

MULTIDIMENSIONAL SCREENING IN INSURANCE MARKETS WITH ADVERSE SELECTION

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ABSTRACT

Bundled coverage of different losses and distinct perils, along with differential deductibles and policy limits, are common features of insurance contracts. We show that, through these practices, insurers can implement multidimensional screening of insurance applicants who possess hidden knowledge of their risks, and thereby reduce the externality cost of adverse selection. Competitive forces drive insurers to exploit multidimensional screening, enhancing the efficiency of insurance contracting. Moreover, multidimensional screening allows competitive insurance markets to attain pure strategy Nash equilibria over a wider range of applicant pools, resolving completely the Rothschild–Stiglitz nonexistence puzzle in markets where the perils space is sufficiently divisible.

INTRODUCTION

When insurance applicants possess hidden knowledge of their risks of incurring a loss, as in the seminal model of insurance contracting with adverse selection analyzed by Rothschild and Stiglitz (1976), the market is driven by competition for those in the low-risk class. These applicants wish to distinguish themselves from high-risk applicants in order to obtain insurance coverage at terms more favorable than would be made available if they were pooled with the high risks. The desired separation is attained through a screening mechanism whereby insurers offer applicants their choice from a menu of contracts, one of which provides high risks with full and fair insurance, while the other incorporates a lower average price for coverage but also includes a deductible that is unacceptable to high risks.

In this article, we show that separation of this sort is accomplished most effectively by multidimensional screening implemented through the contractual bundling of different perils and different losses. Bundled coverage, with differential deductibles and policy limits, allows insurers to screen applicants in several dimensions, thereby reducing the externality cost that low-risk applicants must bear to distinguish themselves from high risks. As a consequence, insurers can compete most effectively for

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the low-risk applicants by offering contracts that exploit efficient multidimensional screening.¹

The prediction that insurance markets will offer contracts with differential coverage for alternative losses and distinct perils is confirmed by commonly observed practices.² For example, a typical homeowner's policy provides bundled indemnification for both liability and property losses, with different policy limits for these losses. Furthermore, within each category of loss, a homeowner faces different levels of coverage for distinct perils that cause the same damage, or different losses caused by the same peril. Generally, there is a deductible for the home being destroyed by fire, but if the same home is destroyed by flood, the deductible is 100%, as the policy explicitly provides no coverage for this peril. Moreover, losses of specific types of property, such as jewelry, rugs, or antiques, are also subject to specific (lower) sublimits.

Similarly, automobile insurance typically bundles different levels of coverage, such as losses resulting from collision, personal injury, uninsured motorists, and medical bills, into the same policy with different sublimits depending on the peril or category of loss. It is also common to observe different deductibles for alternative perils causing the same loss. For example, an automobile lost through theft or fire is typically subject to a lower deductible through the "comprehensive" portion of the policy than would be the case were the loss a result of an accident and therefore covered by the "collision" part of the policy. Coverage limitations are also commonly observed in commercial insurance, where sublimits, coinsurance provisions, deductibles, and exclusions often vary according to the peril or type of loss.³

¹It is worth noting that some authors have examined repeated contracting in adverse selection economies when experience rating is possible, which also introduces a kind of multidimensional screening wherein signaling dimensions correspond to distinct time periods. Multidimensional screening is efficient in these contexts when principals and agents can commit to long-term contracts. Cooper and Hayes (1987) and Dionne and Doherty (1994) pursue analyses in this vein. Fluet (1992) considers a temporal version of partitioning perils, establishing that a time-dependent deductible is efficient when two risk types differ regarding the likely time of incurring the insured loss. Most notable in this regard, however, is the recent work on adverse selection in annuity markets by Rothschild (2007) and Finkelstein, Poterba, and Rothschild (2009). In their setting, the informational asymmetry pertains to the conditional survival probabilities of the agents, and the "low-risk" (from the perspective of the seller of the annuities) agents are those with the lower probability of surviving to the next period. They show that low-risk agents distinguish themselves by exhibiting a willingness to accept front-loaded payment profiles, in essence accepting lower payments in states that they are less likely to experience. Similarly, in our model, low-risk agents distinguish themselves by accepting larger deductibles for perils that they have lower probability of suffering. None of these other studies, however, consider the implications of multidimensional screening for the existence of pure strategy Nash equilibrium.

²Efficient multidimensional screening in the presence of adverse selection is not the only explanation for insurance contracts that bundle multiple perils that are potentially responsible for a given loss, with different deductibles for distinct perils. Information available to an insurer concerning the risk of loss from one peril may be relevant to the risk posed by other perils, giving rise to an economy of scope that can be exploited by bundling several perils, and different deductibles for the perils may arise from moral hazard considerations.

³See the *CPCU Handbook of Insurance Policies* (2001). Notable examples include the declarations of perils covered, and the associated amounts of indemnification, commonly encountered in

Our analysis not only shows that multidimensional screening is efficient but also shows that multidimensional screening provides an avenue for insurance markets to circumvent the nonexistence problem identified by Rothschild and Stiglitz. They establish that a pure strategy Nash equilibrium fails to exist if the proportion of high risks in the applicant pool is sufficiently low.⁴ We demonstrate that by reducing the externality cost of adverse selection, efficient multidimensional screening allows competitive insurance markets to attain pure strategy Nash equilibria over a wider range of applicant pools.⁵ Indeed, by increasing the dimensionality of the screening space, insurers reduce the cost of the adverse selection externality. We conclude that market forces compel each insurer to offer contracts with differential coverage for alternative losses and distinct perils that implement multidimensional screening, thereby enhancing both the efficiency of insurance contracting and the prospects for existence of Nash equilibrium.⁶ Indeed, the Rothschild–Stiglitz nonexistence puzzle

commercial property policies (pp. 111–128), as well as in commercial general liability policies (pp. 340–354).

⁴This finding of market failure has spawned a substantial literature aimed at resolving the existence question, including the non-Nash equilibrium concepts developed by Wilson (1977) and Riley (1979), and the Nash refinements in alternative three-stage games considered by Cho and Kreps (1987) and Hellwig (1987). Although each of these approaches admits a pure strategy equilibrium, their predictions regarding the equilibrium do not agree. Puelz and Snow (1994) provide a summary of the contractual allocations supported by these alternative theories. In a related vein, Rosenthal and Weiss (1984) exhibit a mixed-strategy Nash equilibrium that always exists given a fixed number of insurers. However, as they acknowledge, if existing firms are playing their equilibrium mixed strategies, then an entrant can play a different mixed strategy and earn positive profit. Hence, their solution to the existence problem does not apply in the environment studied by Rothschild and Stiglitz who assume (p. 631) that “[t]he market is competitive in that there is free entry.”

⁵As a result of the potential nonexistence of a pure strategy Nash equilibrium, and of the multiplicity of alternative candidates for equilibrium, positive (equilibrium) analysis of insurance markets with private information has been problematic. For example, Hoy (1982) examines the effect of categorical discrimination on market equilibrium and finds that the results are ambiguous and depend on the precise nature of the equilibrium being considered. In contrast, Crocker and Snow (1986) adopt a normative (efficiency) approach to the problem and conclude that to permit such categorization results in potential Pareto improvements relative to the no-categorization alternative. More recently, Finkelstein, Poterba, and Rothschild (2009) examine the effect of banning gender-based discrimination in annuities. Again, the problematic nature of equilibrium in these settings necessitated that they adopt a normative, rather than positive, analytical approach. Since we demonstrate in this article that competitive pressures force individual insurers to exploit multidimensional screening, and that this practice broadens the range of markets attaining a pure strategy Nash equilibrium, the scope for positive analysis of insurance market reforms is correspondingly broadened.

⁶Only one study of multidimensional screening has considered the implications for existence of Nash equilibrium. Bond and Crocker (1991) introduce a second signaling dimension by assuming correlation between risk type and the intensity of preference for a hazardous good whose consumption can be monitored by the insurer. Their approach fundamentally alters the insurance contracting environment by introducing a moral hazard component through the hidden effects of the hazardous good on the probability of incurring the loss. In contrast, multidimensional screening is introduced in our model through optimal partitioning of the signaling space without otherwise altering the adverse selection environment.

is fully resolved by multidimensional screening in markets where the perils space is sufficiently divisible.

The article proceeds as follows. In the next section, we describe the Rothschild–Stiglitz insurance market, the externality cost of adverse selection, and the potential nonexistence problem. In the section, “Efficient Multidimensional Screening,” multidimensional screening is introduced through differential levels of insurance coverage, and necessary and sufficient conditions are identified under which multidimensional screening is efficient. In the section, “Pure Strategy Nash Equilibrium With Multidimensional Screening,” we show that efficient multidimensional screening in the insurance market enhances the prospects for existence of a pure strategy Nash equilibrium. The final section contains concluding remarks.

EFFICIENCY AND NASH EQUILIBRIUM IN THE ONE-DIMENSIONAL SCREENING MODEL

In the one-dimensional screening model of insurance contracting with adverse selection developed by Rothschild and Stiglitz, competing insurers face two large classes of insurance applicants presenting independent risks. Applicants are expected utility maximizers, each endowed with an exogenous wealth W , a risk-averse von Neumann–Morgenstern utility function $U(\cdot)$, and the possibility of incurring a loss with damage D . They differ only with respect to the probability $p^t \in (0, 1)$ of incurring the loss, and each applicant belongs to one of two risk classes $t \in \{H, L\}$ with p^H greater than p^L . Although insurers know these probabilities and the proportion λ of the applicant pool belonging to the high-risk class, they do not know the risk class to which a particular applicant belongs.

An insurance contract $C \equiv (\alpha, S)$ consists of a premium α and a deductible S that together determine wealth levels $W_1 = W - \alpha - S$ if the damage is incurred and $W_0 = W - \alpha$ otherwise. Since the amount of loss D is fixed, a deductible is simply a policy indemnification that is less than the amount of loss, and this coverage limitation could instead be implemented through policy limits or coinsurance provisions. Therefore, we shall use the term “deductible” to refer to any type of coverage restriction.

The expected utility of a contract C to an insurance applicant of risk type t is defined by

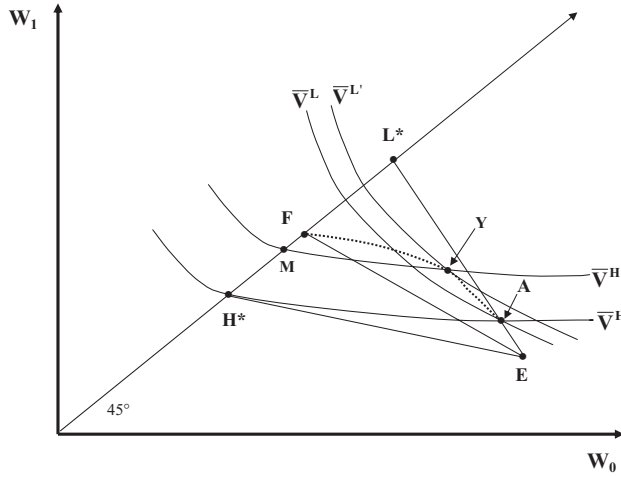
$$V^t(C) \equiv (1 - p^t)U(W_0) + p^tU(W_1). \quad (1)$$

The expected profit earned by an insurer selling contract C to an applicant of risk type t is defined by

$$\pi^t(C) \equiv \alpha - p^t(D - S). \quad (2)$$

Insurers are assumed to be risk neutral and to incur no costs other than indemnified claims.

As a consequence of the informational asymmetry, the only feasible contracts are those that satisfy the incentive compatibility constraints

FIGURE 1 $\{H^*, A\}$ Not Efficient: $\lambda < \lambda^*$ 

$$V^t(C^t) \geq V^t(C^{t'}) \text{ for } t, t' \in \{H, L\}. \quad (3)$$

An *interim incentive efficient* contract solves the problem of maximizing the expected utility of L -types, $V^L(C^L)$, subject to the incentive compatibility constraints (3), the resource constraint for the economy

$$\lambda \pi^H(C^H) + (1 - \lambda) \pi^L(C^L) = 0, \quad (4)$$

and a voluntary participation constraint for H -types,

$$V^H(C^H) \geq \bar{V}^H, \quad (5)$$

that specifies an arbitrary lower bound \bar{V}^H for their expected utilities. The class of interim incentive efficient contracts is generated by varying the participation constraint utility level.

A solution to an interim efficiency problem, which is formally characterized by Crocker and Snow (1985, 1986), can be described with reference to Figure 1. At a solution, the resource constraint (4) holds with equality, as does the incentive constraint (3) for H -type applicants. The contingent wealth position for the H -types entails full insurance, and so lies along the 45° line. The corresponding contingent wealth for L -types provides less than full insurance and is located on the (dotted) locus FA in Figure 1. One member of the class of interim efficient contracts is depicted in Figure 1 by the wealth allocations $\{M, Y\}$, which is a solution to the efficiency problem when the participation constraint utility for H -types is set at or below the level associated with the indifference curve labeled $\bar{V}^{H'}$. Higher levels of reservation utility for the H -types result in efficient wealth allocation pairs closer to point F , which corresponds

to the first-best pooling contract and is itself a solution to the efficiency problem when \bar{V}^H is set equal to $U(W - \bar{p}D)$, where $\bar{p} \equiv \lambda p^H + (1 - \lambda)p^L$.

The case depicted in Figure 1 illustrates the nonexistence problem identified by Rothschild and Stiglitz, which arises when the only candidate for a pure strategy Nash equilibrium, depicted in the figure as $\{H^*, A\}$ (and referred to in what follows as the Rothschild–Stiglitz contracts), is not contained in the set of interim incentive efficient allocations. Since both types of insurance applicants prefer some interim efficient contracts, such as those resulting in $\{M, Y\}$, to those resulting in $\{H^*, A\}$, there are potentially profitable defections that attract both types away from their Rothschild–Stiglitz contracts, which therefore cannot be sustained as a pure strategy Nash equilibrium.⁷ Note that all of the interim efficient contracts for L -types in Figure 1 lie between F and Y along locus FA and entail cross-subsidization, with losses accruing to the H -type contract offset by profits earned on the L -type contract, and that $\{M, Y\}$ is the interim efficient allocation most preferred by the L -types.

As the proportion of L -types in the population falls, however, the profitability of a defecting pair of contracts declines as the insurance pool tilts toward the losing H -type contract. In the figure, point F moves down the 45° line and the slope of locus FA declines in absolute value.⁸ At a critical value for the proportion of H -types, λ^* , the slope of locus FA at point A coincides with the marginal rate of substitution of the L -types at that point so that, for $\lambda \geq \lambda^*$, the Rothschild–Stiglitz contracts, resulting in the wealth allocations $\{H^*, A\}$ are a solution to an interim efficiency problem. In this event, there are no jointly profitable and incentive compatible defections that can attract both types of applicants away from their Rothschild–Stiglitz contracts,

⁷Rothschild and Stiglitz restrict insurers to each offer only a single contract, and their counterexample demonstrating that a pure strategy Nash equilibrium may not exist concentrates on the existence of a profitable pooling contract preferred by both types of customers to $\{H^*, A\}$. While the existence of such a pooling contract is sufficient to demonstrate nonexistence, it is not necessary, as recognized by Rothschild and Stiglitz (p. 643). Indeed, this is the case in Figure 1, where there are no profitable pooling defections of the type considered by Rothschild and Stiglitz (since the pooling fair-odds line FE lies everywhere below the indifference curve \bar{V}^L), yet the pair $\{M, Y\}$ is preferred by both types to $\{H^*, A\}$. The most attractive defections always involve separating pairs, such as $\{M, Y\}$ in which there is a cross-subsidy from L -types to the H -types. This issue is addressed in Crocker and Snow (1985).

⁸To see that the locus FA becomes flatter as λ increases, totally differentiate the binding incentive compatibility constraint (3) for H -types (as an equality) and the resource constraint (4), and then eliminate dW^H to obtain

$$-dW_1^L/dW_0^L = \frac{(1 - p^H)U'(W_0^L) + (1 - p^L)U'(W^H)(1 - \lambda)/\lambda}{p^H U'(W_1^L) + p^L U'(W^H)(1 - \lambda)/\lambda}$$

as the slope (in absolute value) of the locus FA . Partially differentiating this expression with respect to λ yields

$$\partial(-dW_1^L/dW_0^L)/\partial\lambda = \frac{-(1 - p^L)U'(W^H)/\lambda^2}{[p^H U'(W_1^L) + p^L U'(W^H)(1 - \lambda)/\lambda]^2} T,$$

where $T = p^H U'(W_1^L) - p^L(1 - p^H)U'(W_0^L)/(1 - p^L)$. This derivative is negative since we have $p^H > p^L(1 - p^H)/(1 - p^L)$ and $U'(W_1^L) > U'(W_0^L)$ when L -types are underinsured, implying that T is positive.

which as a result are sustainable as a pure strategy Nash equilibrium. Put differently, the pure strategy Nash equilibrium exists when the Rothschild–Stiglitz contracts are interim incentive efficient.

Finally, the critical value, λ^* , is the value of λ at which the Rothschild–Stiglitz contracts are the interim efficient contracts most preferred by the L -types; that is, they solve the interim efficiency problem when the participation constraint (5) is omitted. This relationship will be exploited in the section, “Pure Strategy Nash Equilibrium With Multidimensional Screening,” to demonstrate that multidimensional screening reduces λ^* and thus broadens the range of markets in which pure strategy Nash equilibria exist.

EFFICIENT MULTIDIMENSIONAL SCREENING

We introduce multidimensional screening in the insurance market setting by assuming that D is the expected damage conditional on a loss occurring, and that the potential causes of this expected damage can be partitioned into $n > 1$ mutually exclusive and collectively exhaustive perils.⁹ We use D_i to denote the loss associated with the i th peril and $\theta_i^t \in (0, 1)$ to denote the conditional probability of this loss occurring for an applicant in risk class $t \in \{H, L\}$. Insurers categorize applicants on the basis of observable traits, and we assume that in partitioning the perils space insurers know the damage D_i and the probability θ_i^t for each peril conditional on a loss occurring. The probability of a loss occurring, p^t , remains hidden knowledge, along with the conditional probabilities θ_i^t , but insurers can now exploit n signaling dimensions to screen insurance applicants.¹⁰

In this context, a contract C consists of a premium α and a vector of deductibles $S = (S_1, S_2, \dots, S_n)$ that together determine wealth $W_i = W - \alpha - S_i$ if damage is incurred as a consequence of the i th peril, and $W_0 = W - \alpha$ if no damage is incurred. The expected utility of a contract C to an insurance applicant of risk type t is now given by

$$V^t(C) \equiv (1 - p^t)U(W_0) + p^t \sum_{i=1}^n \theta_i^t U(W_i), \quad (6)$$

and the corresponding expected profit by

$$\pi^t(C) \equiv \alpha - p^t \sum_{i=1}^n \theta_i^t (D_i - S_i). \quad (7)$$

Hence, just as in the model of one-dimensional screening, there are only two types of insurance applicants and they signal their types through their deductible choices.

⁹Note that we are not introducing additional sources of risk, as in Dionne and Gollier (1992) but, rather, decomposing a given risk of loss into its distinct potential causes.

¹⁰Specifically, for peril i the insurer knows D_i , and that the conditional probability of this particular peril occurring is either θ_i^H or θ_i^L , depending on whether the insurance applicant is a type H or L individual, respectively. The knowledge of type, of course, is still privately known only by applicants.

Our innovation is to allow applicants to signal their hidden knowledge of risk type in more than one dimension through the choice of a vector of deductibles.¹¹

It is important to recognize that bundling of coverage for all of the perils into a single insurance policy is efficient. With bundling, there are only two incentive compatibility constraints (3), one for each risk class. However, if the perils were unbundled and covered by separate contracts, then additional incentive compatibility constraints would be required. Since these additional constraints cannot increase the value of efficient contracts, bundling of the perils is efficient.¹² Moreover, as in the one-dimensional screening environment, only the H -type incentive constraint is binding at a solution to an interim efficiency problem, and a solution provides H -types with full coverage and underinsures L -types. Thus, the introduction of multidimensional screening through a partitioning of the perils potentially responsible for damage does not fundamentally alter the structure of the screening environment set out by Rothschild and Stiglitz.

As in the case of one-dimensional screening where there is a single deductible, interim incentive efficient contracts maximize the expected utility of L -types subject to the incentive compatibility constraint for the H -types, the resource constraint (4), and the voluntary participation constraint for H -types (5), but now using definitions (6) and (7) for expected utility and profit in place of (1) and (2).¹³ The Lagrangian expression associated with the interim efficiency problem may be written as

$$L = V^L(C^L) + \xi_n[V^H(C^H) - V^H(C^L)] \\ + \mu_n[\lambda\pi^H(C^H) + (1 - \lambda)\pi^L(C^L)] + \delta_n[V^H(C^H) - \bar{V}^H]. \quad (8)$$

The first-order conditions characterizing the L -type contract are

$$(1 - p^L)U'(W_o^L) - \xi_n(1 - p^H)U'(W_o^L) - \mu_n(1 - \lambda)(1 - p^L) = 0 \quad (9)$$

¹¹Note that, if we were to assume that every applicant has an expected loss, conditional on a loss occurring, equal to D (so that $\sum_{i=1}^n \theta_i^H D_i = \sum_{i=1}^n \theta_i^L D_i \equiv D$), then restricting the deductibles S_i to be the same for all of the perils would reduce the contracting environment to the one-dimensional screening model considered by Rothschild and Stiglitz.

¹²Fluet and Pannequin (1997) show that bundling of independent risks in a single-umbrella policy is efficient when two risk types present an insurer with a bundle of risks. We find that bundling is not only efficient but is also part of any Nash equilibrium with multidimensional screening. Koehl and Villeneuve (2001) point out that wholly comprehensive insurance contracts are not observed in practice and that insurers instead specialize for legal or strategic reasons, as when exclusivity cannot be enforced. Their analysis of adverse selection reveals that there are efficiency losses as a consequence, since specialization weakens the ability of insurers to screen applicants. We follow Rothschild and Stiglitz (p. 632) and assume that insurers can practice exclusivity, and pursue their remark (p. 630) acknowledging that actual insurance contracts offer coverage for many potential losses or perils. While these contracts are not wholly comprehensive, they do offer efficiency gains over complete specialization of coverage offerings as expected, and by exploiting multidimensional screening they also have implications for existence of equilibrium.

¹³The L -type incentive compatibility constraint is never binding in this interim incentive efficiency problem, just as is the case with one-dimensional screening, as shown by Crocker and Snow (1985).

for the no-loss state, and for all screening dimensions i ¹⁴

$$p^L \theta_i^L U'(W_i^L) - \xi_n p^H \theta_i^H U'(W_i^L) - \mu_n (1 - \lambda) p^L \theta_i^L = 0, \quad (10)$$

while those for the corresponding H -type contract are

$$(\xi_n + \delta_n)(1 - p^H)U'(W_o^H) - \mu_n \lambda (1 - p^H) = 0, \quad (11)$$

and for all screening dimensions i

$$(\xi_n + \delta_n)p^H \theta_i^H U'(W_i^H) - \mu_n \lambda p^H \theta_i^H = 0. \quad (12)$$

Equations (11) and (12) imply that marginal utility for H -types is the same in every state, so that $W_o^H = W_i^H \equiv W^H$ and the H -type contract results in full insurance. From Equation (10), it follows that for screening dimension k , we must have

$$(1 - \xi_n p^H \theta_k^H / p^L \theta_k^L) U'(W_k^L) = \mu_n (1 - \lambda), \quad (13)$$

implying that $U'(W_k^L)$ varies directly with θ_k^H / θ_k^L . We thus have

$$U'(W_j^L) > (=) [<] U'(W_i^L)$$

when $\theta_j^H / \theta_j^L > (=) [<] \theta_i^H / \theta_i^L$. Since $U(\cdot)$ is strictly concave, reflecting risk aversion, it follows that $S_i^L < (=) [>] S_j^L$ as $\theta_i^L / \theta_j^L > (=) [<] \theta_i^H / \theta_j^H$. This establishes the following result concerning the pattern of deductibles in an interim efficient L -type contract with multidimensional screening.

Theorem 1: *In a solution to the interim efficiency problem with multidimensional screening, $S_i^L < (=) [>] S_j^L$ as $\theta_i^L / \theta_j^L > (=) [<] \theta_i^H / \theta_j^H$.*

In what follows, we shall say that multidimensional screening is *efficient* if, at a solution to the interim efficiency problem, $S_i^L \neq S_j^L$ for some dimensions i and j , so that L -types bear a deductible that differs according to the peril. Notice that efficient multidimensional screening always follows the pattern of comparative advantage enjoyed by L -types for signaling in one dimension rather than another. Moreover, this pattern depends solely on the exogenous conditional probabilities associated with the perils. This result is analogous to that obtained by Rothschild (2007) and Finkelstein, Poterba, and Rothschild (2009) in an annuity setting, where the “low-risk” annuitants (those with lower survival probabilities) distinguish themselves by their willingness

¹⁴For simplicity, we abstract from corner solutions, in which L -types have a zero deductible in one or more dimensions, and from cases in which any of the natural constraints $S_k^L \leq D_k$ for all k are binding. We note, however, that if $\theta_k^L = 0 < \theta_k^H$ obtains, then $\xi = 0$ when $S_k^L \leq D_k$ is not a binding constraint, and the adverse selection externality is eliminated by a multidimensional screening contract in which the L -type deductible is positive only in the k th dimension.

to accept lower annuity payments in future years. Those that are “high risk” (with higher survival probabilities), however, find such front loading to be more costly since they have a better chance of living long enough to suffer the effects of the lower annuity payments. Put differently, the low risks signal their types by accepting lower annuity payments in states that they do not expect to experience.

Finally, suppose that a finer partitioning of the i th signaling dimension can divide the conditional probability θ_i^t between two perils, one with conditional probability θ_{i1}^t and the other with probability θ_{i2}^t . The following corollary to Theorem 1 shows that this finer partitioning of the i th dimension is efficient if and only if $\theta_{i1}^L/\theta_{i2}^L \neq \theta_{i1}^H/\theta_{i2}^H$.

Corollary: *Partitioning the i th signaling dimension increases the value of the interim incentive efficiency problem if and only if $\theta_{i1}^L/\theta_{i2}^L \neq \theta_{i1}^H/\theta_{i2}^H$.*

Proof: From Theorem 1, we have $S_{i1}^L < (=) [>] S_{i2}^L$ as $\theta_{i1}^L/\theta_{i2}^L > (=) [<] \theta_{i1}^H/\theta_{i2}^H$, implying that screening with different deductibles $S_{i1}^L \neq S_{i2}^L$ is efficient if and only if $\theta_{i1}^L/\theta_{i2}^L \neq \theta_{i1}^H/\theta_{i2}^H$. It follows that constraining the interim efficiency problem by requiring $S_{i1}^L = S_{i2}^L$ would reduce the value of the problem if and only if $\theta_{i1}^L/\theta_{i2}^L \neq \theta_{i1}^H/\theta_{i2}^H$. Q.E.D.

While H -types are unaffected by the introduction of multidimensional screening, since their efficient contract always satisfies their voluntary participation constraint (5) as an equality, L -types benefit in three ways. First, the bundling of separate perils is valuable to the L -types since bundling reduces the number of incentive constraints that must be satisfied, as compared to the unbundled alternative. Second, they obtain relatively more coverage for the perils they are more likely to experience relative to the H -types, and third, this shift in coverage makes their contract less attractive to H -types, relaxing the incentive constraint and thus further reducing the cost of adverse selection to the benefit of L -types. Hence, adopting efficient multidimensional screening is Pareto improving and each insurer, competing for L -types, will be led by market forces to implement efficient multidimensional screening. We shall henceforth assume that multidimensional screening is efficient.

PURE STRATEGY NASH EQUILIBRIUM WITH MULTIDIMENSIONAL SCREENING

In this section, we establish that efficient multidimensional screening improves the prospects for existence of a pure strategy Nash equilibrium by reducing the critical value of λ below which Nash equilibrium does not exist. It is important to recognize that this result is not a trivial consequence of the assumed efficiency of multidimensional screening since, as an implication of Theorem 1, all of the second-best incentive efficient contracts are more valuable to L -types with multidimensional screening.¹⁵ Thus, L -types are better off with multidimensional screening than they are with

¹⁵Increased efficiency does not necessarily improve the prospects for existence of Nash equilibrium, as demonstrated by Hoy (1982) and Crocker and Snow (1986). The latter show that risk categorization by immutable observable traits is efficient when the proportion of H -types varies across the categories, so that categorization is informative. However, the implications for existence are ambiguous. The prospects for existence of Nash equilibrium are enhanced

one-dimensional screening not only at the candidate for Nash equilibrium but also at any potential defection. The intuition behind our existence result is that incentive feasible contracts farther removed from the first-best pool, F , entail higher adverse selection costs for L -types, who then gain more from the introduction of multidimensional screening. With reference to Figure 1, L -types gain more from the introduction of efficient multidimensional screening at point A than at point Y , diminishing the attractiveness of their defecting contract.

Recall that in the case of one-dimensional screening, there exists a critical value, λ^* , at and above which the Rothschild–Stiglitz contracts are interim incentive efficient. A parallel result holds in an environment with multidimensional screening. The analogue to the Rothschild–Stiglitz contracts in such an environment provides the H -types with full and fair insurance, and hence provides them the sure wealth W_{RS}^H that results in point H^* in Figure 1, but the L -type contract incorporates differential deductibles to implement efficient multidimensional screening while still earning zero profit.¹⁶ Furthermore, just as in the one-dimensional case, the multidimensional Rothschild–Stiglitz contracts are sustainable as a Nash equilibrium if and only if they are interim incentive efficient.

Assume that n -dimensional screening is efficient and let λ_n^* denote the critical value for the proportion of H -types such that Nash equilibrium with n -dimensional screening exists if and only if $\lambda \geq \lambda_n^*$. We shall establish that with $n > 1$, $\lambda_n^* < \lambda^*$, so that multidimensional screening permits Nash equilibrium to exist in some markets where it does not exist with one-dimensional screening. To this end, we first show that the interim efficient pair of multidimensional screening contracts most preferred by the L -types is the multidimensional Rothschild–Stiglitz pair if and only if $\lambda = \lambda_n^*$. Recall that, in the case of one-dimensional screening, the Rothschild–Stiglitz contracts are the interim efficient contracts most preferred by L -types if these contracts solve the efficiency problem when the participation constraint is omitted and that this occurs if and only if $\lambda = \lambda^*$. We now formally establish and generalize this result in the following lemma.

Lemma 1: *The multidimensional Rothschild–Stiglitz contracts are the interim efficient contracts most preferred by L -types if and only if $\lambda = \lambda_n^*$.*

Proof: Consider the interim efficiency problem with the profit constraint

$$\pi^H(C^H) = \bar{\pi}^H \quad (14)$$

replacing the participation constraint (5). Observe that when $\bar{\pi}^H = 0$, this additional constraint ensures that the solution value for H -type wealth is W_{RS}^H corresponding to full and fair insurance, implying that the solution for the L -type contract is their

for the category in which H -types predominate but are worsened for the category in which L -types are more prevalent.

¹⁶More precisely, the multidimensional analogue to the Rothschild–Stiglitz contracts provides the H -type with the full and fair insurance wealth level W_{RS}^H , and the deductibles paid by the L -type are a solution to the interim efficiency problem when the participation constraint is $V^H(C^H) = U(W_{RS}^H)$.

multidimensional Rothschild–Stiglitz contract. Introducing the Lagrange multiplier γ for the profit constraint (14), the Lagrangian function for the efficiency problem is

$$\begin{aligned}\tilde{L} = & V^L(C^L) + \xi_n[V^H(C^H) - V^H(C^L)] \\ & + \tilde{\mu}_n[\lambda\pi^H(C^H) + (1 - \lambda)\pi^L(C^L)] + \gamma[\pi^H(C^H) - \bar{\pi}^H].\end{aligned}\quad (15)$$

Setting $\bar{\pi}^H$ equal to zero, and treating wealth levels as the choice variables, the first-order condition characterizing W_{RS}^H is

$$\partial\tilde{L}/\partial W^H = \xi_n U'(W_{RS}^H) - \tilde{\mu}_n\lambda - \gamma = 0.$$

Let \bar{W}^H denote the solution to the incentive efficiency problem when the profit constraint (14) is omitted, so that the solution is the interim efficient pair of contracts most preferred by L -types. If the profit constraint (14) is not binding for the Lagrangian problem (15) when $\bar{\pi}^H = 0$, then $\gamma = 0$ and $\bar{W}^H = W_{RS}^H$, and in that event the interim efficient contracts most preferred by L -types are the multidimensional Rothschild–Stiglitz contracts.

Now suppose the profit constraint is binding when $\bar{\pi}^H = 0$ and that $\gamma > 0$. Then, with \tilde{L}^* denoting the solution value of the Lagrangian function (15), we must have

$$d\tilde{L}^*/d\bar{\pi}^H = -\gamma < 0,$$

indicating that the solution value for V^L would be higher if the profit constraint were relaxed and π^H were allowed to be negative. As this would increase the expected utility of H -types as well, since then $\bar{W}^H > W_{RS}^H$, the Rothschild–Stiglitz contracts with multidimensional screening are not, in this case, interim incentive efficient and so λ must be less than λ_n^* .

On the other hand, if $\gamma < 0$ when $\bar{\pi}^H = 0$ and the profit constraint is binding, then the solution value for V^L would be higher if π^H were allowed to be positive. This change would necessarily make H -types worse off, since then $\bar{W}^H < W_{RS}^H$. In this instance, the Rothschild–Stiglitz contracts with multidimensional screening are interim efficient, and so λ must be greater than λ_n^* , but they are not the interim efficient contracts most preferred by L -types. It follows from these observations that $\bar{W}^H = W_{RS}^H$ if and only if $\lambda = \lambda_n^*$. Q.E.D.

Now consider the interim efficiency problem with the participation constraint omitted and the associated Lagrangian function (8). The solution is the interim efficient allocation with multidimensional screening most preferred by L -types, and the first-order condition characterizing \bar{W}^H can be obtained from Equation (11) by setting $\delta_n = 0$ to eliminate the participation constraint. The resulting first-order condition is

$$\xi_n U'(\bar{W}^H) - \mu_n\lambda = 0,$$

from which it follows that

$$U'(\bar{W}^H) = \mu_n\lambda/\xi_n,$$

where ξ_n and μ_n are the solution multipliers for the Lagrangian (8) when δ_n is set equal to zero. Since the Rothschild–Stiglitz contracts solve this interim efficiency problem when $\lambda = \lambda_n^*$, we have

$$U'(W_{RS}^H) = \mu_n \lambda_n^* / \xi_n$$

as a consequence of Lemma 1.

Now suppose that the conditions of the corollary to Theorem 1 apply to the n th screening dimension, so that there exists a partitioning of the n th dimension that increases the value of any interim efficiency problem. For efficient $(n+1)$ -dimensional screening, the critical value λ_{n+1}^* satisfies

$$U'(W_{RS}^H) = \mu_{n+1} \lambda_{n+1}^* / \xi_{n+1},$$

and therefore

$$\mu_n \lambda_n^* / \xi_n = \mu_{n+1} \lambda_{n+1}^* / \xi_{n+1}, \quad (16)$$

where ξ_{n+1} and μ_{n+1} are the Lagrange multipliers associated with efficient $(n+1)$ -dimensional screening (with $\delta_{n+1} = 0$). Thus, once we demonstrate that

$$\mu_{n+1} / \xi_{n+1} > \mu_n / \xi_n, \quad (17)$$

Equation (16) implies $\lambda_{n+1}^* < \lambda_n^*$, which yields the particular result $\lambda_{n+1}^* < \lambda^*$. Moreover, we shall have established that efficient finer partitioning of the perils space always reduces the critical value of λ and expands the range of markets in which Nash equilibrium exists.

To establish Equation (17), we analyze two programs: (A), which exploits efficient n -dimensional screening, and (B), which efficiently partitions the n th dimension into two distinct perils. Both programs consist in maximizing $V^L(C^L)$ subject to

$$V^H(C^L) \leq \bar{V}^H$$

and

$$\pi^L(C^L) \geq \bar{\pi}^L.$$

Consider first program (A) with solution multipliers $\hat{\xi}_n$ and $\hat{\mu}_n$. We show in Lemma 2 that when \bar{V}^H and $\bar{\pi}^L$ are set equal to their solution values for the interim efficiency problem with the n -dimensional screening of program (A) and with the participation constraint omitted, $\hat{\xi}_n = \xi_n$ and $\hat{\mu}_n = \mu_n(1 - \lambda)$. A parallel result holds for program (B), so that $\hat{\xi}_{n+1} = \xi_{n+1}$ and $\hat{\mu}_{n+1} = \mu_{n+1}(1 - \lambda)$. We then establish that

$$\hat{\mu}_{n+1} / \hat{\xi}_{n+1} > \hat{\mu}_n / \hat{\xi}_n, \quad (18)$$

from which (17) then follows as an immediate consequence of Lemma 2. In proving this and subsequent results, we exploit the fact that Theorem 1 concerning the pattern of efficient multidimensional screening in the interim efficiency problem applies as well to programs (A) and (B).

Lemma 2: *When \bar{V}^H and $\bar{\pi}^L$ are set equal to their solution values for the incentive efficiency problem with the n -dimensional screening of program (A) and with the participation constraint omitted, we have $\hat{\xi}_n = \xi_n$ and $\hat{\mu}_n = \mu_n(1 - \lambda)$, and with the $(n+1)$ -dimensional screening of program (B), we have $\hat{\xi}_{n+1} = \xi_{n+1}$ and $\hat{\mu}_{n+1} = \mu_{n+1}(1 - \lambda)$.*

Proof: The first-order conditions characterizing the L -type contract for program (A) are

$$(1 - p^L)U'(W_o^L) - \hat{\xi}_n(1 - p^H)U'(W_o^L) - \hat{\mu}_n(1 - p^L) = 0 \quad (19)$$

for the no-loss state, and for all screening dimensions i

$$p^L \theta_i^L U'(W_i^L) - \hat{\xi}_n p^H \theta_i^H U'(W_i^L) - \hat{\mu}_n p^L \theta_i^L = 0. \quad (20)$$

The first-order conditions for the corresponding incentive efficiency problem, whose Lagrangian is given by Equation (8) with $\delta_n = 0$, are

$$(1 - p^L)U'(W_o^L) - \xi_n(1 - p^H)U'(W_o^L) - \mu_n(1 - \lambda)(1 - p^L) = 0$$

for the no-loss state, and for all screening dimensions i

$$p^L \theta_i^L U'(W_i^L) - \xi_n p^H \theta_i^H U'(W_i^L) - \mu_n(1 - \lambda)p^L \theta_i^L = 0.$$

Subtracting the second set from the first yields

$$(\xi_n - \hat{\xi}_n)(1 - p^H)U'(W_o^L) + [\mu_n(1 - \lambda) - \hat{\mu}_n](1 - p^L) = 0$$

and, for all screening dimensions i ,

$$(\xi_n - \hat{\xi}_n)p^H \theta_i^H U'(W_i^L) + [\mu_n(1 - \lambda) - \hat{\mu}_n]p^L \theta_i^L = 0.$$

Together the last two equations imply

$$(\xi_n - \hat{\xi}_n)(1 - p^H)U'(W_o^L)/(1 - p^L) = (\xi_n - \hat{\xi}_n)p^H \theta_i^H U'(W_i^L)/p^L \theta_i^L$$

for all screening dimensions i . Suppose $\hat{\xi}_n \neq \xi_n$. Then the preceding equation implies

$$\frac{\theta_i^H}{\theta_i^L} U'(W_i^L) = \frac{\theta_j^H}{\theta_j^L} U'(W_j^L)$$

for all screening dimensions i and j . Without loss of generality, we may assume that $\theta_i^H/\theta_i^L < \theta_j^H/\theta_j^L$, which from Theorem 1 implies that $W_i^L > W_j^L$ and $U'(W_i^L) < U'(W_j^L)$, contradicting the equality. Hence, we must have $\hat{\xi}_n = \xi_n$, which implies $\hat{\mu}_n = \mu_n(1 - \lambda)$. The same line of reasoning applied to program (B) and the case of $(n+1)$ -dimensional screening yields $\hat{\xi}_{n+1} = \xi_{n+1}$ and $\hat{\mu}_{n+1} = \mu_{n+1}(1 - \lambda)$. Q.E.D.

In the final two lemmas, we establish that whenever program (A) provides less than full coverage for L -types, we have $\hat{\xi}_{n+1} < \hat{\xi}_n$ and $\hat{\mu}_{n+1} > \hat{\mu}_n$. Inequality (18) is then an immediate consequence of these results. Let $(\bar{V}^{H*}, \bar{\pi}^{L*})$ denote a pair of values for the constraint parameters such that L -types receive full coverage in the solution to program (A), and hence in the solution to program (B) as well. Note that for any such pair $\hat{\xi}_{n+1} = \hat{\xi}_n$ and $\hat{\mu}_{n+1} = \hat{\mu}_n$. The following Lemma first shows that $\hat{\xi}_{n+1} < \hat{\xi}_n$ when $(\bar{V}^H, \bar{\pi}^{L*})$ is near $(\bar{V}^{H*}, \bar{\pi}^{L*})$ and the solution to program (A) provides L -types with less than full coverage, and then exploits this result to show that $\hat{\xi}_{n+1} < \hat{\xi}_n$ whenever the solution to program (A) entails less than full coverage. In a similar manner, the subsequent Lemma shows that $\hat{\mu}_{n+1} > \hat{\mu}_n$ when $(\bar{V}^{H*}, \bar{\pi}^L)$ is near $(\bar{V}^{H*}, \bar{\pi}^{L*})$, and then extends this result to any pair $(\bar{V}^H, \bar{\pi}^L)$ that result in less than full coverage for L -types.¹⁷

Lemma 3: *Whenever the solution to program (A) entails less than full coverage for L -types, we have $\hat{\xi}_{n+1} < \hat{\xi}_n$.*

Proof: Let the solution values for programs (A) and (B) be denoted by V_n^L and V_{n+1}^L , respectively. Then, with $\bar{\pi}^L = \bar{\pi}^{L*}$, we have

$$\lim_{\bar{V}^H \uparrow \bar{V}^{H*}} (V_{n+1}^L - V_n^L) = 0.$$

Since multidimensional screening is efficient, $V_{n+1}^L - V_n^L > 0$ whenever $\bar{V}^H < \bar{V}^{H*}$ and $\bar{\pi}^L = \bar{\pi}^{L*}$. Hence, $V_{n+1}^L - V_n^L$ must be decreasing in \bar{V}^H somewhere in any neighborhood of $(\bar{V}^{H*}, \bar{\pi}^{L*})$, that is, for any $\delta > 0$ there is some $\varepsilon \in (0, \delta)$ such that, with $\bar{V}^H = \bar{V}^{H*} - \varepsilon$ and $\bar{\pi}^L = \bar{\pi}^{L*}$, we have $d(V_{n+1}^L - V_n^L)/d\bar{V}^H < 0$. As envelope results from the Lagrangian functions for programs (A) and (B), we have

$$dV_n^L = \hat{\xi}_n d\bar{V}^H - \hat{\mu}_n d\bar{\pi}^L \quad \text{and} \quad dV_{n+1}^L = \hat{\xi}_{n+1} d\bar{V}^H - \hat{\mu}_{n+1} d\bar{\pi}^L.$$

With $d\bar{\pi}^L = 0 > d\bar{V}^H$, we then have $d(V_{n+1}^L - V_n^L) = (\hat{\xi}_{n+1} - \hat{\xi}_n)d\bar{V}^H > 0$ given $d(V_{n+1}^L - V_n^L)/d\bar{V}^H < 0$, implying $\hat{\xi}_{n+1} < \hat{\xi}_n$ for parameters $(\bar{V}^H, \bar{\pi}^{L*})$.

Now suppose that for some values of \bar{V}^H and $\bar{\pi}^L$ such that the solution to program (A) entails less than full coverage for L -types, we have $\hat{\xi}_{n+1} \geq \hat{\xi}_n$. Then, since $\hat{\xi}_{n+1} < \hat{\xi}_n$ for some other parameter values for which programs (A) and (B) provide nearly full coverage, and the environment is continuous, there would exist values for \bar{V}^H and

¹⁷This approach to establishing Lemmas 3 and 4 was suggested by an anonymous referee whose insights here and elsewhere have greatly improved the quality of this article.

$\bar{\pi}^L$ such that $\hat{\xi}_{n+1} = \hat{\xi}_n$. Solving Equations (19) and (20) for $\hat{\xi}_n$ when Equation (20) is evaluated for the n th screening dimension, we find

$$\hat{\xi}_n = \frac{1 - \hat{U}'_n / \hat{U}'_o}{\frac{1 - p^H}{1 - p^L} - \frac{p^H \theta_n^H}{p^L \theta_n^L} \frac{\hat{U}'_n}{\hat{U}'_o}}, \quad (21)$$

where we use \hat{U}'_n and \hat{U}'_o denote the solution values for program (A) for marginal utilities in the n th peril dimension and the no-loss state, respectively, to distinguish them from their values at the solution to program (B). Recall that program (B) differs from program (A) as it efficiently partitions the n th dimension into two distinct perils that we shall index by j and k . Solving the equations corresponding to (19) and (20) for program (B), we arrive at

$$\hat{\xi}_{n+1} = \frac{1 - U'_i / U'_o}{\frac{1 - p^H}{1 - p^L} - \frac{p^H \theta_i^H}{p^L \theta_i^L} \frac{U'_i}{U'_o}} \quad \text{for } i = j, k. \quad (22)$$

By Equating (21) and (22) and rearranging terms, we obtain

$$\left(\frac{1 - p^H}{1 - p^L} - \frac{p^H \theta_n^H}{p^L \theta_n^L} \right) \frac{\hat{U}'_n}{\hat{U}'_o} = \frac{U'_i}{U'_o} \left[\frac{1 - p^H}{1 - p^L} - \frac{p^H \theta_n^H}{p^L \theta_n^L} \frac{\hat{U}'_n}{\hat{U}'_o} + \frac{p^H \theta_i^H}{p^L \theta_i^L} \left(\frac{\hat{U}'_n}{\hat{U}'_o} - 1 \right) \right]$$

for $i = j, k$. As the right-hand side therefore has the same value for $i = j$ and $i = k$, we have

$$\begin{aligned} U'_j & \left[\frac{1 - p^H}{1 - p^L} - \frac{\hat{U}'_n}{\hat{U}'_o} \frac{p^H \theta_n^H}{p^L \theta_n^L} + \frac{p^H \theta_j^H}{p^L \theta_j^L} \left(\frac{\hat{U}'_n}{\hat{U}'_o} - 1 \right) \right] \\ & = U'_k \left[\frac{1 - p^H}{1 - p^L} - \frac{\hat{U}'_n}{\hat{U}'_o} \frac{p^H \theta_n^H}{p^L \theta_n^L} + \frac{p^H \theta_k^H}{p^L \theta_k^L} \left(\frac{\hat{U}'_n}{\hat{U}'_o} - 1 \right) \right]. \end{aligned} \quad (23)$$

Without loss of generality, we may assume $\theta_j^H / \theta_j^L > \theta_k^H / \theta_k^L$, which by Theorem 1 implies $U'_j > U'_k$ and then, since the less than full insurance assumption implies that \hat{U}'_n is greater than \hat{U}'_o , the left-hand side of (23) must exceed the right-hand side, contradicting the equality. Hence, the supposition that we can have $\hat{\xi}_{n+1} \geq \hat{\xi}_n$ is false, and so we must have $\hat{\xi}_{n+1} < \hat{\xi}_n$ whenever program (A) provides L -types with less than full coverage. Q.E.D.

Lemma 3 shows that the marginal cost of the incentive constraint declines with finer efficient partitioning of the perils space. Our final lemma establishes a corresponding result for the multipliers of the resource constraint, namely, that the marginal value of relaxing the resource constraint increases with finer efficient partitioning of the perils space.

Lemma 4: *Whenever the solution to program (A) entails less than full coverage for L -types, we have $\hat{\mu}_{n+1} > \hat{\mu}_n$.*

Proof: Following the same line of reasoning used in Lemma 3 to establish that $\hat{\xi}_{n+1} < \hat{\xi}_n$ for some parameter values $(\bar{V}^H, \bar{\pi}^{L*})$ such that programs (A) and (B) provide nearly full coverage, while reversing the roles of \bar{V}^H and $\bar{\pi}^L$ by holding $\bar{V}^H = \bar{V}^{H*}$ constant and reducing $\bar{\pi}^L$ below $\bar{\pi}^{L*}$, leads to the conclusion that $\hat{\mu}_{n+1} > \hat{\mu}_n$ for some parameter values $(\bar{V}^{H*}, \bar{\pi}^L)$ for which programs (A) and (B) provide L -types with nearly full coverage.

Now suppose that, for some values of \bar{V}^H and $\bar{\pi}^L$ such that the solution to program (A) entails less than full coverage for L -types, we have $\hat{\mu}_{n+1} \leq \hat{\mu}_n$. Then, since $\hat{\mu}_{n+1} > \hat{\mu}_n$ for some other parameter values for which programs (A) and (B) provide nearly full coverage, and the environment is continuous, there would exist values for \bar{V}^H and $\bar{\pi}^L$ such that $\hat{\mu}_{n+1} = \hat{\mu}_n$. From the first-order conditions (19) and (20) for program (A), we obtain

$$\hat{\mu}_n = \left[1 - \hat{\xi}_n \frac{1 - p^H}{1 - p^L} \right] \hat{U}'_o, \quad (24)$$

and for each screening dimension

$$\hat{\mu}_n = \left[1 - \hat{\xi}_n \frac{p^H \theta_i^H}{p^L \theta_i^L} \right] \hat{U}'_i. \quad (25)$$

From the corresponding first-order conditions for program (B), we obtain

$$\hat{\mu}_{n+1} = \left[1 - \hat{\xi}_{n+1} \frac{1 - p^H}{1 - p^L} \right] U'_o \quad (26)$$

and for each screening dimension

$$\hat{\mu}_{n+1} = \left[1 - \hat{\xi}_{n+1} \frac{p^H \theta_i^H}{p^L \theta_i^L} \right] U'_i. \quad (27)$$

The supposition that $\hat{\mu}_{n+1} \leq \hat{\mu}_n$, and the continuity of the problem, implies the existence of values for \bar{V}^H and $\bar{\pi}^L$ such that the right-hand sides of Equations (24) and (26) are equal, and the right-hand sides of Equations (25) and (27) are equal for the first $n - 1$ screening dimensions. Since L -types receive less than full coverage, we have $\hat{\xi}_{n+1} < \hat{\xi}_n$ from Lemma 3, which implies that the term within brackets in Equation (26) exceeds the corresponding term in Equation (24), and the term within brackets in Equation (27) exceeds the corresponding term in (25) for the first $n - 1$ screening dimensions. It now follows that we must have $U'_o < \hat{U}'_o$ and $U'_i < \hat{U}'_i$ for $i = 1, \dots, n - 1$, implying that $W_o^L > \hat{W}_o^L$ and $W_i^L > \hat{W}_i^L$ for $i = 1, \dots, n - 1$, where \hat{W}_o^L and \hat{W}_i^L , and

W_o^L and W_i^L , denote solution values for wealth in programs (A) and (B), respectively. Since expected profit is, by construction, the same for both programs, we must have

$$\begin{aligned} (1 - p^L)(W_o^L - \hat{W}_o^L) + p^L \sum_{i=1}^{n-1} \theta_i^L (W_i^L - \hat{W}_i^L) \\ = p^L \theta_n^L \hat{W}_n - p^L (\theta_j^L W_j^L + \theta_k^L W_k^L) > 0, \end{aligned}$$

where $\theta_n^L = \theta_j^L + \theta_k^L$. Hence, \hat{W}_n^L must exceed one or the other, or both, of W_j^L and W_k^L . Without loss of generality, we may assume $\theta_j^H / \theta_j^L > \theta_k^H / \theta_k^L$, which by Theorem 1 implies $W_k^L > W_j^L$, so that we must then have $\hat{W}_n^L > W_j^L$ and $\hat{U}_n' < U_j'$.

Now consider an amended program (B) in which an additional constraint is introduced requiring

$$W_j^L - W_k^L \geq 0. \quad (28)$$

Since the solution to program (B) entails $W_k^L > W_j^L$, the additional constraint is binding at a solution to the amended program, and therefore, the solution is the same as the solution to program (A). The first-order condition for the j th screening dimension in the amended program (B) is

$$p^L \theta_j^L \hat{U}_n' - \xi_n p^H \theta_j^H \hat{U}_n' - \hat{\mu}_n p^L \theta_j^L + \hat{\gamma} = 0, \quad (29)$$

where $\hat{\gamma}$ is the Lagrange multiplier for the additional constraint. Notice that the first-order condition for the k th screening dimension is

$$p^L \theta_k^L \hat{U}_n' - \xi_n p^H \theta_k^H \hat{U}_n' - \hat{\mu}_n p^L \theta_k^L - \hat{\gamma} = 0, \quad (30)$$

and that (29) and (30) sum to

$$p^L \theta_n^L \hat{U}_n' - \xi_n p^H \theta_n^H \hat{U}_n' - \hat{\mu}_n p^L \theta_n^L = 0,$$

which is the first-order condition for the n th screening dimension in program (A). Since constraint (28) is binding, $\hat{\gamma}$ is positive.

From Equation (29), we obtain

$$\left[1 - \xi_n \frac{p^H \theta_j^H}{p^L \theta_j^L} \right] \hat{U}_n' = \hat{\mu}_n - \hat{\gamma} / p^L \theta_j^L, \quad (31)$$

and from the first-order condition for the j th screening dimension in the original program (B),

$$p^L \theta_j^L U_j' - \xi_{n+1} p^H \theta_j^H U_j' - \hat{\mu}_{n+1} p^L \theta_j^L = 0,$$

we obtain

$$\left[1 - \hat{\xi}_{n+1} \frac{p^H \theta_j^H}{p^L \theta_j^L} \right] \hat{U}_j = \hat{\mu}_{n+1}. \quad (32)$$

Since $\hat{\xi}_n > \hat{\xi}_{n+1}$ and $U'_j > \hat{U}'_n$, the left-hand side of Equation (32) exceeds the left-hand side of Equation (31), implying

$$\hat{\mu}_{n+1} > \hat{\mu}_n - \hat{\gamma} / p^L \theta_j^L,$$

and since $\hat{\gamma}$ is positive, we have $\hat{\mu}_{n+1} > \hat{\mu}_n$, contradicting the supposition that $\hat{\mu}_{n+1} \leq \hat{\mu}_n$. Hence, for all values of \bar{V}^H and $\bar{\pi}^L$ that result in less than full coverage in program (A), we must have $\hat{\mu}_{n+1} > \hat{\mu}_n$. Q.E.D.

As a result of Lemmas 3 and 4, inequality (18) must hold, so that

$$\mu_{n+1}(1 - \lambda) / \xi_{n+1} = \hat{\mu}_{n+1} / \hat{\xi}_{n+1} > \hat{\mu}_n / \hat{\xi}_n = \mu_n(1 - \lambda) / \xi_n,$$

where the equalities follow from Lemma 2. Thus, from Equation (16), we obtain the following result.

Theorem 2: *If n -dimensional screening is efficient and it is efficient to further partition one of the screening dimensions into two distinct perils, then $\lambda_n^* > \lambda_{n+1}^*$.*

This theorem shows that the implementation of efficient multidimensional screening in any dimension reduces the critical value λ_n^* and therefore improves the prospects for the existence of Nash equilibrium. As a consequence, implementation of efficient multidimensional screening increases the scope for Nash equilibrium to characterize the outcome of competitive contracting in insurance markets with adverse selection.

To pursue the implications of Theorem 2 further, suppose that the partitioning of the n th signaling dimension increases the value of the incentive efficiency problem. Then, from the corollary to Theorem 1, there exist fractions β^L and β^H such that L -types have a stronger comparative advantage in one of the parts than in the unpartitioned signaling dimension; that is, we have

$$\min \{ \beta^L \theta_n^L / \beta^H \theta_n^H, (1 - \beta^L) \theta_n^L / (1 - \beta^H) \theta_n^H \} < \theta_n^L / \theta_n^H.$$

Without loss of generality, assume $\beta^L < \beta^H$, so that $\beta^L \theta_n^L / \beta^H \theta_n^H < \theta_n^L / \theta_n^H$. For each risk type t , define $\theta_{n+1}^t = \beta^t \theta_n^t$. Assume that partitioning dimension $n + 1$ increases the value of the incentive efficiency problem and that further partitioning of that part in which the L -types have a stronger comparative advantage continues to increase the value of the incentive efficiency problem. In that case, the limit value of the smallest conditional probability ratio is zero, and the first-order condition for efficient signaling in that dimension implies that the limit value for ξ is zero. Then, since W^H is positive,

the first-order condition

$$\xi U'(W_{RS}^H) - \mu \lambda^* = 0$$

must hold, implying that the limit value for λ^* is zero. Hence, with continued finer partitioning of the perils space, efficient screening approaches first best. We also conclude that for any value of $\lambda \in (0, 1)$, efficient multidimensional screening leads to $\lambda_n^* < \lambda$ if the screening space can be sufficiently finely partitioned, eliminating the nonexistence problem identified by Rothschild and Stiglitz.

CONCLUSIONS

Bundled coverage of different losses and distinct perils with differential deductibles and policy limits allows insurers to implement multidimensional screening of insurance applicants who possess hidden knowledge of their risks of loss. As in the case of one-dimensional screening examined by Rothschild and Stiglitz, competition for low-risk applicants governs the level of signaling and the existence of Nash equilibrium, but is now also seen to determine the number of distinct screening dimensions exploited in insurance contracting. When the two types of applicants face different odds of incurring losses of distinct types or a loss caused by distinct perils, multidimensional screening reduces the externality cost of adverse selection that low-risk applicants must bear to distinguish themselves from high risks, as the two types can be more efficiently sorted by offering different coverage limitations for the alternative losses in a pattern advantageous to the low risks. Moreover, by reducing the externality cost of adverse selection, efficient multidimensional screening allows competitive insurance markets to attain pure strategy Nash equilibria over a wider range of applicant pools.

These insights apply equally in other markets where adverse selection limits contracting possibilities. In the job market context examined by Spence (1973), where education serves as the signaling dimension, fields of specialization play the role of distinct perils, as those job applicants best suited for positions as engineers have an advantage in math and the natural sciences, while those best suited for positions as journalists find a more writing-intensive curriculum to their advantage. Similarly, in credit markets, applicants can signal their creditworthiness by accepting a higher collateral requirement, as suggested by Bester (1985). By setting different requirements for different types of collateral, lenders can take advantage of any differences in the willingness of applicants to bear the risk of forfeiture, enhancing efficiency, and the prospects for existence of a pure strategy Nash equilibrium in these markets.

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