(a) \[ \sin z = \frac{e^{iz} - e^{-iz}}{2i} \]

(i) The series expansion for \( \sin z \) in powers of \( z \) is convergent for all \( z \), and remains convergent after differentiation, giving \( \frac{dz}{dz} = \cos z \). The derivative is continuous, and exists for all \( z \), so there are no singularities.

(ii) Zero exist. Let \( z = x + iy \), with \( x \) \& \( y \) real.

\[ \sin z = 0 \iff e^{ix-y} = e^{-ix+y} \]

\[ \iff e^{2ix} = e^{2y} \]

\[ |e^{2ix}| = 1 \iff x \text{ only have a zero of } \sin z \text{ when } e^{2y} = 1, \text{ or } y = 0 \]

We also need \( e^{2ix} = 1 \), which is true exactly when

\[ x = \text{integer} \times \pi \]

So the zeros of \( \sin z \) are at \( z = n \pi i \) for any integer \( n \).

(b) Let \( f(z) = \frac{1}{\sin z} \)

(i) Then \( \frac{df}{dz} = -\frac{\cos z}{\sin^2 z} \). This exists is continuous except when \( \sin z = 0 \).
So the singularities of \( f(z) \) are isolated double pole at \( z = n \pi \). Notice that \( \cos n\pi = (-1)^n \neq 0 \), as at these points, the zeros in the denominator: \( -\frac{\cos z}{\sin^2 z} \) are not cancelled by zeros in the numerator.

(ii) Zeros of \( f(z) \) occur when \( \sin z = 0 \). But since \( \sin z \) is always finite, since its series expansion converges, there are no zeros of \( f(z) \).