Concepts to know

• Hermitian conjugate $A^\dagger$ of linear operator or square matrix.
• Hermitian operator/matrix
• Unitary operator/matrix
• Symmetric, orthogonal matrices
• Eigenvector, eigenvalue
• Determinant
Standard theorems to remember (finite-dimensional case)

- Eigenvalues of hermitian matrix/operator are real
- Eigenvectors for different eigenvalues of hermitian matrix/operator are orthogonal
- Hermitian matrices can be diagonalized by unitary operator
- There’s an orthonormal basis of eigenvectors of hermitian operator
- 2 hermitian matrices $A$ and $B$ can be simultaneously diagonalized if and only if (iff) they commute
- $\det(AB) = \det(A) \det(B)$. $A$ invertible iff $\det(A) \neq 0$
- Eigenvalues obey $\det(A - \lambda I) = 0$
- $n$th order polynomial has $n$ roots, counting multiplicity:
  \[ P(z) \propto \prod_{j=1}^{n} (z - z_j) \]
Diagonalization: Multiple different situations

• Linear operator, like $H$ in quantum mechanics.

  [Diagonalization properties of hermitian operators dominate thoughts about diagonalization for many physicists.]

• Quadratic form(s), as in small oscillation problem in mechanics:

  $$L = \sum_{i,j} \left( \frac{1}{2} q^i K_{ij} \dot{q}^j - \frac{1}{2} q^i V_{ij} \dot{q}^j \right)$$

  (Here we use simultaneous diagonalization of non-commuting real symmetric matrices!)

• Singular-value decomposition (SVD):

  Let $M$ be any matrix, size $n \times m$. Then there exist unitary $U$ ($n \times n$) and $V$ ($m \times m$), and diagonal $\Sigma$ ($n \times m$, $\Sigma_{ii} \geq 0$) such that

  $$M = U \Sigma V^\dagger$$
Issues to be discussed

- Examples of applications
- Commonalities
- Differences: Change of matrix “coordinates” under change of vector coordinates/basis.
- Unitary diagonalization of hermitian matrices as basis for other theorems
- Summaries of improved proofs
- Jordan normal form for general linear operator/diagonal matrix