Remarkable Mathematicians
From Euler to von Neumann

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EMMY NOETHER (1882-1935)
We now come to the first woman mathematician who can undoubtedly be described as 'great'. As Weyl said at her funeral: 'The memory of her work in science and of her personality among her fellows will not soon pass away. She was a great mathematician, the greatest, I firmly believe, her sex has ever produced, and a great woman.' The innovative approach to modern abstract algebra of Emmy Noether not only produced major new results, but also inspired highly productive work by students and colleagues who
emulated her techniques. After being dismissed from her university post at the beginning of Nazi rule in Germany she moved to the United States where a promising start to a new career was cut short by her untimely death.

Amalie Emmy Noether was born on March 23, 1882 in Erlangen, where her father Max Noether was professor of mathematics at the university. His perceptive obituaries of his contemporaries are a valuable source of information. Her mother, Ida Amalia (nee Kaufmann), came from a wealthy Jewish family of Cologne. She had a conventional upbringing, attending the Municipal School for the Higher Education of Daughters until she was eighteen when she was certified as a teacher in French and English at Institutions for the Education and Instruction of Females. Women had been allowed to enroll at universities in France since 1861, England since 1878, and Italy since 1885. However, in Germany as late as 1900, this was still not permitted; professors frequently refused permission for women even to attend their lectures, and only very rarely were women allowed to take university examinations. At the University of Erlangen the academic senate in 1898 passed a resolution declaring that the admission of women students would 'overthrow all academic order'.

However Emmy Noether was determined to undertake university studies and she was not easily deterred. She succeeded in attending courses in mathematics and other scientific subjects at Erlangen and Gottingen, after which she enrolled at the former university in 1904 when it was first possible for women to do so. Her true abilities were quite slow to show themselves. However by 1908 she had completed her dissertation 'On complete systems of invariants for ternary biquadratic forms', under her father's colleague Paul Albert Gordan, who she had known since she was a child, and was awarded a Ph.D., *summa cum laude*.

For the next seven years she remained at Erlangen, without a position, doing a little teaching but mainly engaged in research. During this period she began to work with the algebraist Ernst Fischer, who led her away from the algorithmic methods of Gordan towards the broad theoretical style characteristic of Hilbert. This change of direction led to an invitation from Hilbert and Klein to join them at Gottingen. At that time the new theory of relativity was causing great excitement. Emmy Noether was one of the first to understand its implications and contributed two new results which are important in the general theory. Hilbert tried to secure an academic position for her but there was too much opposition. This provoked his famous outburst at a faculty meeting:
'I do not see that the sex of the candidate is an argument against her admission as privatdozent. After all, we are a university, not a bathing establishment.' However, it is believed the opposition was less because of her sex than because of her radical political convictions. It was not until the reforms of 1919 that she became even a privatdozent at the age of thirty-seven. Three years later she received the honorary title of unofficial associate professor, without official responsibilities, and without stipend. Nevertheless she acted as thesis adviser for a number of Gottingen Ph.D. students.

A keen mind and infectious enthusiasm for mathematical research made Emmy Noether an effective teacher, although she could not be described as a good lecturer. Her classroom technique, like her thinking, was strongly conceptual. Rather than simply lecturing, she conducted discussion sessions in which she would explore some topic with her students. She loved to spend free time with them, especially on long walks. Sometimes she would become so engrossed in the conversation that her students would have to remind her to watch for traffic. On one memorable occasion her slip came down when she was lecturing - she bent down, pulled it off, threw it in the corridor, and kept on lecturing.

Outstanding mathematicians often make their greatest contributions early in their careers; Emmy Noether was an exception. She began producing her most powerful and creative work around the age of forty. Her change of style started with a 1920 paper on non-commutative fields (such as the quaternions). During the years that followed, she developed a very abstract and generalized approach to the axiomatic development of algebra. As Weyl attested, she originated above all a new and epoch-making style of thinking about algebra, later written up by her former student van der Waerden in his well-known textbook on modern algebra. Her revolutionary 1921 paper on ideal theory, where the concept of Noetherian ring originated, is arguably her finest work. 'You'll find it already in Dedekind', she used to say, but although Dedekind may have had the basic ideas it was Emmy Noether who developed in its full richness the theory which has exerted such an enormous influence on modern algebra.

By this time Emmy Noether was at the height of her powers, and her ideas were winning acceptance more and more. She lectured at the 1928 International Congress in Bologna, gave courses at the University of Moscow and the Communist Academy in the winter of 1928/9, and was a principal speaker at the important 1932 International Congress in Zurich. At the Georgia Augusta she had at last been appointed associate
professor. She never attained the top rank of full professor, although she contributed so much to making Göttingen the premier mathematical centre in Europe, many would say in the world. On her fiftieth birthday her group of algebraists, called the Noether family, held a celebration in her honour.

Only a year later the Nazis seized power and, as we know, one of their first acts was to deprive 'non-Aryan' government officials, including university teachers, of their positions, with certain exceptions. Most members of the mathematics faculty at the Georgia Augusta were Jewish; they were henceforth forbidden to teach at the university or even to enter the department. For a time Emmy Noether continued to meet informally with students and colleagues, inviting groups to her apartment, while trying, with her despondent colleagues, to decide what to do. She seriously considered moving to Moscow, where she had a following, but Aleksandrov was unable to convince the university authorities of the need to act quickly. In the meantime efforts were being made on her behalf in America, and before the end of 1933 she had arrived there, with a temporary position at Bryn Mawr College. At first there had been a plan to invite her to Oxford, but in the end it was Bryn Mawr, supported by funds from the Rockefeller Foundation, which offered her a position for 1933/4. Normally the Rockefeller funds were only intended for cases where there was a definite plan to provide a permanent post; because she had no interest in undergraduate teaching there was no prospect of Bryn Mawr offering this. However her supporters, including Birkhoff, Lefschetz, and Wiener, succeeded in persuading the college to extend her appointment.

Although the college had something of a mathematical tradition, as we know, the department only consisted of four faculty members and five graduate students, none of whom had been exposed to any abstract algebra. Emmy Noether took four of the students under her wing and taught them in a mixture of German and English. With her characteristic curiosity and good nature she settled in happily into her new home. She did her best to speak in English from the very first; she wanted to know how things were done in America, whether it was giving a tea party or taking a Ph.D., and she attacked each subject with the disarming candour and vigorous attention which won over everyone who knew her. Her work was as inevitable and natural as breathing, a background for living taken for granted; but that work was only the core of her relation to her students. She lived with them and for them in a perfectly unselfconscious way. She looked on the world
Emmy Noether (1882-1935)

with direct friendliness and unfeigned interest and she wanted them to
do the same. She loved to take walks, and many a Saturday, with five
or six students, she tramped the roads around the college with a fine
disregard for bad weather.

From Bryn Mawr she was able to visit other friends and former
colleagues from Germany who had also come to America. She gave a
course of weekly lectures on algebra at Princeton; the distinguished
audience included A.A. Albert, Richard Brauer, Nathan Jacobson, H.S.
Vandiver, and Oscar Zariski. Olga Taussky, her assistant at Gottingen,
had followed her to Bryn Mawr and often accompanied her on these
visits to Princeton. She recalled: 'I remember that on one occasion she
had to apply the binomial theorem to a very special situation.
Although she had enormous insight into difficult abstract structure
this computation was a great challenge to her. She did however
master it and was very pleased about this. In fact she turned back from
the blackboard about three times to smile proudly at us.'

So what was she like personally? 'Warm like a loaf of bread',
Weyl wrote, 'there radiated from her a broad, comforting, vital
warmth. She was strongly myopic and wore spectacles with thick
lenses. She was fat, rough and loud, but so kind that all who knew her
loved her. She thought little about what she should wear, what she
should eat, and so on. Her intentions could hardly be further removed
from the effects which her appearance had, especially on those who
did not know her. In particular, there was the handkerchief. She kept it
tucked under her blouse. While lecturing she had a way of jerking it
out and thrusting it back, very energetically, and this was very
noticeable to her audience. During the years before she began keeping
her hair cropped short, she wore it up, and little by little, during the
excitement of lecturing, it would fall out of place.'

At the end of that first year in America she returned briefly to
Germany and was appalled to find how much the situation had
deteriorated. Towards the end of her second year at Bryn Mawr she
went into hospital for surgery to remove a uterine tumour. Although
the operation was not without risk it was carried out successfully.
However while still in hospital she developed a high fever and died
suddenly on April 14, 1935 at the age of fifty-three, apparently from
either a post-operative embolism or an infection. Shortly before she
died she had remarked to Veblen at Princeton that the previous year-
and-a-half had been the very happiest in her whole life, for she was
appreciated at Bryn Mawr and Princeton as she had never been
appreciated in her own country. After cremation her ashes were
interred near the college library; Weyl concluded his memorial address with these words:

It was only too easy for those who met her for the first time, or had no feeling for her creative power, to consider her queer or to make fun at her expense. She was heavy of build and loud of voice, and it was not easy for one to get the floor in competition with her. She preached mightily, and not as the scribes. She was a rough and simple soul, but her heart was in the right place. Her frankness was never offensive in the least degree. In everyday life she was most unassuming and utterly unselfish; she had a kind and friendly nature. Nevertheless she enjoyed the recognition paid to her; she could answer with a bashful smile like a young girl to whom one had whispered a compliment. No one could contend that the Graces had stood by her cradle; but if we in Gottingen often chaffingly referred to her as 'der Noether' (with the masculine article) it was also done with a respectful recognition of her power as a creative thinker who seemed to have broken through the barrier of sex. She possessed a rare humour and a sense of sociability; a tea in her apartment could be most pleasurable. She was a kind and courageous being, ready to help, and capable of the deepest loyalty and affection. And of all I have known, she was certainly one of the happiest.

Two traits determined her nature; first the native productive powers of her mathematical genius. She was not clay, pressed by the artistic hands of God into a harmonious form, but rather a chunk of human primary rock into which he had blown his creative breath of life. Secondly her heart knew no malice; she did not believe in evil - indeed it never entered her mind that it could play a role among men. This was never more forcefully apparent to me than in the last stormy summer, that of 1933, which we spent together at Gottingen. A time of struggle like this one ... draws people together; thus I have a particularly vivid recollection of these months. Emmy Noether, her courage, her frankness, her unconcern about her own fate, her conciliatory spirit, were in the midst of all the hatred and meanness, despair and sorrow surrounding us, a moral solace.