Plan for next two lectures

- Integration along “contour” in \( \mathbb{C} \); Cauchy's theorem.

- Examples of use of Cauchy's theorem (plus other tools) to calculate/analyze certain kinds of integral.

- Some important general theorems about analytic functions.
Complex integration

- Integral along a contour $\Gamma, z = z(s)$, is defined by
  \[ \int_\Gamma dz \, f(z) = \int ds \frac{dz}{ds} f(z(s)) \]
  It is independent of the parameterization of $\Gamma$.

- Cauchy’s theorem: Integral around closed contour $C$ is zero, if $f(z)$ analytic in the region $\Omega$ bounded by $C$: $\int_C f(z) \, dz = 0$

- Equivalently: If $\Gamma_0$ and $\Gamma_1$ are two paths between the same endpoints $a$ and $b$, and $f(z)$ is analytic in the region bounded by $\Gamma_0$ and $\Gamma_1$, then
  \[ \int_{\Gamma_0} dz \, f(z) = \int_{\Gamma_1} dz \, f(z) \]

- Proofs:
  - By Stoke’s theorem + Cauchy-Riemann equations.
  - By deforming the contour by a parameter $a$: $\Gamma_a(s)$, and then showing
    \[ \frac{d}{da} \int_{\Gamma_a} f(z) \, dz = 0. \]