Physics 525 (Methods of Theoretical Physics)

- Content
- Overall structure of course
- Grading

Notes:

- No Mathematica for solutions to homework (unless I say otherwise).
- Detailed solutions of homework, with good explanations, are needed.
- Don’t assume that the textbooks or lectures get everything exactly correct. Get accustomed to questioning what you are being told.
Schedule of topics

• (Approx. 10 lectures) Complex variables; complex-analytic functions; . . .

• (Approx. 4 lectures) Finite and infinite dimensional vector spaces . . .

• (Approx. 4 lectures) Linear operators and their properties; self-adjoint operators; eigenvalues and eigen-vectors; unitary operators.

• (Approx. 3 lectures) Calculus of variations and its application to classical mechanics and classical field theory. This introduces ideas of functionals and functional analysis.

• (Approx. 3 lectures) Distributions/generalized functions

• (Approx. 5 lectures) Fourier series; Fourier integrals and their properties; applications to differential equations; distributions; Green’s functions.

• (Approx. 10 lectures) Differential equations that occur commonly in physics; special functions; orthonormality and completeness. Applications of the methods introduced earlier.
How did complex numbers come to be discovered/invented

One issue (among others), from 16th century:

- Initially only “real numbers” known.
- With only real numbers, $\sqrt{x}$ only exists if $x \geq 0$.
- Now a solution of $x^3 - px - q = 0$ for $x$, is given by Tartaglia’s formula:

$$x = \sqrt[3]{\frac{q}{2}} + \sqrt{\left(\frac{q}{2}\right)^2 - \left(\frac{p}{3}\right)^3} + \sqrt[3]{\frac{q}{2}} - \sqrt{\left(\frac{q}{2}\right)^2 - \left(\frac{p}{3}\right)^3}$$

(Check it!)

- Set $p = 1$, $q = 0$. Solution of $x^3 - x = 0$ are $x = \pm 1$, $x = 0$.
- But Tartaglia’s formula gives

$$x = \frac{1}{\sqrt{3}} \left( 3\sqrt[3]{\sqrt{-1}} + 3\sqrt[3]{-\sqrt{-1}} \right)$$

- It makes sense and is correct only if we can give a consistent meaning to $\sqrt{-1}$ (and the rest of the algebra works).
Complex numbers: Quick summary of basics

• Why? Uses?

• Definition: ordered pair of real numbers \( z = (x, y) \), with
  \[
  \begin{align*}
  & (a, b) + (c, d) \overset{\text{def}}{=} (a + c, \ b + d), \\
  & (a, b) \cdot (c, d) \overset{\text{def}}{=} (ac - bd, \ ad + bc).
  \end{align*}
  \]

• Identify:
  \[
  \begin{align*}
  & \text{real number } x = (x, 0), \\
  & i = (0, 1), \text{ so } i^2 = -1, \text{ and } z = x + iy.
  \end{align*}
  \]

• Prove standard algebraic properties same as for real numbers.

• Definitions norm \(|z|\), and conjugate \(z^*\) (or \(\bar{z}\)).

• Argand diagram, \(z = re^{i\theta}\), etc.

• Phasors to represent amplitude and phase of oscillations at definite frequency \(f(t) = \text{Re} \ a e^{i\omega t}\). (AC circuits, diffraction, etc.)