SU(2) x U(1) gauge group with couplings \( g_2, g_1, \).

Covariant derivative

\[ D_\mu = \partial_\mu - ig_2 W_\mu^a T^a - \frac{ig_1}{2} Y B_\mu, \]

where \( T^a \) are matrices that act on the field multiplet being used, and that obey SU(2) algebra, while \( Y \) is hypercharge of multiplet.

Choose a scalar multiplet, \( \phi = (\phi_1, \phi_2, \phi_3) \), in \( I=1 \) rep.

Since it is real, \( Y=0 \). Standard rep. of SU(2) \( I=1 \) is

\[ (T^a)^{bc} = i \varepsilon^{abc} \]

\[ (D_\mu \phi)_a = \partial_\mu \phi_a + g_2 W_\mu^b \varepsilon^{abc} \phi_c. \]

What are possible self-interactions?

Construct products of \( \phi \) in irreps of SU(2).

1. 2 \( \Phi \): Possibilities are \( I=0, 1 \) \( 2 \phi \phi \) or \( I=1 \) is antisymmetric: \( \varepsilon^{abc} \phi \phi \phi = 0 \) for bosonic field.

\[ I=0 \quad \phi \phi \phi = \phi^2 \]

\[ I=2 \] (symmetrized): \( (\phi \phi \phi - \frac{1}{2} \varepsilon^{abc} \phi^2) \)

2. 3 \( \Phi \): Can't get singlet from \( \phi \) times \( I=0 \) or \( \infty \).

3. 4 \( \Phi \): \( I=0 \) from \( (\Phi \phi \phi) \) \( \Phi (I=2 \phi \phi) \).

...
\[
(\phi^a \phi^b - \frac{1}{3} \delta_{ab} \phi^c)^2 = \phi^a \phi^b + \frac{1}{9} \delta_{ab} \delta^{cd} \phi^c \phi^d - \frac{2}{3} \phi^a \phi^b \delta_{ab} \phi^c \phi^d
\]
\[
= (\phi^a)^2 (1 + \frac{1}{3} - \frac{2}{3})
\]
\[
= \phi^a (\phi^a)
\]

General quadratic potential is \(A \phi^a + B (\phi^a)^2 + \text{c.c.}\).

Put this in form \(-A (\phi^2 - \nu^2)^2\).

Higgs Lagrangian density is
\[
L_H = \frac{1}{2} \left( D_\mu \phi \right)^2 - A (\phi^2 - \nu^2)^2
\]

Use unitary gauge to put \(\phi^a\) in 3-direction by an operator gauge transformation.

\[
\phi = (0, 0, \nu + H)
\]

\[
D_\mu \phi_a = \delta_{a3} D_\mu H + g_2 W_\mu^b \epsilon_{ab3} (\nu + H)
\]

\[
D_\mu \phi = (0, 0, 1) D_\mu H + g_2 (\nu + H) (W_\mu^3, -W_\mu^1, 0)
\]

\[
L_H = -\frac{1}{2} (D_\mu H)^2 - g_2^2 (\nu + H)^2 (W_\mu^3, -W_\mu^1, 0)
\]

\[-\lambda (2 \nu H + H^3)^2 \]
Mass terms

\[-\frac{g_2^2 v^2}{2} (W_{\mu}^+ W_{\mu}^- + H^2)\]

\[-\frac{1}{2} 8v^2 H^2.\]

This gives a $W$ mass of $g_2 v$. The remaining 2 gauge bosons are massless. The Higgs mass is $2\sqrt{\frac{v^2}{2}}$.

The unbroken symmetry is $U(1) \otimes U(1)$, where one $U(1)$ is the original $U(1)$, and the other $U(1)$ is the subgroup of $SU(2)$ generated by $T^3$.

Since there are 2 massless gauge bosons, but only one is observed in reality, the model is in disagreement with data.

One could eliminate one of them by setting $g_2 = 0$, so it decouples from physics. But then the remaining massless gauge boson would be $W^\pm$. This has chiral couplings to charged leptons since the $SU(2)$ gauge fields couple to LH fields. So $W^\pm$ cannot be the photon.