Let \( f(z) = z + (z^*)^2 \) \[(1)\]

\[= x + iy + (x - iy)^2 \]

\[= x + x^2 - y^2 + i(y - 2xy) \]

(2)

Where real & imaginary parts of \( z \) are \( x \) & \( y \).

So the real & imaginary parts of \( f \) are

\[u(x,y) = x + x^2 - y^2 \quad v(x,y) = y - 2xy.\]

(a) From general definition

\[\frac{\partial f}{\partial z} = \frac{1}{2} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) + \frac{i}{2} \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)\]

\[= \frac{1}{2} \left( 1 + 2x + 1 - 2x \right) + \frac{i}{2} \left( -2y + 2y \right)\]

\[= 1\] \[(3)\]

\[\frac{\partial f}{\partial z^*} = \frac{1}{2} \left( \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \right) + \frac{i}{2} \left( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right)\]

\[= \frac{1}{2} \left( 1 + 2x - 1 + 2x \right) + \frac{i}{2} \left( -2y - 2y \right)\]

\[= 2x - 2iy\]

\[= 2z^*\] \[(4)\]
(b) If we treat $z$ and $z^*$ as if they were independent, eq. (1) gives

$$\frac{\partial f}{\partial z} = 1 \quad \text{and} \quad \frac{\partial f}{\partial z^*} = 2z^*,$$

which agrees with Eq. (3) or (4).