

**Optimal Compensation with Earnings Manipulation:  
Managerial Ownership and Retention**

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May 6, 2010

### **Abstract**

The optimal managerial compensation contract is characterized in an environment in which the manager influences the distribution of earnings through an unobservable effort decision. Actual earnings, when realized, are private information observed only by the manager, who may engage in the costly manipulation of earnings reports. Retention is modeled explicitly by requiring that the optimal contract satisfy interim individual rationality, so that the manager earns non-negative profit for any earnings realization. We find that the optimal contract induces under-reporting for low earnings and over-reporting for high earnings, and that the optimal contract may be implemented through a compensation package composed of a performance bonus based upon (manipulated) earnings and a stock option that is repriced to be in the money for low earnings realizations.

*“Executives at dozens of public companies, including Starbucks, Google [and] Intel, are taking steps to lower the prices that their employees would have to pay to convert options into stock. The moves are usually described as important for retaining employees, especially as stock options that vest over several years look utterly worthless in the current market....But the moves leave shareholder advocates fuming....The process, in their view, is fundamentally unfair. Modifying the options means employees gain from stock price increases, while investors feel the brunt of stock price declines.” (The New York Times, March 27, 2009, p. B.1)*

## **1. Introduction.**

Managerial compensation arrangements have traditionally included both performance bonuses and stock options, the former often justified by firms as a mechanism to align the interests of the manager with those of the shareholders<sup>1</sup> and the latter as a tool to facilitate managerial retention.<sup>2</sup> Shareholders often disagree with this assessment, however, as evidenced by controversies involving the payment of bonuses to managers, most recently those involving Wall Street banks in the wake of the recent financial meltdown, and particular vitriol is reserved for the common practice of repricing a manager’s under water options to be in the money. Indeed, one representative of the investor community notes disapprovingly that “[o]ur members generally detest [repricing] and consider it antithetical to the whole concept of incentive

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<sup>1</sup>For a detailed description of the observed structure of managerial compensation arrangements, see Murphy (1999). The use of bonuses and options has continued, as indicated by the 2008 *The Wall Street Journal*/Hay Group CEO Compensation Study.

<sup>2</sup>Chen (2004) examines the characteristics of firms that permit options to be repriced.

compensation.”<sup>3</sup> We provide a resolution to this disagreement regarding the role of bonuses and repriced options by modeling formally the optimal managerial incentive contract, which we then show can be implemented through a combination of performance bonuses and stock options that are repriced for low earnings outcomes.

The problem we consider is that of a firm owner who wishes to hire a manager to run the business. The manager takes a private costly action that influences the probability distribution of actual earnings which, when realized, are observed only to the manager. The manager produces an earnings report, which may differ from the actual earnings if the manager is willing to incur falsification costs. Thus, we are examining a contracting environment with both a hidden action and hidden information, and the problem is to characterize the optimal incentive contract that balances the ex ante incentives of the manager to shirk with the ex post incentives to engage in earnings manipulation. We also introduce the problem of managerial retention by requiring that the optimal contract satisfy interim incentive rationality, so that the manager does not wish to leave the firm for any earnings realizations.

Our work is most closely related to Crocker and Slemrod (*RAND*, 2007), who derive the optimal ex ante individually rational contract under costly state (earnings) falsification and moral hazard. In that environment, an optimal contract entails bonuses paid to the manager which are increasing in the size of the earnings report, and the structure of the bonuses reflects an efficiency tradeoff between the effect such bonuses have on inducing higher levels of effort by the manager, on the one hand, and the incentives the bonuses generate for the falsification of earnings reports, on the other. While the ex ante individual rationality constraint permits full

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<sup>3</sup>*The Wall Street Journal*, April 8, 1999, p. R.6.

extraction of the manager surplus by the owner through the use of a lump sum transfer, one feature of the optimal contract is that, for some realized earnings, the manager may prefer to quit rather than continue with the firm. As in Crocker and Slemrod, we will consider the optimal contract under costly earnings falsification and moral hazard where the contract must be ex ante individually rational with respect to the manager's effort choice, but we will also address the retention issue directly by requiring that the contract also be interim individually rational with respect to the manager's earnings report. This latter requirement, which is necessary in order to guarantee that the manager not wish to leave the firm after observing the earnings outcome, introduces a surplus extraction role for the optimal bonus arrangement, which substantially changes the nature of the optimal contracting problem.

The efficient balancing of moral hazard and adverse selection in the presence of interim individual rationality creates countervailing incentives of the type examined by Lewis and Sappington (1989), and Maggi and Rodriguez-Clare (1995). Moderating the hidden action problem requires the owner to share firm profit with the manager, which gives the manager the incentive to over-report earnings, while the extraction of managerial surplus in the presence of hidden information and interim individual rationality requires the owner to engage in differential rent extraction, which gives the manager the incentive to under-report earnings. We show that the optimal contract exploits these competing effects.

In the case where the manager is given no ownership stake in the firm, the optimal contract results in truthful earnings reports and zero gross manager rent (not accounting for the manager's effort cost) for actual earnings levels below a derived earnings threshold. Above this threshold, the manager over-reports earnings and earns positive gross rent. Alternatively,

endowing the manager with ownership shares in the firm through the granting of an option partially alleviates the moral hazard problem through the usual internalization channel, but it also increases the marginal rent the manager must earn to satisfy the incentive compatibility constraints and exacerbates the surplus extraction role of the bonuses in the optimal contract. With such partial ownership, the optimal contract exhibits the under-reporting of earnings and zero gross managerial rent below a certain earnings threshold. Above this threshold, the manager will continue to under-report earnings but will now earn a positive gross rent. Finally, there will be a second earnings threshold above which the manager over-reports earnings and earns gross rents that are increasing in the reported earnings. Thus, increasing the manager's ownership share reduces the extent of over-reporting induced by the optimal contract, and may actually encourage the under-reporting of earnings by the manager.

This structure of the optimal contract (conditional on the level of manager ownership) raises two key issues: The relationship between the manager's ownership share and owner expected profit, and the role of options and repricing. With regard to the former, we derive a simple test to determine the relationship between expected owner profit and the manager's ownership stake. We show that if the expected earnings distortion, which is the expected difference between actual and reported earnings, is positive then the owner's expected profit is increasing in the manager's ownership share. Since, as noted above, with no manager ownership the optimal contract induces either correct reporting or over-reporting of earnings, our analysis implies that it is always optimal for the owner to endow a manager with some ownership.

With regard to the latter issue, we show that compensating the manager with options that are repriced for certain earnings realizations is preferable to the granting of stock to the manager,

an alternative approach that has been suggested by some observers.<sup>4</sup> We find that the use of outright stock grants allows the manager to earn too much rent, and that to generate the rent profile of an optimal contract in such a setting would require that the manager receive negative bonus payments for lower earnings outcomes. In contrast, a contract that uses options that are repriced at some lower earnings levels allows the owner to pay the manager her optimal rent without resorting to negative bonus payments.

The paper proceeds as follows. In the next section we introduce the economic environment of the model and set up the optimal contracting problem. In Section 3 we provide an informal discussion of the structure of the optimal compensation contract, which is formally derived in Section 4. Section 5 contains our analysis of the effect of manager ownership on owner profit, and a final section contains concluding remarks.

## 2. The Model.

In this model there are two people who make decisions for a firm: an owner and a manager. The owner is responsible for setting managerial incentives and the manager is responsible for running the firm, which requires both an effort and the reporting of the firm's earnings to the owner. Conditional on managerial effort,  $a$ , the distribution of the firm's gross earnings,  $x$ , is given by the distribution  $F(x|a)$  with strictly positive density  $f(x|a)$  and support  $[0,1]$ . We assume that  $F_a < 0$ , so higher levels of manager effort shifts the distribution of earnings to the right in the sense of first order stochastic dominance.

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<sup>4</sup>"If Google is going to reprice when things go wrong, it should also limit the upside to employees. It would be easier simply to pay bonuses instead, tied to corporate performance, with a portion in stock that vests over time to aid retention." (*WSJ*, January 22, 2009)

Effort is unobservable by the owner, which means  $a$  is a *hidden action*, and it is costly to the manager. Let  $h(a)$  denote the manager's effort cost. We assume that  $h$  is strictly increasing, strictly convex, and that  $h(0) = h'(0) = 0$ . The conditions on  $h(0)$  and  $h'(0)$  imply that the manager incurs no fixed costs of effort nor has a strictly positive initial marginal cost of effort that could result in the manager exerting zero effort in response to a range of positive incentive levels.

To induce the manager to choose a positive level of effort, the owner must offer an incentive contract which can compensate the manager in two ways: performance-based compensation and endowing the manager with shares in the firm. To reflect the reality of most large corporations, we assume that only the manager observes the firm's true earnings, so that the value taken by  $x$  is *hidden information*. This means the owner cannot contract on the earnings,  $x$ , directly but only on the earnings reported by the manager, which we denote as  $R$ .

Let  $B(R)$  denote the manager's performance-based compensation and let  $\alpha$  denote the share of the firm given to the manager.<sup>5</sup> Reporting earnings,  $R$ , that differs from actual earnings,  $x$ , imposes falsification costs  $g(R-x)$  on the manager as it requires the manager to devote resources to managing the accounting to make such a report credible. In general, one would expect the falsification costs to be strictly convex in  $R-x$ , strictly increasing in  $|R-x|$ , and minimized at 0 such that  $g(0)=0$ . These properties imply the manager incurs no cost to issuing a truthful earnings report, and that under-reporting and over-reporting earnings are costly. To

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<sup>5</sup>We will demonstrate below how this ownership may be conferred formally through the granting of a stock option.



simplify the analysis and to allow us to focus more directly on the features of the optimal contract, we assume quadratic falsification costs, so that  $g(R-x) = (R-x)^2/2$ .<sup>6</sup>

The risk-neutral manager's utility given any value of  $\alpha$  and any compensation contract  $B(R)$  can be written as

$$\hat{V}(x, R, a; \alpha) = \alpha x + B(R) - g(R-x) - h(a). \quad (1)$$

Two comments are in order before proceeding. First, the risk neutrality of manager utility in the payments  $\alpha x$  and  $B$  would seem to imply that the first-best solution for the owner is to sell the firm to the manager by setting  $\alpha = 1$ , since doing so would internalize the effect of the manager's effort and earnings report choices on firm profits. Indeed, this is precisely the result in the analysis of Crocker and Slemrod (2007) who require that the contract be ex ante individual rational, which effectively permits the manager to purchase the firm up front through a lump sum transfer. In this setting, however, we are concerned with managerial retention which is modeled by requiring that the optimal contract also satisfy an interim individual rationality constraint which would necessarily be violated by the lump sum purchase contract for low earnings realizations. Second, the fact that the shares are valued at  $\alpha x$  implies that the earnings report does not fool the stock market. Investors are able to invert the manager's reporting strategy for valuation purposes given the optimal contract even if they are unable to detect the actual falsification.<sup>7</sup>

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<sup>6</sup>However, we will retain the general notation,  $g(R-x)$ , when it helps highlight the role of manipulation costs in the structure of the optimal contract.

<sup>7</sup>Put differently, given the compensation package  $(B(R), \alpha)$ , the selection of the manager's action  $a$  is known since it satisfies the first order condition (6) below, and so the market can use the actual probability distribution function  $F$  to forecast expected profits and the

The owner will present the manager with a compensation package  $(\alpha, B(R))$ . If the manager accepts the package, then the manager will choose an effort level,  $a$ . Following the effort investment by the manager, earnings will be realized, the manager will issue an earnings report,  $R$ , the owner will pay the manager  $B(R)$ , and the risk-neutral owner will earn profit of

$$\Pi(x, R, \alpha) = (1-\alpha)x - B(R). \quad (2)$$

This sequence of events is illustrated in Figure 1.

The owner's objective is to choose the indirect compensation  $(\alpha, B(R))$  to maximize the expected value of  $\Pi(x, R, \alpha)$  subject to several incentive constraints.<sup>8</sup> Because the manager's ownership share is set before the manager chooses her effort and hence before earnings are realized, we can treat  $\alpha$  as a parameter and derive the optimal compensation contract  $B(R)$  for each value of  $\alpha$ . We will determine the optimal value of  $\alpha$  in a later section. We refer to the contract which solves the owner's problem for each value of  $\alpha$  as the optimal *conditional contract*. For each value of  $\alpha$ , a conditional contract induces an allocation that can be described by three components: an effort level,  $a$ , the level of manager utility,  $\hat{V}$ , and an earnings report,  $R$ , where the latter two depend on the firm's realized earnings,  $x$ .

We solve the owner's problem by invoking the Revelation Principle, which is a solution technique in which we recast the owner's problem as one in which the owner chooses a direct conditional contract instead of the indirect conditional contract,  $B(R)$ . Formally, for each value of  $\alpha$ , a direct conditional contract consists of three components that mirror the allocation

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value of the manager's equity stake.

<sup>8</sup>The term "indirect" refers to the fact that the performance-based term  $B(\cdot)$  depends indirectly on the firm's actual earnings through the manager's earnings report.

structure of this problem: a level of managerial effort the owner would like the manager to choose,  $a$ , an earnings report,  $R(\theta)$ , and a compensation schedule,  $B(\theta)$ , where  $\theta$  is the manager's report of his type,  $x$ .<sup>9</sup> To the extent that the optimal indirect contract consists of a compensation schedule  $B(R)$  that induces earnings manipulation, it will show up in the optimal direct contract through the value of  $R - x$ .

By the Revelation Principle, we will restrict attention to direct contracts that induce the manager to choose the desired level of effort,  $a$ , and to issue a truthful type report, so that  $\theta = x$ .<sup>10</sup> Therefore, a direct conditional contract can be described by an effort level,  $a(\alpha)$ , a reporting function,  $R(x; \alpha)$ , and an indirect utility level for the manager,  $V(x, a; \alpha)$ . Given these values, equation (1) may be used to recover the optimal transfer function  $B(x, \alpha)$  associated with the direct conditional contract. While here we have noted explicitly the reliance of the contract on  $\alpha$ , we will for notational convenience drop explicit reference to  $\alpha$  in these contract terms except where the clarification is helpful.

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<sup>9</sup>To apply the Revelation Principle correctly in this model, the earnings report must be part of the contract. Because the earnings report has a direct effect on the manager's utility through the cost term,  $g(\cdot)$ , the correct application of the Revelation Principle does not allow one to restrict attention to truthful earnings reports. Instead, we need to distinguish between the manager's private information,  $x$ , and the earnings report made by the manager that is used to determine managerial compensation,  $R(x)$ . This last point was first made by Dye (1988) and then again by Gresik and Nelson (1994). Dye's paper has been misinterpreted within the accounting literature to imply that one cannot invoke the Revelation Principle in private information problems of this sort. As shown in Gresik and Nelson (1994), in the context of a multinational tax problem, what Dye's analysis implies is that one must include the earnings report as part of the contract. This is the approach taken in this paper, and the one taken by Crocker and Slemrod (2007).

<sup>10</sup>Myerson (1982) refers to such mechanisms as "honest and obedient."

Because our model includes both moral hazard and adverse selection effects, any direct contract must satisfy several incentive compatibility and individual rationality constraints. Incentive compatibility generates four constraints, two of which apply to the manager's selection of  $\theta$ , and two of which apply to the manager's choice of  $a$ . The choice of a type report is made after learning  $x$ , so that the manager chooses  $\theta$  to maximize  $\hat{V}(x, R(\theta), a; \alpha)$ , whereas the ex ante choice of effort means the manager will choose  $a$  to maximize  $E\hat{V}$ .

Let  $V(x, a; \alpha)$  denote the conditional indirect utility of the manager who optimally issues a truthful type report. That is,

$$V(x, a; \alpha) \equiv \hat{V}(x, R(x; \alpha), a; \alpha). \quad (3)$$

Truthful type reporting by the manager ( $\theta = x$ ) will require that  $V$  satisfy two constraints:

$$V_x = \alpha + g'(R(x) - x) \quad (4)$$

and

$$R'(x) \geq 0. \quad (5)$$

In order for  $\theta = x$  to be optimal for the manager, the first order condition  $\hat{V}_R \cdot R' = 0$  must be satisfied at  $\theta = x$  for all  $x \in (0, 1)$ . This implies equation (4) by applying the Envelope Theorem to (1). Thus, the manager will earn a marginal rent that covers the change in the value of her shares plus the change in her manipulation costs.

Inequality (5) is a second-order incentive compatibility condition. Totally differentiating  $\hat{V}_R(x, R(x), a; \alpha) = 0$  with respect to  $x$  implies  $\hat{V}_{RR} \cdot R' + \hat{V}_{Rx} = 0$ . Since  $\hat{V}_{Rx} = g''(R - x) > 0$ , the earnings report function,  $R$ , must be non-decreasing. Thus, an incentive compatible contract will associate higher earnings,  $x$ , with higher earnings reports,  $R$ .

The last two incentive constraints deal with the manager's choice of productive effort,  $a$ . The manager will choose to invest  $a$  units of effort with  $a > 0$  as long as

$$\partial EV(x, a, \alpha) / \partial a = 0 \quad (6)$$

and

$$\partial^2 EV(x, a, \alpha) / \partial a^2 \leq 0. \quad (7)$$

Equation (6) is the first order condition for the manager's choice of  $a$ , and inequality (7) is the associated second order condition.

The manager's contract will also need to satisfy two individual rationality constraints. As in Crocker and Slemrod (2007), any contract must satisfy ex ante individual rationality, so that

$$EV(x, a, \alpha) \geq 0. \quad (8)$$

No manager would accept a contract that violates (8). In addition to (8), we explicitly model retention by adding the interim individual rationality constraint,

$$V(x, a, \alpha) + h(a) \geq 0. \quad (9)$$

This constraint captures the ability of the manager to quit after observing actual earnings,  $x$ , but before issuing an earnings report,  $R$ . Note that, since the manager has already chosen  $a$  prior to observing  $x$ , the effort cost,  $h(a)$ , is a sunk cost.

To highlight the role of (9), define the manager's gross (of effort costs) indirect utility as

$$W(x; \alpha) \equiv \alpha x + B(R(x) - x) - g(R(x) - x) = V(x, a, \alpha) + h(a). \quad (10)$$

Note that  $W_x = V_x$  and that  $W$  does not depend directly on  $a$  since at the earnings report stage the effort choice is sunk. The effort choice,  $a$ , will however effect  $EW$  through the distribution  $F$ .

By using  $V$  to substitute  $B$  out of  $\Pi$  and  $W$  to substitute out  $V$ , the owner's problem can be written as choosing  $(a, W(x), R(x))$  to

$$\begin{aligned}
\max E(x - g - W) \text{ s.t. } & \text{a. } W_x = \alpha + g' \\
& \text{b. } \partial EW / \partial a - h'(a) \leq 0 \\
& \text{c. } W \geq 0 \\
& \text{d. } EW - h(a) \geq 0 \\
& \text{e. } R'(\cdot) \geq 0 \\
& \text{f. } \partial^2 EW / \partial a^2 - h''(a) \leq 0.
\end{aligned} \tag{11}$$

Constraints (a) and (e) are incentive compatibility constraints that ensure truth-telling ( $\theta = x$ ) by the manager in the conditional direct revelation contract. Constraint (b) is the manager's first-order condition for the choice of effort,  $a$ , and constraint (f) is the manager's second-order condition. Constraint (c) is the interim individual rationality constraint, and (d) is the ex ante individual rationality constraint.

Before proceeding, note that constraint (11a) implies

$$W(x; \alpha) = W(0) + \int_{t=0}^x [\alpha + g'(R(t) - t)] dt \tag{12}$$

which, after integrating by parts yields

$$EW(a; \alpha) = W(0) + \int_{t=0}^1 [\alpha + g'(R(t) - t)] (1 - F(t|a)) dt, \tag{13}$$

so that

$$\partial EW(a; \alpha) / \partial a = - \int_{t=0}^1 [\alpha + g'(R(t) - t)] F_a(t|a) dt, \tag{14}$$

and

$$\partial^2 EW(a; \alpha) / \partial a^2 = - \int_{t=0}^1 [\alpha + g'(R(t) - t)] F_{aa}(t|a) dt. \quad (15)$$

We demonstrate below that an optimal contract satisfies  $W_x \equiv \alpha + g'(R(x) - x) \geq 0$  for all  $x$ . Thus, (15) is negative as long as  $F$  is convex in  $a$ , which is a distributional assumption that we will make in Section 4, so that the second order condition (f) will naturally be satisfied at a solution to the optimality problem based on the remaining constraints.

As is commonly done in these settings, we will proceed by solving a modified version of (11) in which constraint (e) is dropped (in addition to (11f)), and then check at the end to make sure that (e) is satisfied. Call this modified problem (11'), which yields the Hamiltonian

$$\mathcal{H} = (x - g(R - x) - W)f + \phi(\alpha + g'(R - x))$$

where  $\phi$  is the co-state variable,  $W$  is the state variable, and  $R$  is the control. Using (14) yields the Lagrangian

$$\mathcal{L} = \mathcal{H} + \tau W - \mu[(\alpha + g'(R - x))F_a + h'(a)f] + \lambda f(W - h) \quad (16)$$

where  $\tau(x)$  is the non-negative multiplier on the interim individual rationality constraint (11c),  $\mu$  is the non-negative multiplier on the effort constraint (11b), and  $\lambda$  is the non-negative multiplier on the ex ante participation constraint (11d).<sup>11</sup>

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<sup>11</sup>The reason  $\mu \geq 0$  is as follows. Replace the right-hand side of (11b) with  $\beta \geq 0$ . An increase in  $\beta$  increases the marginal cost of inducing any given  $a$  and hence reduces owner expected profit. If  $\mathcal{H}$  denotes the owner's value function, then standard optimal control procedures imply  $\partial \mathcal{H} / \partial \beta = -\mu \leq 0$  (with strict inequality if  $a > 0$ ). Thus,  $\mu$  must be non-negative.

### 3. Informal Discussion of Results

Before proceeding to characterize formally a solution to (16), we will describe the nature of our primary result, how this problem relates to others in the extant literature, and the role of countervailing incentives in the optimal contract. Under a set of regularity conditions pertaining to the distribution function  $F$  which are specified in the next section, we demonstrate that the optimal reporting function,  $R$ , satisfies

$$R - x = \frac{(1-\lambda)(F-1)}{f} - \frac{\mu F_a}{f} \quad (17)$$

where  $\lambda$  is the multiplier associated with the ex ante individual rationality constraint (11d) and  $\mu$  is the multiplier associated with the effort constraint (11b). As long as  $\lambda < 1$ , the first term on the right hand side is negative, and the second term is positive, so that an optimal reporting function may entail either over- or under-reporting of earnings depending on which effect dominates.

In the special case where  $F_a = 0$ , manager effort has no effect on firm earnings and the contracting problem reduces to the costly state falsification environment examined by Crocker and Morgan (1998). In this setting, the parties face the standard tradeoff between efficiency and surplus extraction that is commonly observed when contracting in the presence of hidden information. When the parties face only an ex ante participation constraint, the solution to the contracting party is to sell the firm to the manager ( $\alpha = 1$ ) for a lump sum payment equal to the firm's expected profit and then pay a bonus that is uniformly zero in reported earnings. Since the ex ante participation constraint permits full extraction of managerial surplus through lump sum transfers without efficiency cost, it is straightforward to show that  $\lambda = 1$  and, from (17),



$$R = x.$$

If instead the contracting parties face only an interim individual rationality constraint, then  $\lambda = 0$  and (17) reduces to  $R - x = (F - 1)/f$ . As long as  $F$  satisfies the monotone hazard rate property, so that  $\partial((F-1)/f)/\partial x \geq 0$  as depicted in Figure 2, then earnings are under-reported, the amount of under-reporting is monotonically decreasing in  $x$ , and  $R'(x) \geq 0$ . Moreover, as long as  $V_x \geq 0$ , interim individual rationality is satisfied by setting the bonus schedule so that  $V(0) = 0$ , which results in the manager earning information rents that are increasing in actual earnings,  $x$ .<sup>12</sup> Thus, in the presence of interim individual rationality, the optimal contract reflects the tradeoff between efficiency and surplus extraction that is commonly observed in settings with adverse selection, and because of surplus extraction the manager *under-reports* earnings.

In the case where  $F_a \leq 0$ , so that an increase in (unobservable) managerial effort shifts the distribution of earnings to the right, the optimal contract now has a moral hazard component. When the contracting parties face only the ex ante participation constraint, we are in the Crocker and Slemrod (2007) environment in which the use of lump sum transfers permits the frictionless extraction of managerial surplus, so that  $\lambda = 1$ . Then (17) reduces to  $R - x = -\mu F_a / f$  and, as depicted in Figure 2, the optimal reporting function entails earnings *overstatement* by the manager. The optimal contract pays a bonus,  $B$ , to the manager which is increasing in the reported earnings,  $R$ . A bonus structure that is more sensitive to higher earnings reports gives the manager the incentive to take higher levels of the (private) costly action,  $a$ , but also increases

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<sup>12</sup>Crocker and Morgan ensure the monotonicity of  $V$  by assuming that  $|g'| < 1$  and restricting their analysis to the case in which  $\alpha = 1$ . The problems encountered when  $V$  is nonmonotonic are discussed below.

the returns to the overstatement of earnings. Thus, the efficient contract reflects an efficiency tradeoff between the benefits of incentivization and the costs of falsification.

The introduction of interim individual rationality adds a surplus extraction role to the optimal reporting contract and the associated bonus structure. Since the optimal contract in Crocker and Slemrod (2007) violates interim individual rationality, it follows that  $\lambda < 1$  and the optimal reporting function satisfies (17), which is depicted in Figure 3. The optimal contract results in both under- and over-reporting, depending on the actual level of earnings, reflecting a tradeoff between surplus extraction and efficiency. In addition, there is a technical problem in satisfying the interim individual rationality constraint since  $V_x$  (and, hence  $W_x$ ) is necessarily non-monotonic for small values of  $\alpha$ . In the case of quadratic falsification costs,  $W_x = 0$  implies that  $R - x = -\alpha$ , as depicted in Figure 3, while (17) implies that  $R(0) = -(1-\lambda)/f(0)$  and  $R(1)=1$ . Thus, for  $\alpha$  close to zero  $W_x$  will be strictly negative for  $x$  close to zero and strictly positive for some higher earnings.

As a result, the introduction of interim individual rationality introduces countervailing incentives, which require an application of the approach developed by Maggi and Rodriguez-Clare (1995) to characterize an optimal contract. The countervailing incentives imply an optimal contract with three features. First when earnings are below a threshold,  $\hat{x}$ , the manager earns zero gross profit ( $W = 0$ ) and the contract under-reports earnings by the amount  $\alpha$ . Second, when earnings are between  $\hat{x}$  and a second threshold,  $x^+$ , manager profit is increasing in  $x$ , and the amount of under-reporting is decreasing. Third, for earnings levels above  $x^+$ , manager profit is increasing in actual earnings and the contract over-reports earnings.

We now turn to a formal derivation of our results.<sup>13</sup>

#### 4. The Optimal Conditional Contract: A Formal Characterization

In order to characterize a solution to (16), we use several regularity assumptions regarding the behavior of the distribution function,  $F$ .

**Distribution Assumptions:**

- a.  $F(x|a)$  is strictly decreasing, convex and continuously differentiable in  $a$  for all  $x$  and for all  $a \geq 0$ .
- b. There exists  $M > 0$  such that for all  $x$  and for all  $a$ ,  $f(x|a) < M$  and  $f_x(x|a) < M$ .
- c.  $f_x(0|a) > 0$  for all  $a \geq 0$ .
- d.  $(F(x|a)-1)/f(x|a)$  is strictly increasing in  $x$  for all  $a \geq 0$ .
- e.  $(F(x|a)-1)/f(x|a)$  is concave in  $x$  for all  $a \geq 0$  and  $F_a(x|a)/f(x|a)$  is convex in  $x$  for all  $a \geq 0$ .

Assumption (a) implies that higher manager effort induces a first-order stochastic improvement in the distribution of earnings ( $F$  decreasing in  $a$ ) and results in diminishing marginal returns from effort ( $F$  convex in  $a$ ). The convexity of  $F$  with respect to effort will ensure that the first-order approach is valid. Assumptions (b) and (c) are technical assumptions adopted to simplify several of the proofs. Assumption (b) restricts attention to densities that are bounded and have bounded first derivatives with respect to earnings. Assumption (c) requires

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<sup>13</sup>The formal analysis includes the possibility that the reporting function that solves (16) might not satisfy monotonicity condition (11e). Proposition 2, presented below, will show that the modifications to (16) needed to ensure that  $R(x)$  is non-decreasing do not alter the qualitative properties of the optimal contract discussed in this section.

that small but positive earnings are relatively more likely than zero earnings. Our proofs will indicate where these assumptions are used.

Assumption (d) is the standard monotone hazard rate assumption found in most adverse selection models, and is used to guarantee manager indifference curves that exhibit the single-crossing property. For a family of distributions indexed by  $a$ , it will be satisfied, for instance, as long as  $\partial f/\partial x > 0$  for all  $a$ , and example 1 (below) provides an example of one such family. In this paper, assumption (d) is sufficient to support single-crossing only at sufficiently low effort levels. Because of the moral hazard component of this problem, the manager's indifference curves can fail to exhibit the single-crossing property at high enough levels of effort. Finally, assumption (e) is a regularity condition that implies the single-crossing property will only be violated for high earnings levels.

Turning to the Lagrangean expression (16), the term  $\tau W$  is included because constraint (11a) reveals that  $W$  need not be strictly monotonic in  $x$  if the contract induces under-reported earnings (which implies  $g' < 0$ ). In standard contract design problems when  $W$  is monotonic, one can replace the continuum of constraints represented by (11c) with a single constraint that sets either  $W(0)$  or  $W(1)$  equal to 0. Because the manager's indirect utility,  $W$ , may not be monotonic in  $x$ , the manager type that receives zero gross surplus ( $W=0$ ) is endogenously determined. Introducing the term  $\tau W$  formally accounts for this endogeneity.

The non-monotonicity of  $W$  is due to two countervailing incentives created by the moral hazard and adverse selection effects in the presence of an interim individual rationality constraint. The first incentive (moral hazard) comes through the ownership term,  $\alpha$ . Increasing  $\alpha$  gives the manager a greater share of actual firm earnings and hence induces the manager to

invest in higher effort. The second incentive (adverse selection) comes through the earnings report term,  $g'$ . When the direct contract reflects incentives to over-report earnings ( $R(x) > x$ ), marginal manipulation costs will be increasing in  $x$ . This means that the owner can pay the manager a rent either by increasing the manager's ownership share *or* by inducing more over-reporting of earnings. When the direct contract reflects incentives to under-report ( $R(x) < x$ ), marginal manipulation costs will be decreasing in  $x$ . Now the ownership incentives and the under-reporting incentives work in opposite directions. These countervailing incentives give the owner the ability to combine increases in the manager's ownership share with incentives to under-report (via  $R(\cdot)$ ) that result in zero marginal rent being paid to the manager. We will show that this type of countervailing incentive structure plays a key role in the optimal contract. Formally, the presence of the countervailing incentives means our analysis will employ the same techniques as found in Maggi and Rodriguez-Clare (1995).

**Proposition 1.** *Given Distribution Assumptions a-c, if a conditional contract  $(a, W, R)$  satisfies*

$$R - x = (\phi - \mu F_a)/f, \quad (18)$$

$$-(1-\lambda)f + \tau = -\phi' \text{ (almost everywhere),} \quad (19)$$

$$a \left[ \int_{t=0}^1 (\alpha + g'(R(t)-t)) F_a(t|a) dt + h'(a) \right] = 0, \quad (20)$$

$$\lambda(EW - h(a)) = 0 \text{ and } \lambda \geq 0, \quad (21)$$

$$\phi(0) \leq 0, \phi(1) \geq 0, \phi(0)W(0) = \phi(1)W(1) = 0, \tau \geq 0, \text{ and } \tau(x)W(x) = 0, \text{ and} \quad (22)$$

$$R'(x) \geq 0, \quad (23)$$

*then it is an optimal conditional contract.*<sup>14</sup>

Proposition 1 is a translation of Theorems 1 and 2 (chapter 6) in Seierstad and Sydsæter (1987) to the specifics of (11') with constraint (11e) added as (23) for completeness. Eq. (18) is the Euler equation and defines the optimal reporting function. The sign of the term  $(\phi - \mu F_a)/f$  determines for which earnings levels the contract induces over-reporting and for which earnings levels the contract induces under-reporting. The co-state variable,  $\phi$ , will capture both the ownership and manipulation distortions in the contract. To determine the manipulation incentive (captured by  $R - x$ ), one must subtract out the ownership effect, measured by the term  $\mu F_a/f$ . Thus, contracts that create strong incentives for the manager to invest in a larger amount of effort than she would otherwise correspond to a high value of  $\mu$  and for a given value of  $\phi$ , a large manipulation incentive. Eq. (20) is the manager's first-order condition with respect to effort. Condition (21) represents the complementary slackness conditions with regard to the ex ante individuality constraint. The conditions in (22) are the transversality conditions that will help determine which actual earnings level correspond to zero manager rents.

The countervailing incentives allow for the possibility that the manager earns zero marginal rent over a range of earnings. To determine if such an outcome can be the result of an optimal contract, suppose the contract implies zero marginal rent for the manager on a non-degenerate interval of earnings, i.e.,  $W_x = 0$ . Then (11a) implies  $\alpha + g'(R-x) = 0$  or

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<sup>14</sup>Proposition 1 provides sufficient conditions for an optimal conditional contract and thus does not rule out the possibility that the solutions to (18)-(22) may violate (23). We use this simpler formulation in the body of the paper to emphasize the key economic trade-offs in the optimal conditional contract. The more general formulation that explicitly incorporates monotonicity constraint (23) is developed in the appendix in the proof of Proposition 2 where the solution can involve the standard ironing techniques.

$R(x) - x = -\alpha \leq 0$  for all  $x$  in this interval. For an incentive compatible contract to result in zero marginal manager rents, it must induce the manager to under-report earnings. Only in the case in which the manager owns no shares in the firm will incentive compatibility and zero marginal rents imply truthful reporting. Let  $\hat{\phi}(x)$  denote the value of the co-state variable in this case when the countervailing ownership and manipulation incentives exactly offset each other. Thus, (18) implies

$$\hat{\phi}(x) = \mu F_a(x|a) - \alpha f(x|a). \quad (24)$$

For all  $\alpha$ ,  $\hat{\phi}(x) < 0$  for all  $x \in (0,1)$ . Eq. (24) defines a feasible co-state variable as long as it also satisfies (19) and (22). With  $\tau \geq 0$ , (19) implies that  $\phi' \leq (1-\lambda)f$  which, in conjunction with (22) implies that<sup>15</sup>

$$(1-\lambda)(F - 1) \leq \phi(x) \leq (1-\lambda)F. \quad (25)$$

As long as  $\hat{\phi}$  falls within this range defined by (25), an optimal conditional contract can induce zero marginal rents for a range of earnings. If  $\alpha$  and  $\mu$  are sufficiently close to zero,  $\hat{\phi}$  will satisfy (25) for earnings below a level we denote by  $\hat{x}$ . For earnings above  $\hat{x}$ ,  $\hat{\phi}$  will fall below  $(1-\lambda)(F-1)$ . Exploiting the countervailing incentives by setting  $\phi = \hat{\phi}$  for  $x \leq \hat{x}$  and setting  $\phi = (1-\lambda)(F-1)$  for  $x > \hat{x}$  results in the reporting function from (18) of

$$R(x,a;\alpha) = \begin{cases} x - \alpha & \text{if } x \leq \hat{x} \\ x + \frac{(1-\lambda)(F(x|a) - 1) - \mu F_a(x|a)}{f(x|a)} & \text{if } x > \hat{x} \end{cases} \quad (26)$$

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<sup>15</sup>Integrating both sides from 0 to  $x$ , and noting that  $\phi(0) \leq 0$  from (22), yields the left inequality, while integration of both sides from  $x$  to 1 and noting that  $\phi(1) \geq 0$  from (22) yields the right hand inequality.

where the value of  $\hat{x}$  is endogenous and calculated as part of the optimal contract. Note that for any  $\alpha > 0$ , if  $\mu$  is large enough, then  $\hat{\phi}(x) < (1-\lambda)(F-1)$  for all  $x < 1$ . In this case,  $\hat{x} = 0$ .

For  $\alpha = 0$ , (26) implies the contract results in no distortion of low earnings and an upward distortion of high earnings. For  $\alpha > 0$ , (26) implies that the contract results in a downward distortion of low earnings and an upward distortion of high earnings. Truthful reporting when  $\alpha > 0$  will only occur at  $x = 1$  and at one other earnings level greater than  $\hat{x}$ . In addition for all  $\alpha$ , the manager earns zero rent ( $W = 0$ ) and not just zero marginal rent ( $W_x = 0$ ) when  $x < \hat{x}$  and positive rent when  $x > \hat{x}$ . Incentives that induce under-reporting can be attractive to the owner because they reduce the manager's information rent but they also reduce the manager's marginal effort incentive. By adjusting  $\alpha$ , the owner can control the balance between these countervailing effects.

*Example 1.* Let  $F(x|a) = (1-a)x + ax^2$ ,  $h(a) = a^3/3$ ,  $\alpha = 0$ , and  $\lambda = 0$ .<sup>16</sup> Figure 4 plots  $F$ ,  $F - 1$ , and  $\hat{\phi}$ . For all  $a$ ,  $F$ , and  $F - 1$  are increasing functions of  $x$  while  $\hat{\phi}$  must be decreasing near  $x = 0$  and increasing near  $x = 1$ . For this specific family of distributions, the convexity of all three curves ensures that  $\hat{\phi}$  and  $F - 1$  intersect once on  $(0,1)$ . The point of intersection of  $\hat{\phi}$  and  $F-1$  is  $\hat{x}$ . Using (26) to define the reporting function, the optimal conditional contract for  $\alpha = 0$  induces an effort level of .049 and results in the earnings manipulation shown in the top curve in Figure 5. Increasing  $\alpha$  has the effect of shifting the  $\hat{\phi}$  curve down thus reducing the range of earnings over which the manager earns zero rent. Now zero manager rent is associated with

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<sup>16</sup>More precisely, we solved for an optimal contract in the example ignoring the ex ante participation constraint, and then checked to ensure that the solution satisfied (11d).



under-reported earnings as illustrated by the lower curve in Figure 5 for  $\alpha = .05$ . In addition, under-reporting persists above  $\hat{x}$  even while the manager starts to earn positive rent. Over-reported earnings arise only for the highest earnings levels but notice that the magnitude of the earnings manipulation is reduced. Effort rises to .055. *End of example.*

Example 1 highlights three interesting properties of an optimal contract: zero manager rent at low earnings levels induced by exploiting countervailing ownership and manipulation incentives, incentives for both under-reporting and over-reporting earnings, and a compensation schedule that incorporates both insurance and options features. We now prove that these are general properties of optimal conditional contracts.

**Proposition 2.** *Assume the Distribution Assumptions are satisfied. The optimal conditional contract induces a strictly positive level of effort and there exists earnings  $x^+$  such that for all  $x^+ < x < 1$  the contract induces over-reported earnings ( $R > x$ ) and the manager earns positive rent ( $W > 0$ ). If the level of effort induced by the optimal conditional contract is sufficiently small, then there exists an earnings level,  $\hat{x} > 0$ , such that for all  $x < \hat{x}$ , the contract induces weakly under-reports earnings ( $R \leq x$ ) and the manager earns zero gross rent ( $W = 0$ ) (with strict under-reporting for  $\alpha > 0$ ) while for earnings sufficiently close to 1, the contract induces over-reported earnings and the manager earns positive rent.*

Proposition 2 establishes that over-reporting of high earnings is a robust feature of an optimal contract. Moreover, if the targeted level of effort is not too costly for the owner to

induce ( $\mu$  is small), then the optimal contract will also induce under-reporting of low earnings and a range of earnings over which the manager earns zero rent. One can implement the pattern of over- and under-reporting described in Proposition 2 with the compensation schedule,  $B(x)$ , which given (1) and (10) is

$$B(x) = W(x) + g(R(x)-x) - \alpha x. \quad (27)$$

For  $x \leq \hat{x}$ ,  $W(x) = 0$  so  $B(x) = g(-\alpha) - \alpha x$  and  $B'(x) = -\alpha$ . And for  $x > \hat{x}$ ,

$$B(x) = g(R(x)-x) + \int_{\hat{x}}^x g'(R(t)-t)dt - \alpha \hat{x}$$

and  $B'(x) = g'(R(x)-x)R'(x)$ , which is negative when the contract induces under-reported earnings and positive for over-reported earnings. Thus,  $B(\cdot)$  is decreasing for earnings up to  $x^+$  and increasing for earnings above  $x^+$  and  $B(x^+)$  is strictly negative for all  $\alpha > 0$ . This non-monotonic compensation function helps induce the desired level of effort as it effectively insures the manager against very low earnings levels ( $x \leq \hat{x}$ ) and rewards the manager for high earnings ( $x > x^+$ ). This discussion leads to the following proposition.

**Proposition 3.** *If the optimal contract induces under-reporting of earnings below  $x^+$ , the optimal compensation schedule will be decreasing in earnings up to  $x^+$  and increasing in earnings above  $x^+$ .*

Although  $B(x) + \alpha x$  will be strictly positive for all  $x$  when  $\alpha > 0$ , negative values of  $B(x)$  imply that the optimal contract requires the manager to pay the owner for intermediate values of  $x$ . One way to interpret (27) to avoid explicit payments from the manager to the owner is to view it as a combination of an option that allows the manager to purchase  $\alpha$  shares at a specified price

after earnings are realized, a bonus schedule, and a repricing of the option's strike price for certain earnings realizations. The contract will not include a base wage since the fact that the manager is risk neutral and has an outside option equal to zero implies the base wage should equal zero. Viewed in this way, we will show that negative compensation shows up as a decrease in the value of the manager's shares.

The direct compensation and asset valuation of the manager's shares is described by  $B(x) + \alpha x$ . For  $x \leq \hat{x}$ ,

$$B(x) + \alpha x = g(-\alpha) + \alpha(x - x) \quad (28)$$

which reduces to  $g(-\alpha)$  and, for  $x > \hat{x}$ , (27) can be written as

$$B(x) + \alpha x = g(R(x) - x) + \alpha[x - (\hat{x} - (1/\alpha) \int_{t=\hat{x}}^x g'(R(t) - t) dt)]. \quad (29)$$

Since the manager's information rent for  $x > \hat{x}$  equals  $\int_{t=\hat{x}}^x (\alpha + g'(R(t) - t)) dt$  and is strictly positive, the bracketed term in (29) is also strictly positive for all  $x > \hat{x}$ . Thus, changes in the value of the manager's shares can provide a channel through which manager rents are paid.

To observe the role of bonuses and options repricing in the optimal contract, consider a contract that includes giving the manager an option to purchase  $\alpha$  shares of the firm at the strike price  $x^+$ . For all  $x \geq x^+$ , (29) can be rewritten as

$$B(x) + \alpha x = g(R(x) - x) + \int_{t=x^+}^x g'(R(t) - t) dt + \int_{t=\hat{x}}^{x^+} (\alpha + g'(R(t) - t)) dt + \alpha(x - x^+). \quad (30)$$

The first three terms on the right-hand side of (30) constitute a (positive) bonus payment that depends on the level of realized earnings. These terms serve to compensate the manager for the manipulation costs she incurs and to pay part of the manager's rent. The last term in (30) is the manager's gain from exercising the option which is in the money. This gain represents the

remainder of the rent due the manager at earnings above  $x^+$ . For earnings above  $x^+$ , the use of a bonus payment and gains from an option in the money are complements as both increase with  $x$ .

For  $\hat{x} < x < x^+$ , the option is no longer in the money so a decomposition similar to (30) does not provide the manager with any rent via the option. For this range of earnings, the first term on the right-hand side of (29) represents a bonus to cover the manager's manipulation costs and the second term represents the gain to the manager from executing an option with a revised strike price of  $\hat{x} - \int_{\hat{x}}^x g'(R(t)-t)dt$  which is less than actual earnings,  $x$ . By repricing the option to a new lower price that is strictly in the money, the owner pays all of the manager's rent without resorting to a negative bonus. Note that the owner could avoid the need to reprice the option in this range of earnings by initially issuing an option with a strike price of  $\hat{x}$ . With a strike price of  $\hat{x}$ , (29) implies at  $x^+$  that

$$B(x^+) + \alpha x^+ = \int_{\hat{x}}^{x^+} g'(R(t)-t)dt + \alpha(x^+ - \hat{x}). \quad (31)$$

Since the optimal contract induces under-reported earnings on  $(\hat{x}, x^+)$ , the first term on the right-hand side of (31) is strictly negative. This means an option with a strike price of  $\hat{x}$  allows the manager to earn too much rent when earnings are just below  $x^+$  and would necessitate a *negative* bonus payment equal to  $\int_{\hat{x}}^{x^+} g'(R(t)-t)dt$  to achieve the desired level of rent implied by the optimal contract. In fact,  $x^+$  is the lowest strike price that does not require the use of negative bonus payments for some earnings levels.

Finally, for  $x \leq \hat{x}$  manager rent is zero. As (28) indicates, this can be accomplished by paying the manager a bonus equal to  $g(-\alpha)$  to cover her manipulation costs.<sup>17</sup> In addition, by not repricing the option, it remains out of the money and is not exercised.<sup>18</sup>

## 5. Optimal Firm Ownership by the Manager

The previous section demonstrated that the optimal contract conditional on the percent of the firm owned by the manager has certain features that are robust to the manager's stake in the firm. However, changes in  $\alpha$  can be expected to affect not only the manager's behavior under the contract but also the owner's expected profit. In this section, we study the effect of changes in  $\alpha$  and calculate the optimal level of manager ownership.

A change in  $\alpha$  will affect expected owner profit via three channels: the direct change in the owner's share of the firm, the change in  $R$  and  $W$  associated with the optimal conditional contract to reflect a change in the manager's reporting incentives, and the change in the level of effort the owner wishes to induce. The effort channel includes the manager's response to stronger effort incentives created by increased ownership as well as the change in reporting incentives due to a shift in the earnings distribution. Since the owner chooses the level of effort to maximize her expected profit, the Envelope Theorem implies that the first-order effect of a

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<sup>17</sup>Although  $g(-\alpha)$  is constant on  $[\underline{x}, \hat{x}]$ , we refer to it as a constant bonus and not a fixed wage. If it was a fixed wage, it would have to be paid for all  $x$  which would make sense if  $g(-\alpha)$  was the minimum bonus paid to the manager. This is not the case as  $g(R(x^+) - x^+) = 0$ .

<sup>18</sup>An equivalent outcome may be obtained by repricing the options in this region so that they are exactly in the money.

change in  $\alpha$  through the effort channel will be zero. Thus, expected owner profit from the optimal conditional contract for each  $\alpha$  is

$$\hat{\Pi}(\alpha) \equiv E(x - g(R(x; a^*(\alpha), \alpha) - x) - W(x; a^*(\alpha), \alpha)), \quad (32)$$

where  $a^*(\alpha)$  denotes the level of effort induced by the optimal conditional contract for each  $\alpha$ , and the Envelope Theorem implies

$$\hat{\Pi}'(\alpha) = E(-g'(R(x; a^*(\alpha), \alpha) - x)R_\alpha - W_\alpha(x; a^*(\alpha), \alpha)) \quad (33)$$

where  $R_\alpha$  and  $W_\alpha$  refer to the partial derivatives of  $R$  and  $W$  holding  $a$  fixed.

Given the Distribution Assumptions, Proposition 2 implies that  $W = 0$  on  $[0, \hat{x}]$  and that the reporting function  $R$  is continuous with respect to  $x$  on  $[0, 1]$ . The fact that  $W=0$  below  $\hat{x}$  means that (13) implies

$$EW_\alpha(a^*(\alpha); \alpha) = \int_{x=\hat{x}(\alpha)}^1 [1 + R_\alpha(x; a^*(\alpha), \alpha)](1 - F(x|a^*(\alpha)))dx \quad (34)$$

where the notation  $\hat{x}(\alpha)$  reflects the effect of  $\alpha$  on the earnings level at which the manager begins to earn positive rent. The continuity of  $R$  then allows us to write (33) as

$$\hat{\Pi}'(\alpha) = \int_0^{\hat{x}(\alpha)} (R-x)f(x|a^*(\alpha))dx - \int_{\hat{x}(\alpha)}^1 [(R-x)R_\alpha f(x|a^*(\alpha)) + (1+R_\alpha)(1-F(x|a^*(\alpha)))]dx. \quad (35)$$

Adding and subtracting  $\int_{\hat{x}}^1 (R-x)f dx$  to the right-hand side of (35) then implies

$$\hat{\Pi}'(\alpha) = E(R-x) - \int_{\hat{x}}^1 (1+R_\alpha)((R-x)f + 1-F)dx, \quad (36)$$

where  $R_\alpha$  is calculated from (26).<sup>19</sup>

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<sup>19</sup>Eq. (26) is the correct formula for  $R$  if it yields a monotonic reporting function. The formula for  $R$  that accounts for the need to "iron" the solution to (26) is derived in the appendix and may involve a constant report  $\bar{R}$  at the highest earnings levels. For expositional purposes,

With effort fixed at  $a^*(\alpha)$ , the multipliers  $\lambda$  and  $\mu$  must adjust to maintain equality of the manager's effort first-order condition, (20). Using (26), (20) implies for  $a^*(\alpha) > 0$  that

$$-\int_{\hat{x}(a^*(\alpha), \alpha)}^1 (\alpha + (1 - \lambda(a^*(\alpha), \alpha))(F-1)/f - \mu(a^*(\alpha), \alpha)F_a/f)F_a dx - h'(a^*(\alpha)) \equiv 0 \quad (37)$$

where (37) makes explicit the dependence of  $\lambda$  and  $\mu$  on  $a$  and  $\alpha$ . Differentiating (37) with respect to  $\alpha$  holding  $a^*(\alpha)$  constant then implies

$$-\int_{\hat{x}}^1 (1 - \lambda_{\alpha}(F-1)/f - \mu_{\alpha}F_a/f)F_a dx \equiv 0$$

or that

$$\int_{\hat{x}}^1 (1 + R_{\alpha}(x; a, \alpha))F_a(x|a)dx \equiv 0. \quad (38)$$

Finally, substituting the definition of  $R$  from (26) into the integrand in (36) yields

$$\hat{\Pi}'(\alpha) = E(R-x) + \int_{\hat{x}}^1 (1 + R_{\alpha})(\lambda(F-1) + \mu F_a)dx. \quad (39)$$

Eq. (38) implies that (39) reduces down to

$$\hat{\Pi}'(\alpha) = E(R-x) + \int_{\hat{x}}^1 \lambda(1 + R_{\alpha})(F-1)dx. \quad (40)$$

If  $\lambda = 0$ , then (40) implies  $\hat{\Pi}'(\alpha) = E(R-x)$ . However, if  $\lambda > 0$ , then  $EW(x; a^*(\alpha), \alpha) = h(a^*(\alpha))$

and (34) imply

$$EW_{\alpha} = \int_{\hat{x}}^1 (1 + R_{\alpha})(1-F)dx = 0, \quad (41)$$

so even if the ex ante individual rationality constraint binds,  $\hat{\Pi}'(\alpha) = E(R-x)$ . This means that the change in expected owner profit from increasing the manager's shares is solely a function of

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we will assume in the text that the optimal reporting function is defined by (26). The more general derivation of  $R$  and  $R_{\alpha}$  yields the same result but with more complicated expressions. A copy of this more general derivation is available from the authors on request.

the average earnings distortion induced by the optimal conditional contract. From Proposition 2, this distortion is strictly positive when  $\alpha = 0$ .

**Proposition 4.** *Given the Distribution Assumptions, the owner will want to endow the manager with a strictly positive share of the firm.*

The expression  $\hat{\Pi}'(\alpha) = E(R-x)$  can be seen to reflect the trade-offs associated with making the manager a shareholder by rewriting it as

$$\hat{\Pi}'(\alpha) = -\alpha F(\hat{x}|\alpha^*) + \int_{\hat{x}}^1 (R-x)dx. \quad (42)$$

The cost,  $-\alpha F$ , reflects the misreporting costs the manager would incur from under-reporting earnings. The benefit to the owner of increasing  $\alpha$  is the savings from paying lower marginal rents through overstated earnings. Thus, the optimal percentage of shares for the manager trades off the cost of earnings management at low earnings versus lower marginal rents paid to the manager at high earnings. It is achieved when on average the contract induces no earnings distortions.

Returning to Example 1, it is straightforward to demonstrate that the expected manipulation,  $E(R-x)$ , when  $\alpha=0$  is strictly positive and when  $\alpha=.05$  equals  $-.027$ . Thus, in the case of the example, the optimal ownership share of the manager is less than 1.

## 5. Conclusions

We have in this paper characterized an optimal compensation contract for a manager who must take a hidden action which affects the probability distribution of firm profits. These



profits, when realized, are themselves hidden information observable only to the manager, who may engage in earnings manipulation by making earnings reports which differ from the actual level of profits. In contrast with previous work, we model explicitly the managerial retention problem and recognize that the manager may leave the firm whenever it is in her best interest to do so.

In this setting, the optimal compensation arrangement consists of an option to purchase shares of the firm that will be repriced to be in the money for lower earnings realizations, as well as performance bonuses based on (manipulated) earnings reports that are increasing in reported earnings once those reports exceed a well-defined earnings threshold. We also find that the optimal compensation arrangement results in managers under-reporting earnings for small levels of profit, and over-reporting earnings when actual profits are higher. Interestingly, and in contrast to conventional wisdom, giving managers a stake in the firm does not create the incentive to over-report earnings. Indeed, we find that an ownership share both reduces over-reporting for high earnings, and also induces under-reporting for lower earnings realizations.

We also find that the use of repriced options is a key tool in providing appropriate levels of managerial compensation. Specifically, a contract that uses options that are repriced at certain earnings levels allows the owner to pay the manager her optimal rent without the use of negative bonus payments. For low earnings levels that are associated with zero manager rent, the optimal contract implies that the underwater options remain worthless or alternatively are repriced to be exactly in the money. For intermediate earnings levels that are associated with under-reporting and positive rent, the optimal contract can be implemented by paying all of the manager's rent by repricing the options to be strictly in the money. Finally, for high earnings associated with

over-reporting, the options will be in the money and the manager will earn her rent through a combination of the profit from exercising the options and a bonus payment.

## Appendix

### *General information for the proofs.*

Problem (11') is solved in two steps. In step 1,  $a$  is fixed and we use (16) to solve for the optimal earnings report function,  $R$ , and the optimal indirect manager utility function,  $W$ , as a function of  $a$ . In step 2, the optimal value of  $a$  is derived. We may write the owner's expected profit solely as a function of the conditionally optimal earnings report function,  $R$ . Noting that  $\Pi = x - g - W$  and substituting from (13), in step 2 we need only choose  $a$  to maximize

$$E[x - g(R(x;a) - x) - W] = E[x - g(R(x;a) - x)] + \frac{F(x|a) - 1}{f(x|a)}(\alpha + g'(R(x;a) - x)) - W(0). \quad (\text{A.1})$$

Step 1 is completed by using Theorems 1 and 2 (chapter 6) in Seierstad and Sydsæter (1987). Because of the pure state constraint,  $W \geq 0$ , it is possible for  $\phi(x)$  to be discontinuous. Seierstad and Sydsæter (p. 319) allow for this possibility but indicate that with the optimal contract  $\phi(x)$  can only jump down at a finite set of points. At all other points,  $\phi$  must be continuous. Given  $\alpha$ , sufficient conditions for an optimal conditional contract are (18)-(23).

From the discussion in the text, (19) and (22) imply (25), hence  $\lambda \leq 1$  (since  $\lambda > 1$  would lead to a contradiction). Recall also from the text that if  $W_x = 0$  on a non-degenerate interval, then

$$\hat{\phi}(x) = \mu F_a(x|a) - \alpha f(x|a). \quad (\text{A.2})$$

If  $\alpha = 0$ , then  $\hat{\phi}(0) = \hat{\phi}(1) = 0$  and for  $x \in (0,1)$ ,  $\hat{\phi}(x) < 0$ . If  $\alpha > 0$ ,  $\hat{\phi}(x) < 0$  for all  $x$ . Thus,  $\hat{\phi}(1) \leq 0 = (1-\lambda)(F(1|a) - 1)$  and, since  $f$  is bounded by Assumption (b), it follows that, for  $\alpha$  sufficiently small and for each  $\lambda < 1$ ,  $\hat{\phi}(0) \geq \lambda - 1 = (1-\lambda)(F(0|a) - 1)$ . Then by continuity there must exist an actual earnings  $\tilde{x} > 0$  such that for all  $x < \tilde{x}$ ,  $\hat{\phi}(x) > (1-\lambda)(F - 1)$ . However, for  $\alpha$

and  $\lambda$  sufficiently large, it is possible that  $\hat{\phi}(0) \leq \lambda - 1 = (1 - \lambda)(F(0|a) - 1)$ . Holding  $\lambda$ ,  $\mu$ , and  $a$  fixed, this possibility suggests two cases.

Case 1.  $\hat{\phi}(0) \leq \lambda - 1$ .

There are two possibilities in this situation. First, suppose that  $\hat{\phi}$  lies below  $(1 - \lambda)(F - 1)$  for every  $x$ . Since (25) requires that the optimal value of the costate variable satisfy  $\phi(x) \geq (1 - \lambda)(F(x|a) - 1)$ , and we know that  $\phi(\cdot)$  cannot jump up, the only feasible co-state function is  $\phi(x) = (1 - \lambda)(F(x|a) - 1)$ . Alternatively, suppose that  $\hat{\phi}$  crosses  $(1 - \lambda)(F - 1)$ . Then the optimal value of the costate variable cannot be  $\hat{\phi}$  until  $\hat{\phi}$  lies above  $(1 - \lambda)(F - 1)$ , by the argument above. And, we also know that, since  $\tau \geq 0$ , (19) and (22) require  $\phi'(x) \leq (1 - \lambda)f(x|a)$  and  $\phi'(x) = (1 - \lambda)f(x|a)$  whenever  $W(x) > 0$ , which implies that the slope of the costate variable cannot be greater than that of  $(1 - \lambda)(F - 1)$ . Thus, for the costate variable to be  $\hat{\phi}$  would require that the costate variable jump up, which is not permitted. So, the only feasible co-state function is (again)  $\phi(x) = (1 - \lambda)(F(x|a) - 1)$ . The result in Case 1 is that  $W(0)$  will equal zero and  $W(x) > 0$  for all  $x > 0$ .  $R(x)$  will be strictly greater than  $x$  on  $[0, 1)$ .

Case 2.  $\hat{\phi}(0) > \lambda - 1$ .

Define  $\tilde{x} = \min\{x | \hat{\phi}(x) = (1 - \lambda)(F(x|a) - 1)\}$ . Since both  $\hat{\phi}$  and  $F$  are continuous and  $\hat{\phi}(1) \leq 0$ ,  $\tilde{x}$  is well-defined. Also, define  $\bar{x} = \inf\{x | \hat{\phi}'(x) > (1 - \lambda)f(x|a)\}$ . If  $\hat{\phi}'(x) \leq (1 - \lambda)f(x|a)$  for all  $x$ , define  $\bar{x} = 1$ . Finally, define  $\hat{x} = \min\{\tilde{x}, \bar{x}\}$ .

From Assumption (c),  $\hat{\phi}'(0) < 0$  so  $\bar{x}$  must be strictly greater than zero. In this case, define the costate variable as  $\phi(x) = \hat{\phi}(x)$  for  $x \leq \hat{x}$ , and define  $\phi(x) = (1 - \lambda)(F(x|a) - 1)$  for  $x > \hat{x}$ . The case in which  $\tilde{x} < \bar{x}$  is depicted in Figure 4 of the paper. Note that if  $\hat{x} = \bar{x} < \tilde{x}$ , then  $\phi$

will jump down at  $\hat{x}$ . This is depicted in Figure A.1, and in Figure A.2 for the case in which  $\hat{x}=1$ . Thus,  $W(x) = 0$  for all  $x \leq \hat{x}$  and  $W(x) > 0$  for all  $x > \hat{x}$ .

(Note: Without Assumption (c), it is possible for  $\hat{x} = 0$ . In this situation,  $\phi$  can neither equal  $\hat{\phi}$  nor  $(1-\lambda)(F-1)$  for  $x$  close to zero. Then there will exist earnings  $0 < x_0 < x_1$  such that for  $x < x_0$ ,  $\phi(x) = (1-\lambda)(F-1) + k$  where  $k$  is a positive constant and  $W(x)$  will be positive. For  $x > x_1$ ,  $\phi(x) = (1-\lambda)(F-1)$ . For some  $x > x_1$ ,  $W(x) = 0$ .)

***Proof of Proposition 2.***

Based on the above analysis, for any  $a > 0$  and for any  $\lambda \leq 1$ , (12) and (26) define a conditional contract that satisfies (18), (19), and (22) and hence also satisfies the properties of optimal conditional contracts described in Proposition 2 for Cases 1 and 2. (For Case 1,  $\hat{x} = 0$ .)

For  $x \leq \hat{x}$ ,  $R'(x) = 1$  and for  $x > \hat{x}$ ,

$$R'(x) = 1 + \partial((F-1)/f)/\partial x - \mu \cdot \partial(F/f)/\partial x. \quad (\text{A.3})$$

By Assumption (d), the second term in (A.3) is strictly positive. The third term can be either positive or negative. For  $a$  sufficiently close to zero,  $\mu$  will be small enough to ensure that  $R'(x) > 0$ . The distribution in example 1 can be used to show that for  $a$  sufficiently large, (26) implies  $R'(x) < 0$  for some (high)  $x$ , which would violate the second order condition (23). Thus, this proof must also show that the solution to (11) when (23) binds for some  $x$  has the same features as contracts based on (26). A solution in this case will require the use of “ironing” techniques (Fudenberg and Tirole, 1999) to characterize an optimal contract.

The more general formulation of the optimal control problem associated with (11) has the Hamiltonian

$$\mathcal{H} = (x - g - W)f + \phi_W(\alpha + g') + \phi_R R' \quad (\text{A.4})$$

and the Lagrangean

$$\mathcal{L} = \mathcal{H} + \tau W - \mu[(\alpha + g')F_a + h'f] + \lambda_1 f(W-h) + \lambda_2 R'$$

where  $W$  and  $R$  are now treated as state variables and  $R'$  is the control.  $\phi_W$  and  $\phi_R$  are the new co-state variables and  $\lambda_2 \geq 0$  is the multiplier for constraint (23). Sufficient conditions analogous to those in Proposition 1 (again following Theorems 1 and 2 from chapter 6 in Seierstad and Sydsæter) are (20), (23),

$$R - x = \frac{\phi_W - \mu F_a + \phi_R'}{f}, \quad (\text{A.5})$$

$$-(1-\lambda_1)f + \tau = -\phi_W', \quad (\text{A.6})$$

$$\phi_R + \lambda_2 = 0, \quad (\text{A.7})$$

$$\lambda_1(EW - h(a)) = 0 \text{ and } \lambda_1 \geq 0, \quad (\text{A.8})$$

$$\phi_W(0) \leq 0, \phi_W(1) \geq 0, \phi_W(0)W(0) = \phi_W(1)W(1) = 0, \tau(x) \geq 0, \tau(x)W(x) = 0, \quad (\text{A.9})$$

$$\phi_R(0) = \phi_R(1) = 0, \text{ and} \quad (\text{A.10})$$

$$\lambda_2(x) \geq 0 \text{ and } \lambda_2(x)R'(x) = 0. \quad (\text{A.11})$$

As with Proposition 1,  $\phi_W$  and  $\phi_R$  can jump down but not up at a countable number of values of  $x$ . Note that if the solution to (11') implies (23), then  $\phi_R(x) \equiv 0$  and (A.5)-(A.11) plus (20) and (23) collapses down to (18)-(23).

Note also that by (A.7),  $\phi_R \neq 0$  only if  $R' = 0$  as then  $\lambda_2 > 0$ . By Distribution Assumption (e), solutions to (11') that violate (23) must do so on an interval from some  $x'$  to 1. And, since  $R(1) = 1$  as defined by (26), it follows that, on  $(x', 1)$ , we must have  $R(x) > 1$ . That is, the "ironed" solution to (11) will induce  $R = \bar{R} > 1$  on some interval  $\underline{x}$  to 1 where  $\underline{x} < x'$ .

The optimal conditional contract is constructed as follows. First, define  $\hat{x}$  and  $\hat{\phi}$  as above.  $\hat{\phi}$  now represents the value of the co-state variable,  $\phi_w$ , for which  $W_x = 0$  when  $\phi'_R = 0$ . Then define  $\phi_w = \hat{\phi}$  on  $[0, \hat{x}]$  and define  $\phi_w = (1-\lambda)(F-1) - \mu F_a$  on  $(\hat{x}, 1]$ . This is identical to the definition of  $\phi$  above.

Second, define  $x_1 = \min\{x | x + ((1-\lambda)(F-1) - \mu F_a)/f = 1\}$ .  $x_1$  represents the first earnings level at which the reporting function which solves (11') first equals 1. Since (26) implies  $R(1) = 1$ ,  $x_1$  always exists. If  $x_1 = 1$ , then (26) will define an increasing reporting function on  $[0, 1)$  and  $\phi_R \equiv 0$ . If  $x_1 < 1$ , then (26) must define a reporting function that is decreasing for  $x$  close to 1.

Third, choose  $\bar{R}$  from the interval  $(1, \max_x x + ((1-\lambda)(F-1) - \mu F_a)/f)$ . For each  $\bar{R}$ , there will be two solutions to the equation:  $x + ((1-\lambda)(F-1) - \mu F_a)/f = \bar{R}$  on  $(x_1, 1)$ . Denote these two solutions by  $\bar{x}_1$  and  $\bar{x}_2$  such that  $\bar{x}_1 < \bar{x}_2$ . The various values of  $x$  that have been identified are noted in Figure A3.

Define  $\phi_R = 0$  for  $x \leq \bar{x}_1$  and for  $x > \bar{x}_1$  solve (A.5) for  $\phi'_R$  when  $R = \bar{R}$ . This implies  $\phi'_R(x) = (\bar{R} - x)f - (1-\lambda)(F-1) + \mu F_a$ .  $\phi'_R(x)$  will be negative on  $(\bar{x}_1, \bar{x}_2)$  and it will be positive on  $(\bar{x}_2, 1]$ . As a result,

$$\phi_R(x) = -\lambda_2(x) = \int_{t=\bar{x}_1}^x [\bar{R} - t - ((1-\lambda)(F-1) - \mu F_a)/f] f dt \quad (\text{A.12})$$

on  $(\bar{x}_1, 1]$ .  $\bar{R}$  must be chosen so that  $\phi_R(\bar{x}_1) = \phi_R(1) = 0$ . Using (A.12),  $\bar{R} = 1$  implies  $\phi_R(1) < 0$  and  $\bar{R} = \max_x x + ((1-\lambda)(F-1) - \mu F_a)/f$  implies  $\phi_R(1) > 0$ . Thus, there must exist a value of  $\bar{R}$  for which  $\phi_R(1) = 0$ . Thus, ensuring that (23) is satisfied does not alter the quantitative properties of optimal conditional contracts given  $a$  and  $\mu$ .

To check (20), the manager's first-order condition for effort, note that (11b) is strictly negative for  $\mu = 0$  and strictly positive for  $\mu = \infty$ . Thus, for any  $a > 0$ , there exists a  $\mu$  that satisfies (20).

To summarize, for any  $a > 0$ , there exists  $\mu > 0$  such that the optimal conditional contract must fall into one of four categories: Case 1 without ironing at the top, Case 1 with ironing at the top, Case 2 without ironing at the top, and Case 2 with ironing at the top. The transition between these four cases is continuous and hence expected owner profit is a continuous function of  $a$ . If  $\bar{a}$  denotes the first-best level of effort, then the optimal level of effort will be less than or equal to  $\bar{a}$ . This means the owner's choice of effort to induce exists since it maximizes a continuous function on  $[0, \bar{a}]$ . Since  $h'(0) = 0$ , the optimal value of  $a$  must be strictly positive.

***Q.E.D.***

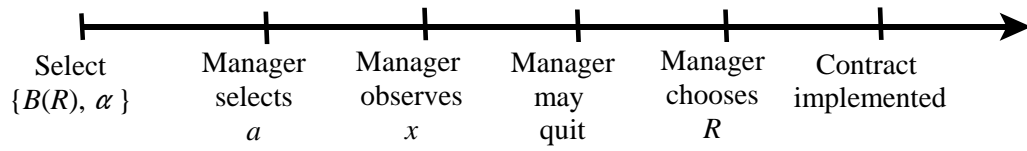


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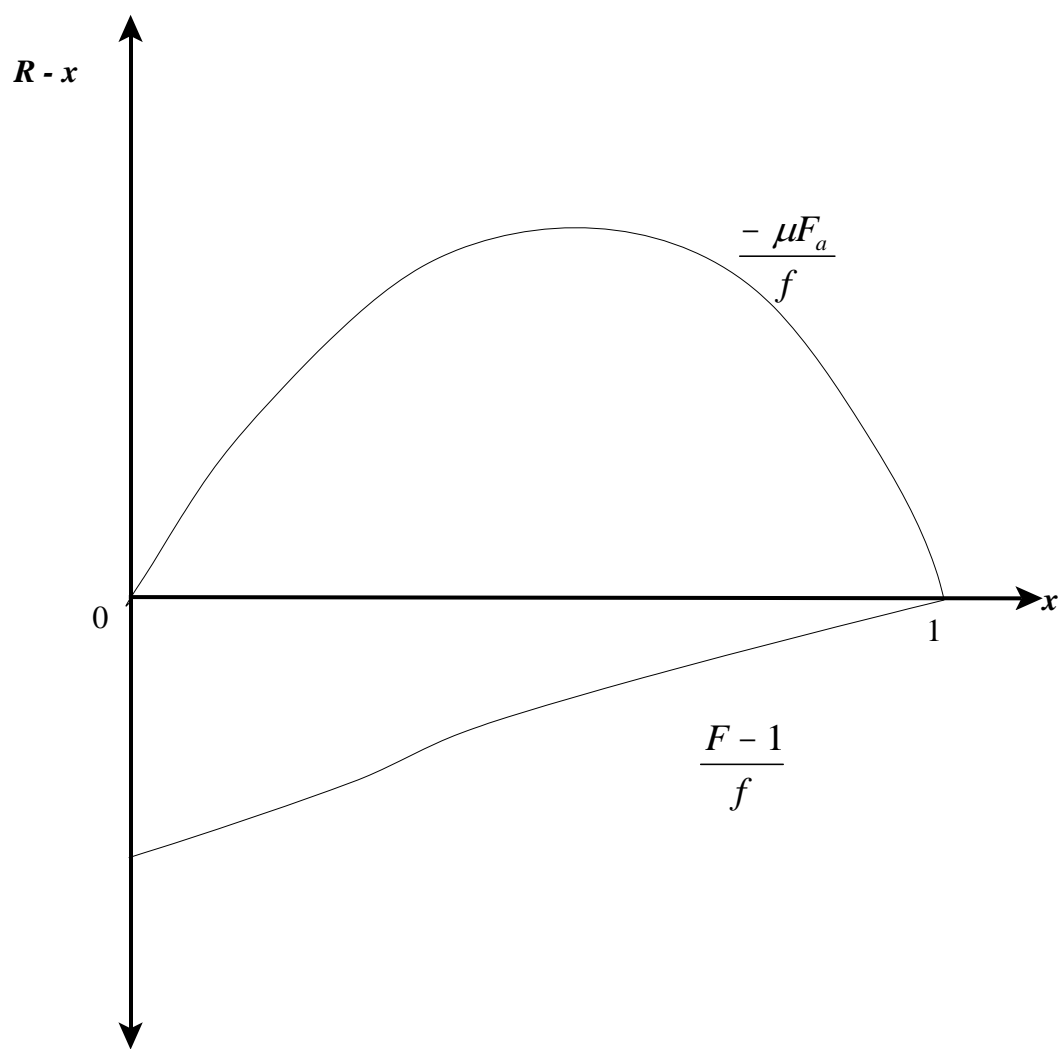
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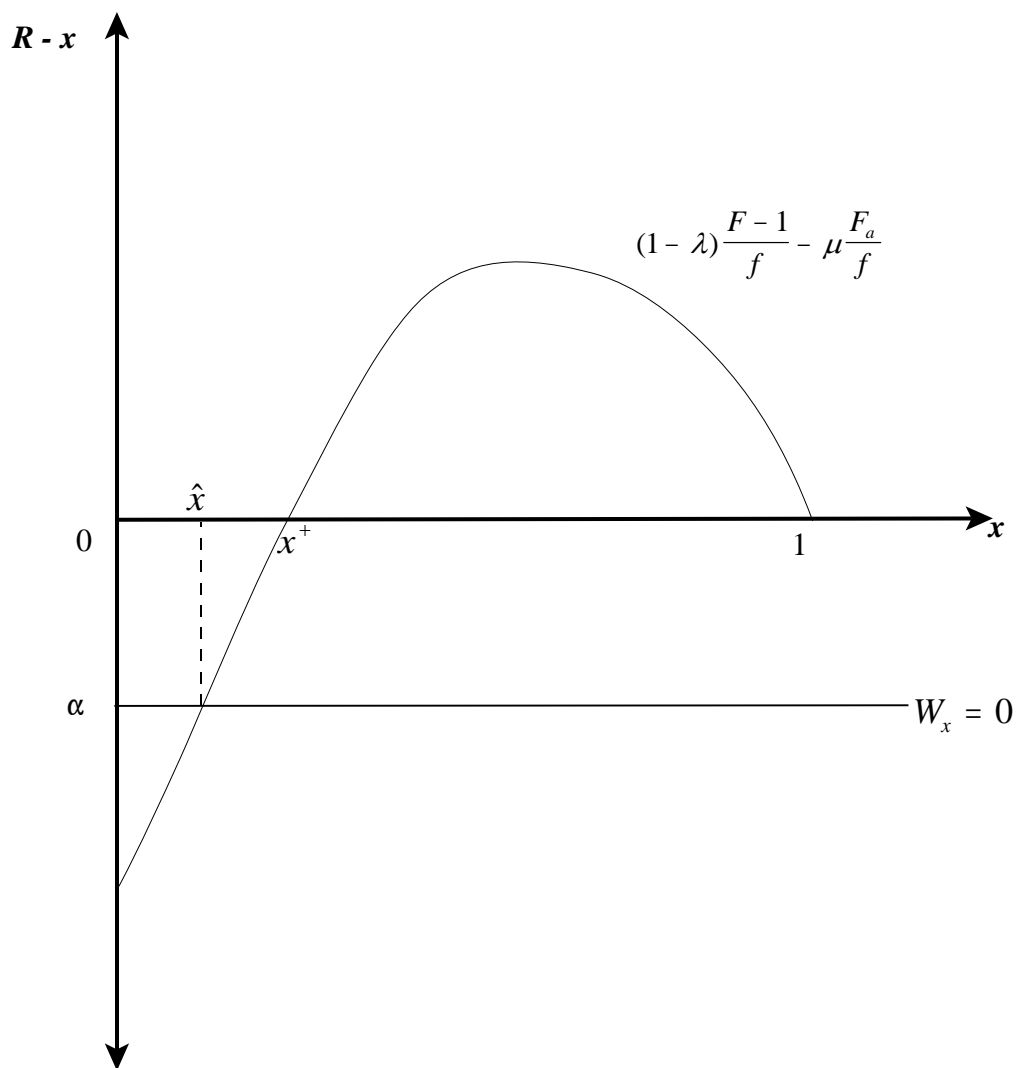
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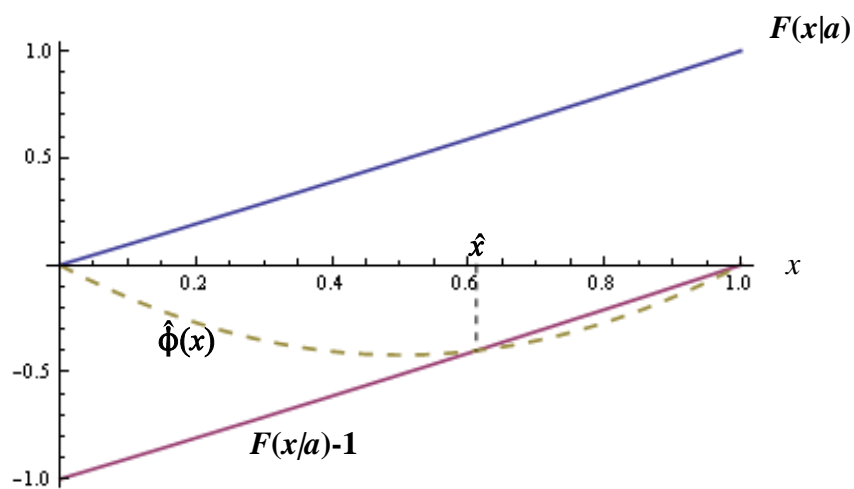
**Figure 1: Sequence of Events**



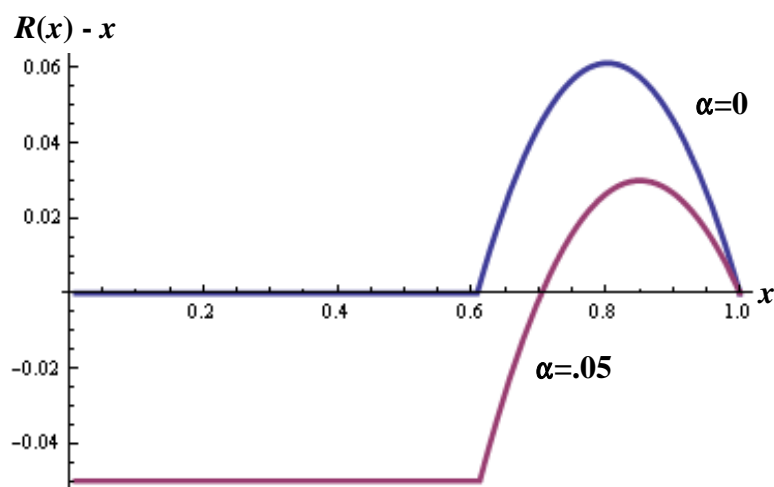
**Figure 2: Rent extraction vs. effort incentive effects in the absence of an interim individual rationality constraint.**



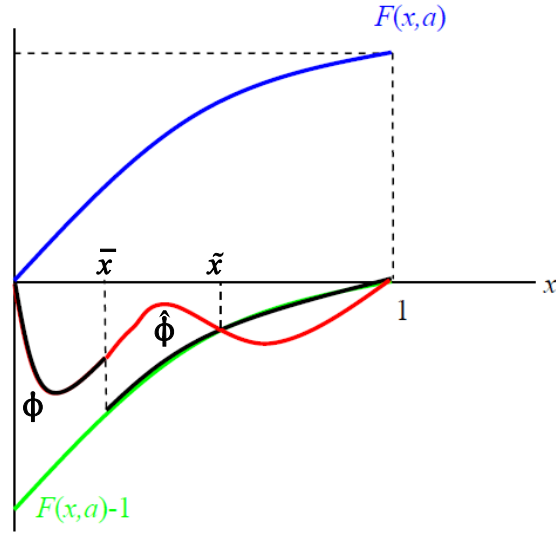
**Figure 3: Rent extraction vs. effort incentive effects with an interim individual rationality constraint.**



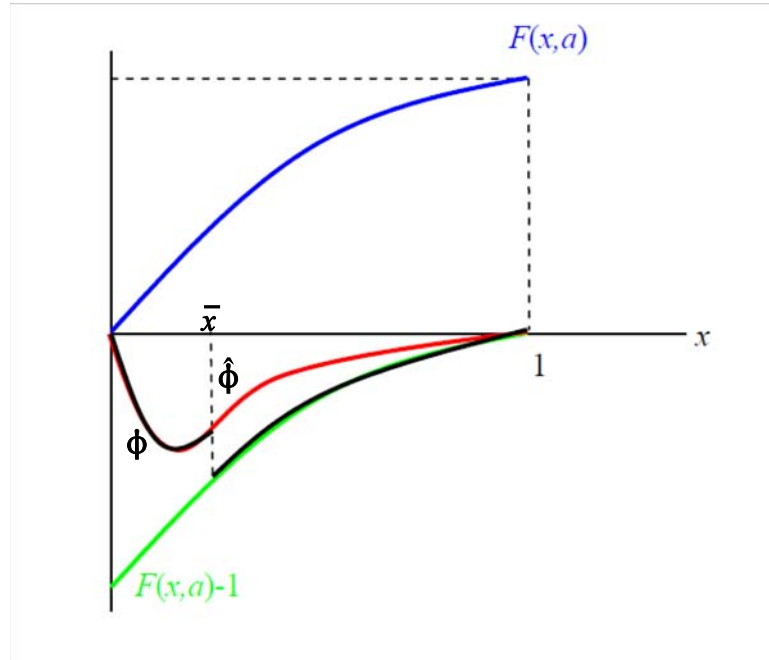
**Figure 4: The trade-off between zero manager rent and earnings manipulation in Example 1**



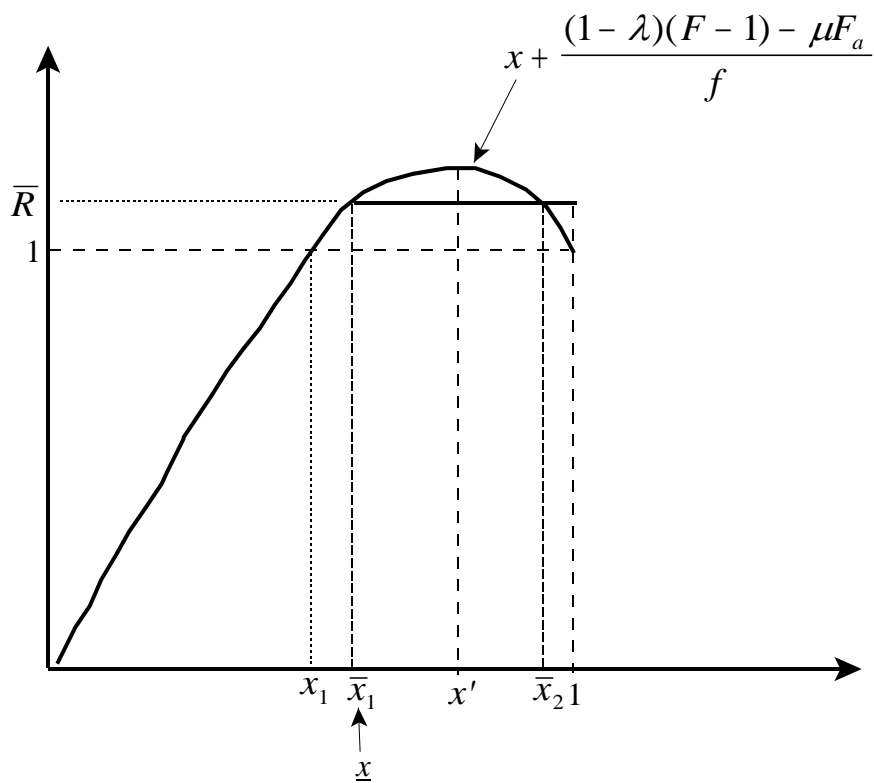
**Figure 5: Optimal earnings manipulation when  $\alpha = 0$  and when  $\alpha = .05$**



**Figure A.1:** An example of a discontinuous co-state function due to a violation of (19) when  $\bar{x} < \tilde{x} < 1$ .



**Figure A.2:** An example of a discontinuous co-state function due to a violation of (19) when  $\bar{x} < \tilde{x} = 1$ .



**Figure A3: The optimal earnings manipulation when ironing at the top of the distribution is required.**