Cross section for $e^+e^- \rightarrow \text{hadrons}$: many resonances


**Width and decay rate of unstable particles**

- Scalar propagator $S(p^2)$ near pole for unstable particle:
  \[ S(p^2) = \frac{ic^2}{-p^2 - m^2 + im\Gamma} + \text{smaller} \]

- Momentum space: Define “width” of “resonance”; it is $\Gamma$

- Compute exponential decay in time. Find
  \[ \Gamma = \text{decay rate} = (\text{proper lifetime})^{-1} \]

- Use Källén-Lehmann representation
  \[ S(p^2) = \int_0^\infty d\mu^2 \rho(\mu^2) \frac{i}{-p^2 - \mu^2 + i\epsilon} \]
  near particle pole, and derive $\Gamma$ from cut self-energy.

- Note spectral function is
  \[ \rho(\mu^2) = \frac{1}{2\pi} |S(-\mu^2)|^2 \Sigma_{\text{cut}}(-\mu^2) \]
List of interaction vertices

\[-\frac{1}{4}W_{\mu\nu}^a W^{a,\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}\]

\[i\bar{\psi}D\psi\]

\[-D_{\mu}\phi^\dagger D^\mu\phi\]

\[-\lambda(\phi^\dagger\phi)^2\]

Yukawa

Z decay and measuring number of neutrinos

- Main decay is to lepton pairs
- Principles to measure number of light neutrino flavors
- Calculation of decay width to lowest order
  - \(\Gamma = 2.4952 \pm 0.0023\) GeV
  - \(N_\nu = 2.9840 \pm 0.0082\)