Abstract definition of vector space (D&K: Ch. II; B&F: Ch. 3, . . . )

- Uses in physics.
- Definition of vector space $V$ over “field” ($\mathbb{R}$ or $\mathbb{C}$ in physics). Has addition and multiplication by scalars, with certain requirements
  
  $|v_1\rangle + |v_2\rangle$ commutative, associative

  null vector $|0\rangle$, negative $-|v\rangle$

  $\lambda|v\rangle$ associative, distributive, $1|v\rangle = |v\rangle$

For the moment, I’ll use Dirac notation for vectors (cf. QM)

- Basic properties follow: Problem set 6#1.

- Examples
  - Displacements
  - Column vectors (incl. infinite dimensional)
  - Function spaces (and quantum state spaces)

N.B. Many (but not all) cases have an “inner product”
Key concepts

- Basis vectors, dimension of space
  Effect of change of basis
- Subspace
- Linear operators
  Matrix representations of linear operators
    (Space of linear operators: $V \rightarrow W$ is itself a vector space.)
  - Dual space $V^*$ of vector space $V$
    Derivative of functional (or function) is a dual vector.
- Kernel and target (sub)spaces for a linear operator.
Basis of a vector space

- Definition
- Dimension of vector space.
- Subsidiary concepts: spanning set, over-complete basis, linear dependence.
- Representation of vectors by column vectors.
- Change of basis.
- Basic issues with infinite dimensional case
- Give illustration by Fourier series and Fourier integral.
- N.B. The concept of a basis can shift for infinite dimensional cases: Finite sum over basis vectors, or infinite sum, or integral.