Plan for next two lectures

- Integration along "contour" in \( \mathbb{C} \); Cauchy's theorem.
- Examples of use of Cauchy's theorem (plus other tools) to calculate/analyze certain kinds of integral.
- Some important general theorems about analytic functions.

Complex integration

- Integral along a contour \( \Gamma \), \( z = z(s) \), is defined by
  \[
  \int_{\Gamma} dz \, f(z) = \int ds \frac{dz}{ds} f(z(s))
  \]
  It is independent of the parameterization of \( \Gamma \).
- Cauchy's theorem: Integral around closed contour \( C \) is zero, if \( f(z) \) analytic in the region \( \Omega \) bounded by \( C \):
  \[
  \int_{C} f(z) \, dz = 0
  \]
- Equivalently: If \( \Gamma_0 \) and \( \Gamma_1 \) are two paths between the same endpoints \( a \) and \( b \), and \( f(z) \) is analytic in the region bounded by \( \Gamma_0 \) and \( \Gamma_1 \), then
  \[
  \int_{\Gamma_0} dz \, f(z) = \int_{\Gamma_1} dz \, f(z)
  \]
- Proofs:
  - By Stoke's theorem + Cauchy-Riemann equations.
  - By deforming the contour by a parameter \( a \): \( \Gamma_a(s) \), and then showing
    \[
    \frac{d}{da} \int_{\Gamma_a} f(z) \, dz = 0.
    \]