Undefined Terms. point, line, plane

Postulate 1. (Line Uniqueness) Given two different points there is exactly one line that contains them.

Postulate 2. (Distance Postulate) To every pair of different points there corresponds a unique positive number, called the distance between the two points.

Postulate 3. (Ruler Postulate) The points of a line can be placed in a correspondence with the real numbers in such a way that

(i) to every point of the line there corresponds exactly one real number,
(ii) to every real number there corresponds exactly one point of the line, and
(iii) the distance between two distinct points is the absolute value of the difference of the corresponding numbers.

Postulate 4. (Ruler Placement Postulate) Given points P and Q of a line, a coordinate system can be chosen in such a way that the coordinate of P is zero and the coordinate of Q is positive.

Postulate 5. (Existence of Points)
(a) Every plane contains at least three non-collinear points.
(b) Space contains at least four non-coplanar points.

Postulate 6. (Points on a Line Lie in a Plane) If two points lie in a plane, then the line containing them lies in the same plane.

Postulate 7. (Plane Uniqueness) Three points lie in at least one plane, and three non-collinear points lie in exactly one plane.

Postulate 8. (Plane Intersection) If two planes intersect, then their intersection is a line.

\(^1\) (Reproduced with permission from School Mathematics Study Group, Geometry, New Haven: Yale University Press, 1961.)
Postulate 9. (Plane Separation Postulate) Given a line and a plane containing it, the points of the plane that do not lie on the line form two sets such that

(i) the sets are convex and
(ii) if \( P \) is in one set and \( Q \) is in the other, then the segment \( \overline{PQ} \) intersects the line.

Postulate 10. (Space Separation Postulate) The points of space that do not lie in a given plane form two sets such that

(i) each of the sets is convex and
(ii) if \( P \) is in one set and \( Q \) is in the other, then the segment \( \overline{PQ} \) intersects the plane.

Postulate 11. (Angle Measurement Postulate) To every angle \( \angle BAC \) there corresponds a real number between 0 and 180.

Postulate 12. (Angle Construction Postulate) Let \( \overrightarrow{AB} \) be a ray on the edge of the half plane \( H \). For every \( r \) between 0 and 180, there is exactly one ray \( \overrightarrow{AP} \) with \( P \) in \( H \) such that \( m(\angle PAB) = r^\circ \).

Postulate 13. (Angle Addition Postulate) If \( D \) is a point in the interior of \( \angle BAC \), then \( m(\angle BAC) = m(\angle BAD) + m(\angle DAC) \).

Postulate 14. (Supplement Postulate) If two angles form a linear pair, then they are supplementary.

Postulate 15. (SAS Postulate) Given a correspondence between two triangles (or between a triangle and itself), if two sides and the included angle of the first triangle are congruent to the corresponding parts of the second triangle, then the correspondence is a congruence.

Postulate 16. (Parallel Postulate) Through a given external point there is at most one line parallel to a given line.

Postulate 17. (Area of a Polygonal Region) To every polygonal region there corresponds a unique positive real number, called the area.

Postulate 18. (Area of Congruent Triangles) If two triangles are congruent, then the triangular regions have the same area.

Postulate 19. (Summation of Areas of Regions) Suppose that the region \( R \) is the union of two regions \( R_1 \) and \( R_2 \). If \( R_1 \) and \( R_2 \) intersect at most in a finite number of segments and points, then the area of \( R \) is the sum of the areas of \( R_1 \) and \( R_2 \).

Postulate 20. (Area of a Rectangle) The area of a rectangle is the product of the length of its base and the length of its altitude.

Postulate 21. (Volume of a Rectangular Parallelepiped) The volume of a rectangular parallelepiped is the product of the length of its altitude and the area of its base.

Postulate 22. (Cavalieri's Principle) Given two solids and a plane, if for every plane that intersects the solids and is parallel to a given plane the two intersections have equal areas, then the two solids have the same volume.