A note on different Predictors of Lifetimes of Censored Items in Progressively Censored Samples from Normal Distribution

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Outline

- Progressive Censoring
- Prediction Problem
- Different Predictors

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Progressive Type II Right Censoring

- n units placed on an experiment
- m completely observed until failure
- Censoring occurs progressively in m stages
  - First failure (the first stage): r_1 of the $n-1$ surviving units randomly withdrawn,
  - Second failure (the second stage): r_2 of the $n-2-r_1$ surviving units are withdrawn,
  - and so on.
  - Finally, the m-th failure (the m-th stage): remaining $r_m = n - m - r_1 - \cdots - r_{m-1}$ are withdrawn.

Type-II right censoring : $r_1 = r_2 = \cdots = r_{m-1} = 0$ and $r_m = n - m$.
Complete sampling scheme: $n = m$ and $r_1 = r_2 = \cdots = r_m = 0$.

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Prediction

- $X_1, \cdots, X_n$: failure times of n independent units from cdf $F(x, \theta)$ and pdf $f(x, \theta)$ with parameter θ.
- We observe only $\boldsymbol{Y} = (Y_1, \cdots, Y_m)$ where $Y_1 \leq \cdots \leq Y_m$: m progressively censored order statistics.
- Purpose of this article: to predict life-lengths
$$Y_{j:r_i};\ j = 1, 2, \cdots, r_i;\ i = 1, 2, \cdots, m$$
of all censored units in all m stages of censoring.
- Prediction of times to failure of only the last r_m units still surviving at the observation Y_m has been considered by Balakrishnan and Rao (1997) for Exponential Distribution.
- BLUP, MLP, MMLP, AMLP and CMP are derived for normal distribution for all $Y_{j:r_i}; j = 1, 2, \cdots, r_i, i = 1, 2, \cdots, m$.

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Notations

$Y_{j:r_i}$: jth order statistic out of r_i units of Y

$Y^{B}_{j:r_i}$: BLUP of $Y_{j:r_i}$

$Y^{OSM}_{j:r_i}$: OSMLP of $Y_{j:r_i}$

$Y^{TSM}_{j:r_i}$: TSMLP of $Y_{j:r_i}$

Notations – Continued

$$Y_{j:r_i}^{OSMM} \quad : \quad \text{OSMMLP of } Y_{j:r_i}$$

$$Y_{j:r_i}^{TSMM} \quad : \quad \text{TSMMLP of } Y_{j:r_i}$$

$$Y_{j:r_i}^{OSAM} \quad : \quad \text{OSAMLP of } Y_{j:r_i}$$

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$$Y_{j:r_i}^{C} \quad : \quad \text{CMP of } Y_{j:r_i}$$

The Normal distribution

- $f(x;\mu,\sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$, $-\infty < \mu < \infty$, $\sigma > 0$.
- $\boldsymbol{Y} = (Y_1, \cdots, Y_m)$.
- For the sake of simplicity, assume $\sigma = 1$ and and will cover only the case of unknown scale parameter μ.
- The case when both parameters μ and σ are unknown is handled with little additional difficulty.

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The BLUP $Y^B_{j:r_i}$ of $Y_{j:r_i}$ is given by

$$Y^B_{j:r_i} = \mu^* + \alpha_1 + \boldsymbol{w}'\boldsymbol{\Sigma}^{-1}(Y - \mu^*\mathbf{1} - \alpha).$$

where the BLUE μ^* of μ:

$$\mu^* = \frac{\mathbf{1}'\boldsymbol{\Sigma}^{-1}(\boldsymbol{Y} - \alpha)}{\mathbf{1}'\boldsymbol{\Sigma}^{-1}\mathbf{1}}.$$

- α and Σ are the vector of means and dispersion matrix, respectively, of the m progressively Type-II right censored order statistics from the standard normal distribution.

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$$\mu^* = \frac{\mathbf{1}'\boldsymbol{\Sigma}^{-1}(\boldsymbol{Y} - \alpha)}{\mathbf{1}'\boldsymbol{\Sigma}^{-1}\mathbf{1}}.$$

- $\boldsymbol{\alpha}$ and $\boldsymbol{\Sigma}$ are the vector of means and dispersion matrix, respectively, of the m progressively Type-II right censored order statistics from the standard normal distribution.

BLUP

- α_1 is the expected value of the j-th order statistic out of r_i units from the standard normal conditional distribution, with the condition that it is greater than Y_i.

$$\alpha_1 = c_{i-1} \sum_{k=1}^{i} a_{k,i} \sum_{l=0}^{\gamma_k - r_i - 1} (-1)^l \binom{\gamma_k - r_i - 1}{l} \times \frac{r_i!\, l!}{(r_i + l + 1)!} E(X_{j+l+1:r_i+l+1}).$$

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BLUP

- $\boldsymbol{w}' = (0, 0, \cdots, \sigma_{ij}, \cdots, 0)$ with $\sigma_{ij} = \mathrm{Cov}(Y_i, Y_{j:r_i})$.

$$\mathrm{Cov}(Y_i, Y_{j:r_i}) = E(Y_i Y_{j:r_i}) - E(Y_i)E(Y_{j:r_i}) = E(Y_i Y_{j:r_i}) - \alpha_1 E(Y_i),$$

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$$E(Y_i Y_{j:r_i}) = c_{i-1} \sum_{k=1}^{i} a_{k,i} \sum_{l=0}^{\gamma_k - r_i - 1} (-1)^l \binom{\gamma_k - r_i - 1}{l} \times \frac{r_i!\, l!}{(r_i + l + 1)!} \times E(X_{l+1:r_i+l+1} \cdot X_{j+l+1:r_i+l+1}),$$

$$E(Y_i) = c_{i-1} \sum_{k=1}^{i} a_{k,i} \frac{1}{\gamma_k} E[X_{1:\gamma_k}],$$

BLUP

where

$$
\begin{aligned}
\gamma_1 &= n \\
\gamma_k &= n - r_1 - \cdots - r_{k-1} - k + 1; \;\; k = 2, \cdots, m \\
c_{l-1} &= c_{l-1}(\gamma_1, \cdots, \gamma_l) = \prod_{k=1}^{l} \gamma_k; \;\; l = 1, \cdots, m \\
a_{k,l} &= a_{k,l}(\gamma_1, \cdots, \gamma_l) = \prod_{u=1, u \neq k}^{l} \frac{1}{\gamma_u - \gamma_k}. \;\; 1 \leq k \leq l \leq m
\end{aligned}
$$

Outline

1. Progressive Censoring
2. Prediction Problem
3. Different Predictors
   - BLUP
   - MLP
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   - MMLP
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     - TSAMLP
   - CMP

MLP

- Let $\boldsymbol{y} = (y_1, y_2, \cdots, y_m)$ with $y_1 \leq y_2 \cdots \leq y_m$ and $y = y_{j:r_i}$ denote the observed value of $\boldsymbol{Y}$ and the unobserved value of $Y_{j:r_i}$, respectively.
- The predictive likelihood function (PLF) of $Y_{j:r_i}$ and $\boldsymbol{\theta}$ is given by

$$\begin{aligned} L &= L(y, \boldsymbol{\theta}; \boldsymbol{y}) \\ &= cf(y)\,[F(y) - F(y_i)]^{j-1}\,[1 - F(y)]^{r_i - j} \\ &\times \prod_{l=1}^{m} f(y_l) \prod_{l=1, l \neq i}^{m} [1 - F(y_l)]^{r_l}, \quad y \geq y_i, \end{aligned}$$

where c denotes a constant factor.

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MLP

- If $Y^L_{j:r_i} = t(\boldsymbol{Y})$ and $\boldsymbol{\theta}^{**} = u(\boldsymbol{Y})$ are statistics for which

$$L(t(\boldsymbol{y}), u(\boldsymbol{y}); \boldsymbol{y}) = \sup_{y, \boldsymbol{\theta}} L(y, \boldsymbol{\theta}; \boldsymbol{y}),$$

then $t(\boldsymbol{Y})$ is said to be the MLP of $Y_{j:r_i}$, and $u(\boldsymbol{Y})$ the predictive maximum likelihood estimator (PMLE) of $\boldsymbol{\theta}$.

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MLP

- The PLF of $Y_{j:r_i} = Y$ and μ is given by

$$L(Z,\mu;\mathbf{Z}) = c_1 c_2 f(Z) \prod_{l=1}^{m} f(Z_l) \prod_{l=1, l\neq i}^{m} [1-F(Z_l)]^{r_l} \times [F(Z)-F(Z_i)]^{j-1} [1-F(Z)]^{r_i-j}, \quad Z \geq Z_i,$$

where
$Z = Y-\mu, Z_l = Y_l - \mu \ (l = 1, \cdots, m), \mathbf{Z} = (Z_1, Z_2, \cdots, Z_m)$,
$c_1 = c_1(j, r_i) = \frac{r_i!}{(j-1)!(r_i-j)!}$ and
$c_2 = c_2(n, m, r_1, \cdots, r_m) =$
$n(n-r_1-1)(n-r_1-r_2-2)\cdots(n-r_1-\cdots-r_{m-1}-m+1)$.

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MLP

- This PLF can be written as a product of two likelihood functions

$$L_1(\mu; \mathbf{Z}) = c_2 \prod_{l=1}^{m} \{f(Z_l)\,[1 - F(Z_l)]^{r_l}\} \text{ and}$$

$$L_2(Z; \mu, \mathbf{Z}) = c_1 f(Z) \frac{[F(Z) - F(Z_i)]^{j-1}}{[1 - F(Z_i)]^{r_i}} [1 - F(Z)]^{r_i - j}, \quad Z \geq Z_i.$$

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MLP

We need:

$$
\begin{aligned}
h(Z) &= \frac{\phi(Z)}{1-\Phi(Z)}, \\
h(Z_l) &= \frac{\phi(Z_l)}{1-\Phi(Z_l)}; l = 1, \cdots, m, \\
h_1(Z_i, Z) &= \frac{\phi(Z_i)}{\Phi(Z)-\Phi(Z_i)}; \quad Z > Z_i, \\
h_2(Z_i, Z) &= \frac{\phi(Z)}{\Phi(Z)-\Phi(Z_i)}; \quad Z > Z_i.
\end{aligned}
$$

OSMLP

- OSMLP $Y_{j:r_i}^{OSM}$ is given by

$$\begin{aligned}\frac{\partial \log L}{\partial \mu} &= Z + \sum_{l=1}^{m} Z_l + \sum_{l=1, l\neq i}^{m} r_l h(Z_l) \\ &\quad + (j-1)\left[h_1(Z_i, Z) - h_2(Z_i, Z)\right] + (r_i - j)h(Z) = 0, \\ \frac{\partial \log L}{\partial Y} &= -Z + (j-1)h_2(Z_i, Z) - (r_i - j)h(Z) = 0,\end{aligned}$$

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TSMLP

- TSMLP $Y_{j:r_i}^{TSM}$ is given by

$$\frac{\partial \log L_1}{\partial \mu} = \sum_{l=1}^{m} Z_l + \sum_{l=1}^{m} r_l h(Z_l) = 0,$$

$$\frac{\partial \log L_2}{\partial Y} = -Z + (j-1)h_2(Z_i, Z) - (r_i - j)h(Z) = 0.$$

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2. Prediction Problem
3. Different Predictors
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   - MLP
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MMLP

Replacing $h(Z), h(Z_i), h_1(Z_i, Z), h_2(Z_i, Z)$ by their expected values. For MMLP, we need:

- Lemma 1

$$E[h(Z_l)] = B_l = c_{l-1} \sum_{k=1}^{l} \frac{a_{k,l}}{(\gamma_k - 1)\gamma_k} E(Z_{1:\gamma_k}); l = 1, 2, \cdots, m.$$

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MMLP

- Lemma 2
  $E[h(Z)] = \frac{1}{r_i - j} E_i(j+2)$ where,

$$E_i(a) = c_{i-1} \sum_{k=1}^{i} a_{k,i} \sum_{l=0}^{\gamma_k - r_i - 1} (-1)^l \binom{\gamma_k - r_i - 1}{l} \times \frac{r_i! l!}{(r_i + l + 1)!} \sum_{m=l+a}^{r_i + l + 1} E(Z_{m:r_i+l+1}).$$

- Lemma 3
  $E[h_1(Z_i, Z)] = \frac{1}{j-1} E_i(2)$.
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OSMMLP

OSMMLP $Y_{j:r_i}^{OSMM}$ is given by

$$Y_{j:r_i}^{OSMM} = \begin{cases} \hat{\mu}^{OSMM} + D_i(j+1) & \text{if} \quad 2 \le j \le r_i - 1 \\ \hat{\mu}^{OSMM} - E_i(3) & \text{if} \quad j = 1 \\ \hat{\mu}^{OSMM} + E_i(r_i+1) & \text{if} \quad j = r_i \end{cases}$$

where

$$\hat{\mu}^{OSMM} = \frac{1}{m}\left[\sum_{l=1}^{m} Y_l + \sum_{l=1, l\neq i}^{m} r_l B_l\right]$$

and

$$\begin{aligned} D_i(c) &= c_{i-1} \sum_{k=1}^{i} a_{k,i} \sum_{l=0}^{\gamma_k - r_i - 1} (-1)^l \binom{\gamma_k - r_i - 1}{l} \\ &\quad \times \frac{r_i! l!}{(r_i+l+1)!} \frac{1}{r_i - j} E(Z_{l+c:r_i+l+1}). \end{aligned}$$

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2. Prediction Problem
3. Different Predictors
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AMLP

Replacing $h(Z_l); l = 1, \cdots, m$, $h(Z), h_1(Z_i, Z), h_2(Z_i, Z)$ by their Taylor Series expansions around the point $F^{-1}(p_l); l = 1, \cdots, m$, $F^{-1}(\pi_{i,j})$, $(F^{-1}(p_i), F^{-1}(\pi_{i,j}))$ and $(F^{-1}(p_i), F^{-1}(\pi_{i,j}))$ respectively, where $\pi_{i,j} = \frac{j}{r_i+1}$ and p_i's can be obtained by

$$\left.\begin{array}{lcl} p_1 & = & p_1^*, \\ p_2 & = & p_1 + p_2^* \cdot \frac{1-p_1}{1-q_1-p_1^*}, \\ & & \\ p_3 & = & p_2 + p_3^* \cdot \frac{1-p_2}{1-q_1-q_2-p_1^*-p_2^*}, \\ & \cdots & \\ p_m & = & p_{m-1} + p_m^* \cdot \frac{1-p_{m-1}}{1-\sum_{i=1}^{m-1} q_i - \sum_{i=1}^{m-1} p_i^*}. \end{array}\right\}$$

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q_i and $p_i^*(i = 1, 2, \cdots, m)$ are defined as follows:

- Suppose in a progressive censoring, r_i items are randomly withdrawn after s_i (additional) failures for $i = 1, 2, \cdots, m$.
- Now assume $\frac{s_i}{n} \to p_i^*$ and $\frac{r_i}{n} \to q_i$.

The Taylor series approximations are given by

$$\begin{aligned} h(Z_l) &\simeq \alpha_l + \beta_l Z_l; l = 1, \cdots, m, \\ h(Z) &\simeq \alpha^* + \beta^* Z, \\ h_1(Z_i, Z) &\simeq \gamma^* - \rho^* Z_i - \nu^* Z, \\ h_2(Z_i, Z) &\simeq \gamma + \rho Z_i - \nu Z. \end{aligned}$$

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$$\begin{aligned} h(Z_l) &\simeq \alpha_l + \beta_l Z_l; l = 1, \cdots, m, \\ h(Z) &\simeq \alpha^* + \beta^* Z, \\ h_1(Z_i, Z) &\simeq \gamma^* - \rho^* Z_i - \nu^* Z, \\ h_2(Z_i, Z) &\simeq \gamma + \rho Z_i - \nu Z. \end{aligned}$$

q_i and $p_i^*(i = 1, 2, \cdots, m)$ are defined as follows:

- Suppose in a progressive censoring, r_i items are randomly withdrawn after s_i (additional) failures for $i = 1, 2, \cdots, m$.
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The constants $\alpha_l, \beta_l, \alpha^*, \beta^*, \gamma^*, \rho^*, \nu^*, \gamma, \rho$ and ν are given by

$$\begin{aligned}
\alpha_l &= f(\zeta_l)\left[(1+\zeta_l^2)(1-p_l) - \zeta_l f(\zeta_l)\right]/(1-p_l)^2, \\
\beta_l &= f(\zeta_l)\left[f(\zeta_l) - (1-p_l)\zeta_l\right]/(1-p_l)^2, \\
\alpha^* &\equiv \alpha^*_{j:r_i} = f(\eta_j)\left[(1+\eta_j^2)(1-\pi_{i,j}) - \eta_j f(\eta_j)\right]/(1-\pi_{i,j})^2, \\
\beta^* &\equiv \beta^*_{j:r_i} = f(\eta_j)\left[f(\eta_j) - (1-\pi_{i,j})\eta_j\right]/(1-\pi_{i,j})^2, \\
\gamma^* &\equiv \gamma^*_{j:r_i} = f(\zeta_i)\left[(1+\zeta_i^2)(\pi_{i,j}-p_i) + \eta_j f(\eta_j) - \zeta_i f(\zeta_i)\right]/(\pi_{i,j}-p_i)^2,
\end{aligned}$$

$$\rho^* \equiv \rho^*_{j:r_i} = f(\zeta_i)\left[-f(\zeta_i) + \zeta_i(\pi_{i,j} - p_i)\right]/(\pi_{i,j} - p_i)^2,$$

$$\nu^* \equiv \nu^*_{j:r_i} = f(\zeta_i)f(\eta_j)/(\pi_{i,j} - p_i)^2,$$

$$\gamma \equiv \gamma_{j:r_i} = f(\eta_j)\left[(1+\eta_j^2)(\pi_{i,j} - p_i) + \eta_j f(\eta_j) - \zeta_i f(\zeta_i)\right]/(\pi_{i,j} - p_i)^2,$$

$$\rho \equiv \rho_{j:r_i} = f(\zeta_i)f(\eta_j)/(\pi_{i,j} - p_i)^2,$$

$$\nu \equiv \nu_{j:r_i} = f(\eta_j)\left[\eta_j(\pi_{i,j} - p_i) + f(\eta_j)\right]/(\pi_{i,j} - p_i)^2,$$

where $\zeta_i = F^{-1}(p_i)$, $\zeta_l = F^{-1}(p_l)$, $\eta_j = F^{-1}(\pi_{i,j})$.

OSAMLP

OSAMLP $Y_{j:r_i}^{OSAM}$ is given by

$$Y_{j:r_i}^{OSAM} = \begin{cases} \hat{Y}_{j:r_i} & \text{if} \quad \hat{Y}_{j:r_i} > Y_i \\ Y_i & \text{if} \quad \hat{Y}_{j:r_i} \leq Y_i \end{cases}$$

where $\hat{Y}_{j:r_i}$ is given by

$$\frac{\hat{\mu}^{OSAM}\left[(j-1)(\nu-\rho)+(r_i-j)\beta^*+1\right]+(j-1)\left[\gamma+\rho Y_i\right]-(r_i-j)\alpha^*}{(j-1)\nu+(r_i-j)\beta^*+1},$$

and

$$\hat{\mu}^{OSAM} = \frac{m\bar{Y}+\sum_{l=1,l\neq i}^{m} r_l(\alpha_l+\beta_l Y_l)+(j-1)\left[\gamma^*-\rho^* Y_i-\nu^*\hat{Y}_{j:r_i}\right]}{m+\sum^{m} r_l\beta_l-(j-1)(\rho^*+\nu^*)}.$$

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Outline

1. Progressive Censoring
2. Prediction Problem
3. Different Predictors
   - BLUP
   - MLP
     - OSMLP
     - TSMLP
   - MMLP
     - OSMMLP
     - TSMMLP
   - AMLP
     - OSAMLP
     - TSAMLP
   - CMP

CMP

A statistic T is called the CMP of $Y_{j:r_i}$ if it is the median of the conditional distribution of $Y_{j:r_i}$ given Y_i.

The CMP $Y^C_{j:r_i}$ of $Y_{j:r_i}$ is such that

$$\int_{y_i}^{Y^C_{j:r_i}} f(y|y_i)dy = \frac{1}{2}$$

in which $f(y|y_i)$ is given by

$$\frac{r_i!}{(j-1)!(r_i-j)!} \frac{\left[\int_{y_i}^{y} e^{-\frac{(x-\mu^*)^2}{2}} dx\right]^{j-1} e^{-\frac{(y-\mu^*)^2}{2}} \left[\int_{y}^{\infty} e^{-\frac{(x-\mu^*)^2}{2}} dx\right]^{r_i-j}}{\left[\int_{y_i}^{\infty} e^{-\frac{(x-\mu^*)^2}{2}} dx\right]^{r_i}}.$$

where μ^* is the BLUE of μ.

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THANK YOU