MODELING THE OCCURRENCE OR NON-OCCURRENCE OF BELIEF-BASED “RUNS”

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1. Introduction

“Runs” on banks and on similar financial intermediaries have been a recurrent phenomenon. In some cases, a run has occurred when depositors or investors have received information that the intermediary lacks sufficient resources to honor all of its claims. Then there is a race to present claims to the intermediary before those resources run out. But many people—including market participants and economics researchers who have studied runs—believe that not all runs can be explained simply as a response to insufficiency of resources.

Motivated by that suspicion, Bryant [1] and Diamond and Dybvig [2] have formulated theories of an expectations-based run. These theories emphasize that a depositor’s decision whether or not to withdraw funds from the bank, if there is not actually an immediate need for money, depends on beliefs about whether or not other depositors without urgent needs will keep their money in the bank. If so, then there will be enough resources for everyone. If not, then the bank’s resources will be exhausted, so each depositor will race to withdraw his/her funds before that happens. Either belief can be self fulfilling. The upshot is that, for no particular reason, a run on a conservatively managed bank can occur.

A bank can structure and operate itself as a narrow bank—one that can honor all of its claims, even if a run occurs. Essentially, to do so, it must not pay interest on deposits. But prospective depositors generally are willing to take a small risk of there being a run, in order to receive interest. Diamond and Dybvig, especially, have emphasized that a narrow bank would unanimously be considered worse than an interest-paying bank, if everyone were confident that no one else would make withdrawals that they did not genuinely need to make.

To restate Diamond and Dybvig’s point in the language of game theory, each way of structuring and operating the bank entails a specific game among the depositors. The narrow-bank game has a unique equilibrium. The interest-paying-bank game has multiple equilibria, one of which is unanimously better than, and the other of which (that is, equilibrium in which a run occurs) is unanimously worse than, the narrow-bank equilibrium, when viewed from an ex ante perspective.

Thus, Diamond and Dybvig suggest that, acting in accord with the preferences of their depositors, who initially are confident that everyone will “play by the rules of the game,” bankers opt to pay interest. In their terminology, bankers opt to offer a standard deposit contract. A run occurs when, possibly without any good reason, depositors lose confidence that other depositors will exercise their withdrawal options under a standard deposit contract only when they have genuine need to do
so. On Diamond and Dybvig’s view, occasional runs are the price that we (bankers and depositors) pay for using the contract that works in everyone’s best interest most of the time. Those runs may be purely belief based, occurring in the absence of any cause having to do with “economic fundamentals.” They occur because the economy with financial intermediation has the structure of a game with multiple equilibria.

Diamond and Dybvig’s logic involves an implicit assumption. They assume that the narrow-bank contract (that is, the regime of non-interest-bearing deposits) and the standard debt contract are the only contracts that the banker could possibly offer. But, what if there were a contract that would as good, or better, for depositors than the standard debt contract is, and that would entail a game with a unique equilibrium. In that case it would be inexplicable, within Diamond and Dybvig’s theoretical framework, why bankers would offer a standard deposit contract rather than offering a different contract that would be both as good for depositors and also immune to the possibility of a run.

Green and Lin [4], [5] have modeled an environment closely similar to Diamond and Dybvig’s model, in which the standard debt contract studied by Diamond and Dybvig is not the the optimal banking contract. In fact, the optimal contract can be characterized by iterated elimination of strictly dominated strategies, and thus has a unique equilibrium. In that environment, a sound analysis concludes that a belief-based run would not occur. Subsequently other researchers, such as Ennis and Keister [3], have formulated other variants of the environment in which the contract having the best equilibrium also has another equilibrium that resembles Diamond and Dybvig’s run equilibrium. Taken together, this research clarifies the logic of Diamond and Dybvig’s argument and makes a start towards answering the question of under what circumstances belief-based runs will or will not occur.

Diamond and Dybvig model the bank essentially as an intermediary that insures depositors against the risk of having an urgent need for immediate consumption, and who offers a contract with the objective of maximizing the depositors’ welfare. In this note, I formulate a very simple model of such an intermediary. For some values of the model’s parameters, the optimal contract will entail a game with both an intended equilibrium and also another, welfare-inferior, equilibrium that resembles Diamond and Dybvig’s run equilibrium. For other values of the parameter, the optimal contract will entail a game having a unique equilibrium in dominant strategies.

2. AN INSURANCE ENVIRONMENT

Instead of depositors and a banker, let’s think two workers and their boss. Hard work is being done on a hot day. There are two workers, named A and B. The boss insures the workers against the risk of being thirsty.

With probability \( p \), either worker will be thirsty at the time when the boss gives them a break. Let \( A_θ \) be the event that A will be thirsty, and let \( A_ν \) be the event that A will not be thirsty. Let \( B_θ \) and \( B_ν \) be the corresponding events for B. The workers’ risks of being thirsty are independent. Probabilities of compound events are shown in the following table.
The two workers have identical utility functions for consuming water. The utility of taking a drink of size \( d \geq 0 \), when in physical state \( \sigma \in \{\theta, \nu\} \), is determined by two parameters: marginal utility, \( m_\sigma \), and a satiation point, \( s_\sigma \). The marginal utility of water consumption beyond the satiation point is \(-1\). Thus, the utility function is

\[
(2) \quad u(d, \sigma) = m_\sigma \min\{d, s_\sigma\} - \max\{d - s_\sigma, 0\}
\]

Assume that a thirsty worker gets higher marginal utility (up to satiation) than a non-thirsty worker does, and that the thirsty worker’s satiation point is higher. That is,

\[
(3) \quad m_\theta > m_\nu \text{ and } s_\theta > s_\nu
\]

The following graph shows \( u(d, \theta) \) and \( u(d, \nu) \). (The slopes of the utility functions at the right side of the graph should be \(-1\), so the scale of the vertical axis is compressed.) As the graph is drawn, both workers can enjoy consumption of water up to their satiation points if neither is thirsty, but not otherwise. If the two-unit aggregate endowment were drawn further to the right, both could consume as much water as they like, even if both are thirsty. Alternately, if \( s_\nu \) were drawn just a tiny bit to the left of \( s_\theta \), then consumption to the satiation point would be infeasible even if neither worker were thirsty. Another special feature of the graph, not implied by the assumptions that have been made here, is that workers attain non-negative utility in every feasible allocation. If a worker could feasibly be forced to drink an amount of water that was far above his/her satiation point, then the worker’s utility would be negative.
The boss has wisely arranged for a two-unit jug of water and two cups to be brought to the job site at break time. (A unit of water might perhaps be interpreted to be half a liter.) In the vocabulary of economics, there is a two-unit aggregate endowment that the boss must allocate between the two workers.

3. The boss’s optimization problem

The workers and the boss in this environment correspond to the depositors and the bank in Diamond and Dybvig’s model environment. The boss’s objective is to maximize the expected sum of the workers’ utilities, subject to the resource constraint and an information constraint. I will also consider only allocations that satisfy a symmetry condition, which can be shown not to be a binding constraint on the level of the objective function that can be achieved.

Formally, let \( D(\sigma, \tau) \) be the amount of water that is given to a worker whose physical state is \( \sigma \) and whose partner’s physical state is \( \tau \). Note that \( D(\sigma, \tau) \) is the same for worker \( A \) as it is for worker \( B \). This is the symmetry condition that has just been mentioned. Below, I will refer to \( D \) as being the contract between the boss and the workers.

Using the probabilities specified in table (1), the boss’s objective function is

\[
\begin{align*}
&= p^2[u(D(\theta, \theta), \theta) + u(D(\theta, \theta), \theta)] \\
&+ p(1 - p)[u(D(\theta, \nu), \theta) + u(D(\nu, \theta), \nu)] \\
&+ (1 - p)p[u(D(\nu, \theta), \nu) + u(D(\theta, \nu), \theta)] \\
&+ (1 - p)^2[u(D(\nu, \nu), \nu) + u(D(\nu, \nu), \nu)]
\end{align*}
\]
Taking advantage of the symmetry of $D$, and dividing by 2 for convenience (since dividing an objective function by a positive constant does not change its optimal solution), the objective function can be re-written as

$$p^2 u(D(\theta, \theta), \theta) + p(1 - p)[u(D(\theta, \nu), \theta)$$
$$+ u(D(\nu, \theta), \nu)] + (1 - p)^2 u(D(\nu, \nu), \nu)$$

(5)

3.1. **Resource-feasible allocations.** An allocation of water to the two workers is resource feasible if the total amount of water that they are given to drink never exceeds two units. Now the condition of being resource feasible can be formalized: For all $\sigma$ and $\tau$ in $\{\theta, \nu\}$,

$$0 \leq D(\sigma, \tau) \quad \text{and} \quad D(\sigma, \tau) + D(\tau, \sigma) \leq 2$$

(6)

Let’s briefly consider how the boss should specify $D$ to optimize the objective function (5), if the resource-feasibility constraint were the only constraint. The answer is very simple: give the water to the worker(s) whose marginal utility for it is highest. Specifically,

$$D(\theta, \theta) = \min\{1, s_\theta\} \quad D(\nu, \nu) = \min\{1, s_\nu\}$$
$$D(\theta, \nu) = \min\{2, s_\theta\} \quad D(\nu, \theta) = \min\{2 - D(\theta, \nu), s_\nu\}$$

(7)

The contract, $D$, specified in equation (7) is called the public-information-optimum contract.

3.2. **Workers’ private information and voluntary self reporting.** Besides resource feasibility, another constraint on the boss’s flexibility to allocate water to the workers is that each worker’s knowledge of whether or not he/she is thirsty is private, and the workers cannot be coerced to report it truthfully. The boss must induce them to do so voluntarily. Below, I will refer to the worker’s situation of being thirsty or not thirsty as being the worker’s utility type. Sometimes I will simply write ‘type’ as a shorthand for ‘utility type’.

In effect, the boss makes the following deal with each worker: You tell me whether you are thirsty or not. Your co-worker will tell me the corresponding information. If you report being of type $\hat{\sigma}$ and your co-worker reports being of type $\hat{\tau}$, then you will receive $D(\hat{\sigma}, \hat{\tau})$ units of water, and you must drink all of it. (The “hats” on $\sigma$ and $\tau$ indicate that these are reports, rather than necessarily being the workers’ true utility types.) If these deals induce the workers to report their respective types truthfully, then the allocation is incentive compatible. Beginning in the next section, two specific versions of incentive compatibility will be defined.

In general, the privacy of the workers’ information is a binding constraint on what the boss can accomplish on their behalf. For example, section 5 will begin with an example in which the boss cannot successfully administer the public-information-optimum contract because the workers will lie about their types. However, if the public-information-optimal contract remains feasible to administer in an environment where workers’ information is private, then it is also optimal in that environment. This observation is an instance of the general principle that, if an optimal solution of a constrained optimization problem remains feasible when a further constraint is added, then it also remains optimal in that situation.
4. Bayesian Nash equilibrium and Bayesian incentive compatibility

Each worker knows privately whether he/she is thirsty, although the co-worker and the boss cannot observe it. A worker’s strategy is to report (either truthfully or falsely) a utility type, knowing his/her true type. A strategy will be denoted by \((\tilde{\theta}, \tilde{\nu})\), where \(\tilde{\theta}\) is the report that the worker gives if thirsty, and \(\tilde{\nu}\) is the report given if not thirsty. For example, \((\tilde{\theta}, \tilde{\theta})\) is the strategy of always claiming to be thirsty, and \((\tilde{\nu}, \tilde{\theta})\) is the strategy of always lying.

Suppose that \(A\)’s strategy is \((\tilde{\theta}, \tilde{\nu})\) and that \(B\)’s strategy is \((\tilde{\tau}, \tilde{\tau})\). Also, suppose that \(A\)’s true utility type is \(\sigma\) and that \(B\)’s true utility type is \(\tau\). \(A\) understands that his/her co-worker’s true type is \(\theta\) with probability \(p\) and \(\nu\) with probability \(1 - p\). Therefore, \(A\)’s expected utility from playing his/her strategy against \(B\)’s strategy is

\[
U_A((\tilde{\theta}, \tilde{\nu}), (\tilde{\tau}, \tilde{\tau}), \sigma) = pu(D(\tilde{\theta}, \tilde{\tau}), \sigma) + (1 - p)u(D(\tilde{\nu}, \tilde{\tau}), \sigma)
\]

Symmetrically, \(B\)’s expected utility from playing his/her strategy against \(A\)’s strategy is

\[
U_B((\tilde{\theta}, \tilde{\nu}), (\tilde{\tau}, \tilde{\tau}), \tau) = pu(D(\tilde{\theta}, \tilde{\tau}), \tau) + (1 - p)u(D(\tilde{\nu}, \tilde{\tau}), \tau)
\]

A noteworthy feature of equations (8) and (9) is that, when a worker has a particular utility type, that worker’s report corresponding to the other type does not affect his/her utility. For example, setting \(\sigma = \theta\) in equation (8), \(U_A((\tilde{\theta}, \tilde{\nu}), (\tilde{\theta}, \tilde{\nu}), \theta)\) depends on \(\tilde{\theta}\), \(\tilde{\nu}\), and \(\tilde{\nu}\), but not on \(\tilde{\nu}\).

Thus, when \(A_\theta\) (that is, worker \(A\) of type \(\theta\)) chooses a utility-maximizing strategy for 
\(A\), all that matters is \(\tilde{\theta}\). Because of this feature, we can regard the contract as being a game among four players, \(A_\theta\), \(A_\nu\), \(B_\theta\), and \(B_\nu\), each of whom makes a report in \(\{\tilde{\theta}, \tilde{\nu}\}\). Then, we can find Nash equilibria of that four-player game. Such an equilibrium is called a Bayesian Nash equilibrium of the two-player game between \(A\) and \(B\). That is, the strategy profile \((((\tilde{\theta}, \tilde{\nu}), (\tilde{\nu}, \tilde{\nu}))\) is a Bayesian Nash equilibrium of the game entailed by contract \(D\) if the following conditions (corresponding to optimization by players \(A_\theta\), \(A_\nu\), \(B_\theta\), and \(B_\nu\) in the four-player game) are satisfied.

\[
pu(D(\tilde{\theta}, \tilde{\theta}), \theta) + (1 - p)u(D(\tilde{\theta}, \tilde{\nu}), \theta) \geq pu(D(\tilde{\nu}, \tilde{\theta}), \theta) + (1 - p)u(D(\tilde{\nu}, \tilde{\nu}), \theta)
\]

\[
pu(D(\tilde{\theta}, \tilde{\nu}), \nu) + (1 - p)u(D(\tilde{\nu}, \tilde{\nu}), \nu) \geq pu(D(\tilde{\theta}, \tilde{\theta}), \nu) + (1 - p)u(D(\tilde{\theta}, \tilde{\nu}), \nu)
\]

\[
pu(D(\tilde{\nu}, \tilde{\nu}), \theta) + (1 - p)u(D(\tilde{\nu}, \tilde{\nu}), \theta) \geq pu(D(\tilde{\theta}, \tilde{\theta}), \theta) + (1 - p)u(D(\tilde{\theta}, \tilde{\nu}), \theta)
\]

\[
pu(D(\tilde{\theta}, \tilde{\nu}), \nu) + (1 - p)u(D(\tilde{\nu}, \tilde{\nu}), \nu) \geq pu(D(\tilde{\nu}, \tilde{\theta}), \nu) + (1 - p)u(D(\tilde{\nu}, \tilde{\nu}), \nu)
\]

Contract \(D\) is Bayesian incentive compatible if the truthful-revelation strategy profile, \(((\tilde{\theta}, \tilde{\nu}), (\tilde{\theta}, \tilde{\nu}))\) is a Bayesian Nash equilibrium.
5. Dominant-strategy incentive compatibility

Suppose that \( \{\hat{\sigma}, \hat{\tau}\} = \{\hat{\theta}, \hat{\nu}\} \), and that \( \sigma \in \{\theta, \nu\} \). Then \( \hat{\sigma} \) is a strictly dominant strategy for player \( A_\sigma \) if \( \hat{\sigma} \) provides \( A_\sigma \) strictly higher utility than \( \hat{\tau} \) does, regardless of what player \( B \) reports. That is,

\[
\begin{align*}
\text{(11)} & \quad u(D(\hat{\sigma}, \hat{\sigma}), \sigma) > u(D(\hat{\tau}, \hat{\sigma}), \sigma); \quad \text{and} \\
\text{Symmetrically,} & \quad u(D(\hat{\sigma}, \hat{\sigma}), \sigma) > u(D(\hat{\tau}, \hat{\sigma}), \sigma)
\end{align*}
\]

Contract \( D \) is dominant-strategy incentive compatible if truth telling is always a strictly dominant strategy, that is, if \( \hat{\theta} \) is a strictly dominant strategy for \( A_\theta \) and \( B_\theta \) and \( \hat{\nu} \) is a strictly dominant strategy for \( A_\nu \) and \( B_\nu \).

6. Is the public-information-optimum contract Bayesian incentive compatible?

Now I will show that there are some parameter values of the model for which the public-information-optimum contract specified in equation (7) is not Bayesian incentive compatible, and that there are other parameter values for which that contract is Bayesian incentive compatible.

Consider the following parameter values:

\[
(13) \quad p = 0.5 \quad m_\nu = 1 \quad m_\theta = 2 \quad s_\nu = 3 \quad s_\theta = 4
\]

According to equation (7), these values imply that

\[
(14) \quad D(\hat{\theta}, \hat{\theta}) = 1 \quad D(\hat{\nu}, \hat{\theta}) = 0 \quad D(\hat{\theta}, \hat{\nu}) = 2 \quad D(\hat{\nu}, \hat{\nu}) = 1
\]

That is, given either report (thirsty or not thirsty) by worker \( B \), worker \( A \) receives one more unit of water by reporting to be thirsty than by reporting not to be thirsty. Even if worker \( A \) is not thirsty and receives the entire, two-unit, water endowment, the worker will not reach his/her satiation point. That is, for both utility types of worker \( A \), claiming to be thirsty is the strictly dominant strategy. Since that report is untruthful for \( A_\nu \), and since no player can play a strictly dominated strategy in a Bayesian Nash equilibrium, the public-information-optimum contract is not Bayesian incentive compatible at parameter values (13).

For an example in which the public-information-optimal contract is Bayesian incentive compatible, consider the following parameter values.

\[
(15) \quad p = 0.5 \quad m_\nu = 1 \quad m_\theta = 2 \quad s_\nu = 1 \quad s_\theta = 3
\]

According to equation (7), these values imply that

\[
(16) \quad D(\hat{\theta}, \hat{\theta}) = 1 \quad D(\hat{\nu}, \hat{\theta}) = 0 \quad D(\hat{\theta}, \hat{\nu}) = 2 \quad D(\hat{\nu}, \hat{\nu}) = 1
\]

Again, regardless of what the co-worker reports, a worker always gets one unit more by claiming to be thirsty than by claiming not to be thirsty, and the satiation point of a thirsty worker is larger than the aggregate endowment. Thus, truthful reporting is a thirsty worker’s dominant strategy.

If the co-worker will report truthfully, then a worker who claims not to be thirsty receives 0 if the co-worker is thirsty and receives 1 if the co-worker is not thirsty. A non-thirsty worker who claims not to be thirsty receives expected utility 0.5, then,
if the co-worker reports truthfully. A worker who claims to be thirsty receives 1
if the truthful co-worker is thirsty, and receives 2 if the truthful co-worker is not
thirsty, so the expected utility of a non-thirsty worker who claims to be thirsty is
$0.5 \times 1 + 0.5 \times (1 - (2 - 1)) = 0.5$. That is, the non-thirsty worker is indifferent between
reporting being thirsty and reporting being not thirsty. Reporting truthfully is a
best reply, albeit not the unique one, to a truthful co-worker.

Since truthful reporting is a best reply to truthful reporting for both types of
worker, it is a Bayesian Nash equilibrium. That is, the public-information-optimum
contract is Bayesian incentive compatible in the environment with parameter values
(15).

7. A RUN EQUILIBRIUM

Diamond and Dybvig define a run equilibrium of the banking system to be a
Bayesian Nash equilibrium, of the game entailed by the system, in which the pri-
vately informed depositors always report needing urgently to make withdrawals,
regardless of whether or not they actually experience such needs. Correspond-
ing to that definition, the run-equilibrium strategy profile in the present model is
$((\hat{\theta}, \hat{\theta}), (\hat{\theta}, \hat{\theta}))$.

It turns out that the run-equilibrium strategy profile is a Bayesian Nash equilib-
rium in the environment with parameter values (15). If the co-worker will always
claim to be thirsty, then a worker who also claims to be thirsty will always receive
one unit of water, while a worker who claims not to be thirsty will never receive any
water. Since one unit does not exceed the satiation point of either type of worker,
both types prefer more water to less, so claiming to be thirsty is the best reply to
a co-worker who always claims to be thirsty.

To recapitulate, the environment with parameter values (15) possesses both a
truth-telling Bayesian Nash equilibrium and also a run equilibrium. Since a contract
is defined to be Bayesian incentive-compatible if it possesses a truth-telling Bayesian
Nash equilibrium—whether or not it also possesses a different equilibrium—the
public-information-optimum contract is incentive compatible.

Note that the expected utility of a non-thirsty worker is higher (1.0 versus 0.5)
in the run equilibrium than in the truth-telling equilibrium, although the expected
utility of a thirsty worker is higher (3 versus 2) in the truth-telling equilibrium.
However, it is generally considered most appropriate to make welfare comparisons
from an ex ante perspective, corresponding to the boss’s objective function.\footnote{An ex ante comparison is made from “behind the veil of ignorance,” as advocated, for example, by Harsanyi [6] and Rawls [7].}

8. DOMINANT-STRATEGY INCENTIVE COMPATIBILITY

Finally, consider the model with the following parameter values.

\[(17)\]

\[
p = 0.5 \quad m_\nu = 1 \quad m_\theta = 2 \quad s_\nu = 0.4 \quad s_\theta = 0.8
\]

Here, since feasibility is never a binding constraint on providing satiation-point
consumption to both workers, what a worker receives depends only on his/her own
report. Truthful reporting provides a worker with consumption at exactly his/her
true satiation point, while false reporting results in consumption at the “wrong”
satiation point. Thus, truthful reporting is the strictly dominant strategy for both
types of agent. The public-information-optimum contract is dominant-strategy
incentive compatible in the environment with parameter values (17).
Since the profile of strictly dominant strategies must be unique if it exists, and
since there can be no other Bayesian Nash equilibrium in that case, there is no run
equilibrium in this environment.

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