\( M \) is linear op.: \( V \to V \).

\( H = M^*M \) is a linear op. from \( V \) to \( V \).

\( H^* = (M^*M)^* = M^*M = H \).

\( H \) is Hermitian.

\( H \) can be diagonalized. Eigenvectors \( |i\rangle \) can be chosen to form an orthonormal basis of \( V \); the eigenvalues \( \lambda_i \) with \( H |i\rangle = \lambda_i |i\rangle \) are real. This all comes from standard theorems.

\[ \langle i | j \rangle = \delta_{ij} \] is the orthonormality condition.

\[ \langle i | H | i \rangle = \langle i | \lambda_i | i \rangle = \lambda_i \langle i | i \rangle = \lambda_i. \]

But

\[ \langle i | H | i \rangle = \langle i | M^*M | i \rangle = (M|i\rangle)^* M |i\rangle \]

\[ = |H |i\rangle |i\rangle \]

\[ \geq 0. \]

\[ \therefore \lambda_i \geq 0. \]

Define \( |\epsilon_i \rangle = M |i\rangle \).
\[ \langle e_i | e_j \rangle = (m_i^j)^+ m_i^j \]
\[ = \langle i | m_i^j m_i^j | i \rangle \]
\[ = \langle i | m_i^j | i \rangle \]
\[ = \delta_j^i \langle i | i \rangle \]
\[ = \delta_j^i \delta_{ij} \quad \text{(no summation convention!)} \]

If \( i \neq j \) then \( \langle e_i | e_j \rangle \), i.e., \( |e_i\rangle \) and \( |e_j\rangle \) are orthogonal.

If \( i = j \)
\[ \| e_i \| = \sqrt{\langle e_i | e_i \rangle} = \sqrt{\lambda_i} \]