Textbook sections

• Dennery & Krzywicki Ch. 1:
  – Sec. 1 gives a useful summary of some mathematical concepts that are prerequisite for this course. We’ll use the ideas repeatedly.
  – But note:
    ∗ Their notation \( A + B \) for the union of two sets is not the most common one. The more common notation is \( A \cup B \).
    ∗ Their definition of “neighborhood” is too restricted compared with the standard one.
  – The remainder of the chapter contains material for this course in approximately the order I will cover it.
  – However, I plan to omit the material (Secs. 9 & 10) on conformal transformations. (Even though this can be useful and important for some purposes.)
  – We will also not cover Secs. 27–30, 33.
  – It is worth skimming through the omitted sections, nevertheless.

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• Byron & Fuller, Ch. 6:
  – This covers the material on the theory of analytic functions, but without the preceding material in D & K.
  – But we will not cover Secs. 6.5, 6.6.
  – Although we will cover most of Sec. 6.7 (“Residue Theory”), we will omit the material on summation of series.
  – We will omit Sec. 6.8 (“Applications to special functions”). However, some of contents of that section will be encountered in a later part of the course.

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Linearity; continuation of material from last time

• Linearity:
  Last time I talked about the derivative of a function being a linearization of the function for small changes in the function’s argument. However, that was ambiguous.
  What I meant was what I will here call “real-linearity”, as opposed to “complex linearity”. Explanation on board . . . .

• Singularities, etc
  – Definitions on slide 4 from last time.
  – Illustrate with examples
    – . . .

Analyticity of Fourier transform

• Define Fourier transform, as usual, by
  \[ \tilde{f}(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} \, dt \]
  It is initially defined for real \( \omega \).

• Suppose that \( f(t) \) vanishes for \( t < t_0 \) for some constant \( t_0 \), and that it decreases exponentially when \( t \to +\infty \). Then:
  – Integral for \( \tilde{f}(\omega) \) exists for complex \( \omega \) if \( \text{Im} \, \omega \leq 0 \), and also for \( \text{Im} \, \omega \) positive and not too big.
  – The complex derivative \( \frac{d\tilde{f}(\omega)}{d\omega} \) exists.
  – Hence \( \tilde{f}(\omega) \) is analytic in the lower half plane, including the whole of the real axis.
  – From examples, we expect that the nearest singularities in the upper half \( \omega \)-plane correspond to large \( t \) asymptotics.

• Generalizations possible: e.g., if \( f(t) \) has power-law behavior at large \( t \), then singularities occur on real axis, but not for \( \text{Im} \, \omega \) strictly less than 0.

\[ ^1 \text{N.B. when } \omega \text{ is real, the ordinary real derivative and the complex derivative are equal.} \]