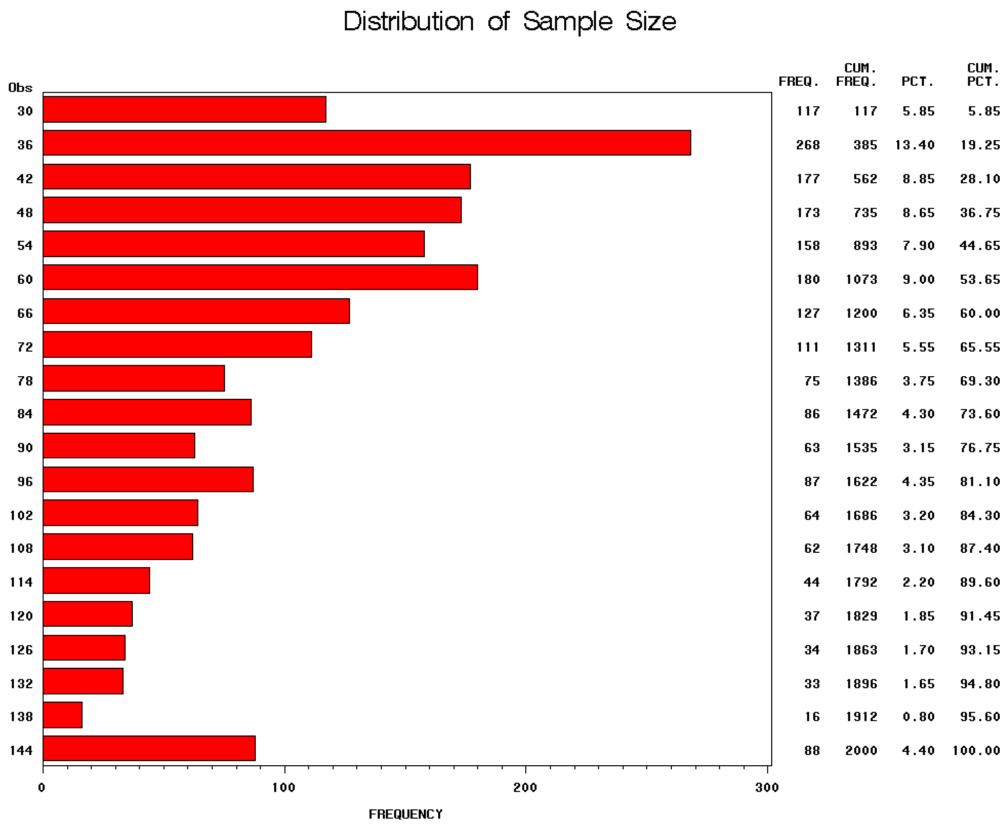


Figure 2: Distribution of Hedge Funds' Sample Size

This figure presents the distribution of hedge funds' life-spans, measured in months, in our sample.

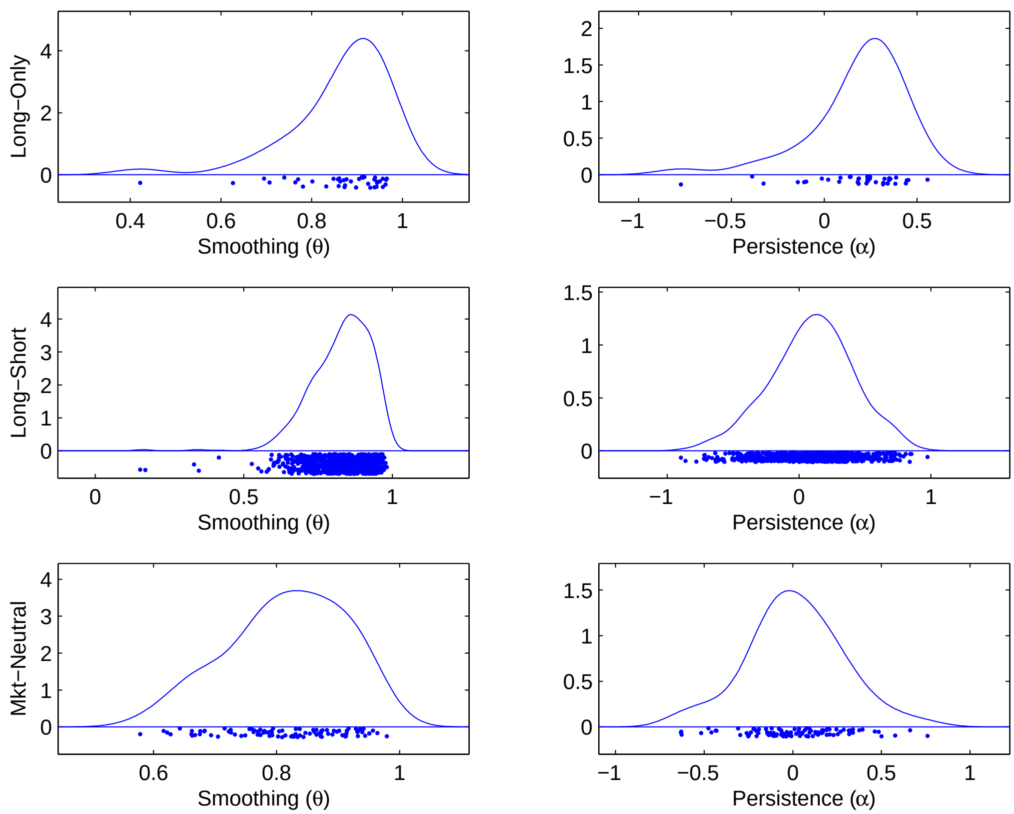


Figure 3: Smoothing and Persistence Estimates
This figure presents the kernel density of the fund-level posterior means of the smoothing parameter $\theta$ and the performance persistence parameter $\rho_\alpha$.

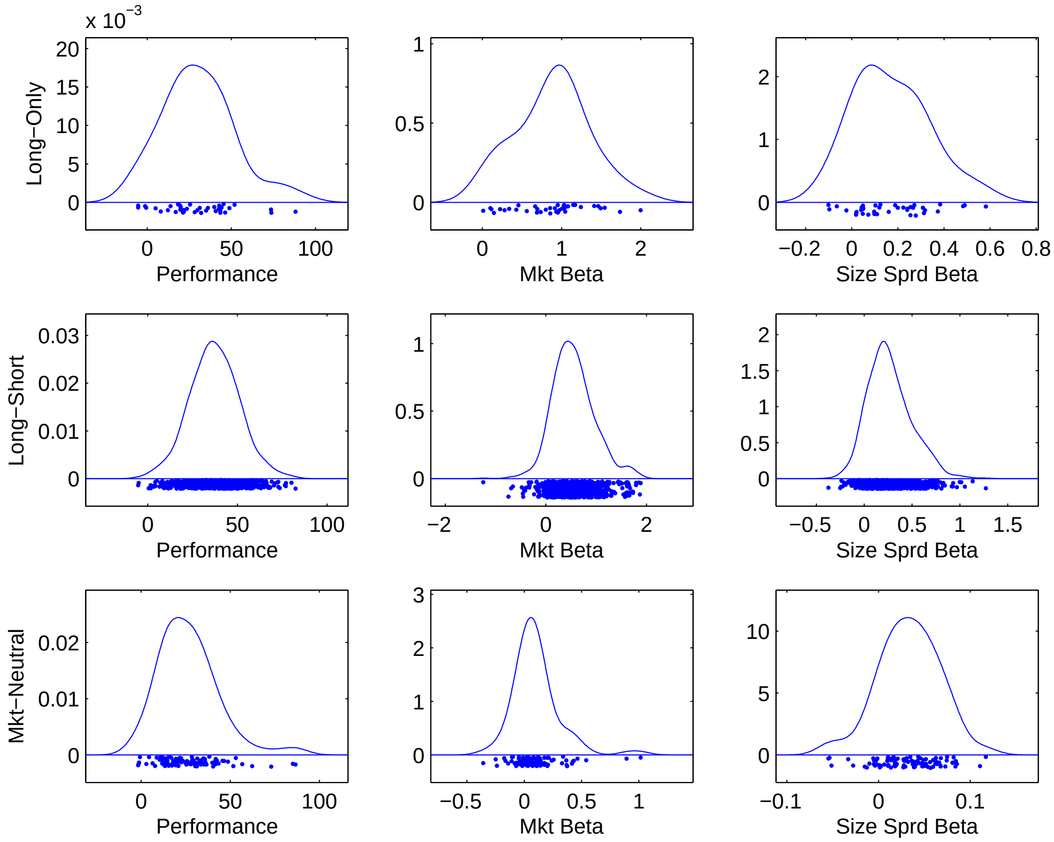


Figure 4: Long-run Coefficients and Shrinkage

This figure presents the kernel density of the fund-level posterior means of $\bar{\alpha}$ and $\bar{\beta}'s$ (long-run abnormal performance and factor loadings). Abnormal performance is measured in basis points.

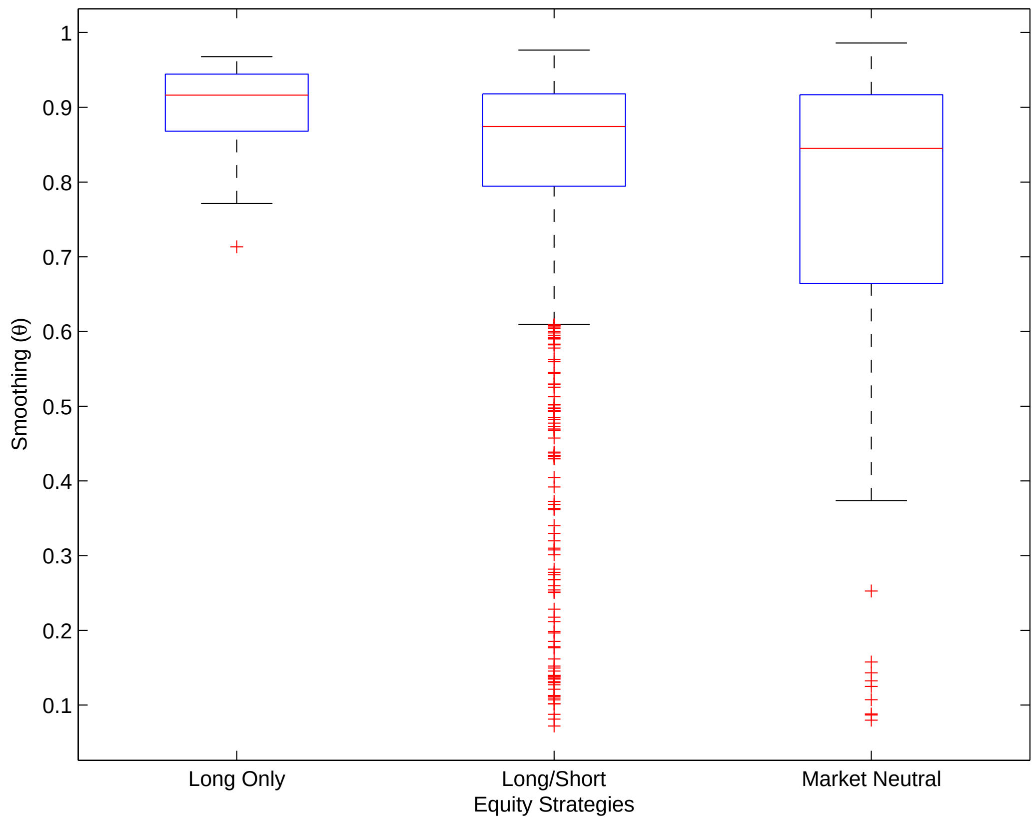


Figure 5: Smoothing by Strategy
This figure presents the distribution, by investment strategy, of the estimated smoothing parameters $\theta$.

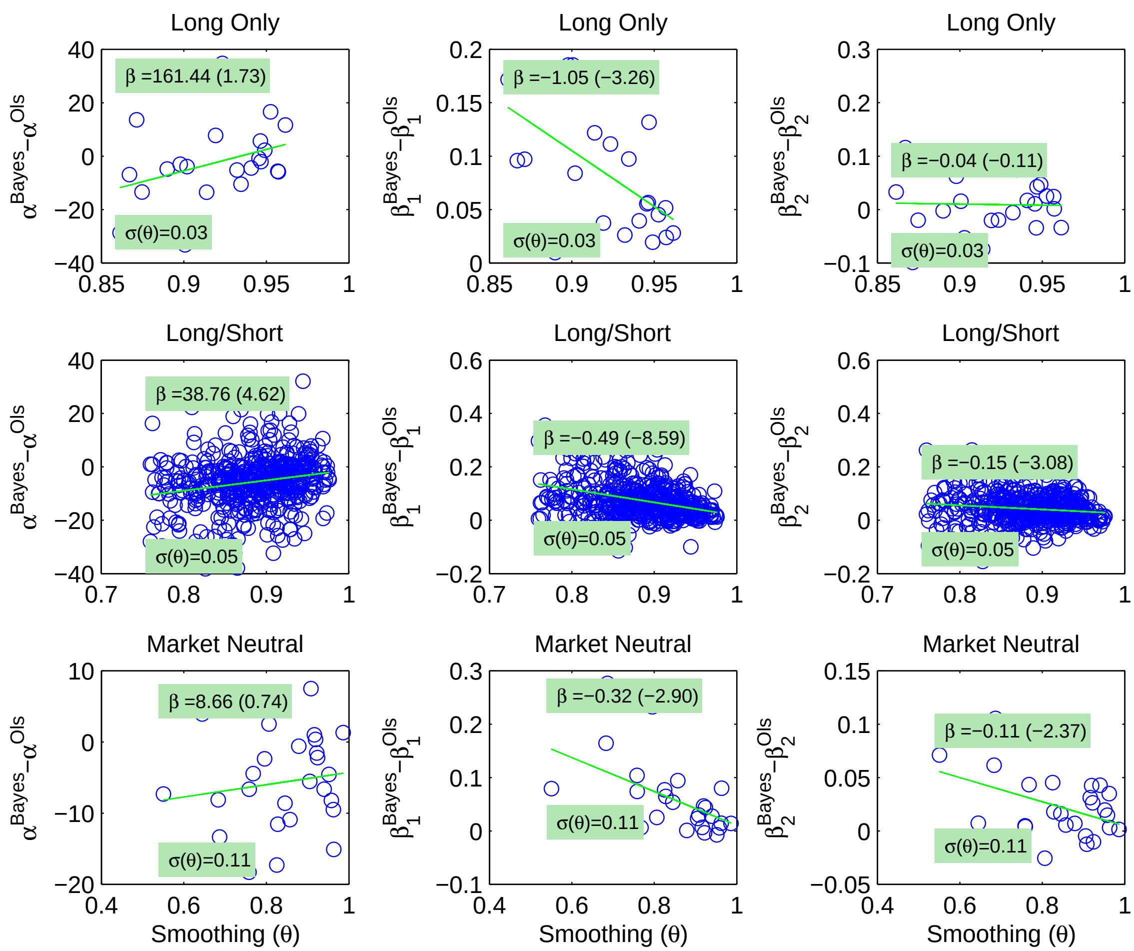


Figure 6: Bias Vs Smoothing

This figure presents scatter plots, by investment strategy, of the difference between OLS and Bayesian alphas and betas. The scale on the alpha graphs is expressed in basis points. The graphs report observations for which $\theta$ is above the $25^{th}$ percentile of the strategy group and for which the funds loadings are both positive. To reduce the influence of outliers, the fitted lines are fit to the data after eliminating observations with a studentized residual in excess of 3, and a Cook's distance ten times bigger than the median Cook's distance in the strategy group.