Book Problems:
   Chapter 4, Problems 6, 13, 14, 15, 16

Additional Problems:

1. **State Diagrams.** In each of the following questions, draw a transition probability diagram or specify a transition probability matrix for a three-state Markov Chain \((X_t \in \{0, 1, 2\})\) with the specified properties.
   (a) The chain is irreducible.
   (b) State 0 is transient and the other two states are recurrent.
   (c) State 0 and state 1 are transient and state 3 is recurrent.
   (d) The chain has two recurrent classes.
   (e) The chain has three recurrent classes.

2. **Transient Markov Chain.** Give an example of a Markov Chain in which all states are transient. Describe the state-space (the values that \(X_t\) can take on) and the transition probabilities between states. Show that all states are transient.

3. **HIV Progression and Prediction.** Use the HIV stage data (“hiv.Rdata”) from Homework 3 for this problem. Individuals with HIV are classified into 6 states. The first 5 states (1-5) are states with progressively fewer T4 cells, and the last state (6) is a deceased state.
   (a) Use the HIV stage data **hiv.Rdata** or **hiv.csv** from my website to estimate the transition probability matrix \(P\). This is identical to the question asked in Homework 3.
   (b) Classify each state (1-6) as being transient or recurrent.
   (c) Calculate \(P^{(3)}\).
   (d) Estimate the median lifespan of an individual with HIV by finding a number of time steps \(t^*\) such that the probability of an individual being deceased at time \(t < t^*\), given that the individual was in state 0 at time \(t = 0\), is less than 0.5, AND the probability of an individual being deceased at time \(t \geq t^*\), given that the individual was in state 0 at time \(t = 0\), is greater than 0.5. Describe how you found \(t^*\).