Quiz 1 Review Problems Set

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1 Linear Equation

1.1
Solve this linear equation,

\[
\frac{4}{x+3} + 2 = \frac{x+7}{x+3}
\]  

(1)

Proof. This equation is equivalent to,

\[
\frac{2x+10}{x+3} = \frac{x+7}{x+3}
\]  

(2)

Then, by numerator, we have \(2x+10 = x+7\), which means that \(x+3 = 0\), thus we have \(x = -3\).

But we also have \(x+3\) as denominator, thus this equation does not have any solution.

1.2
Solve this linear equation,

\[
\frac{4}{x+5} + 2 = \frac{4}{x+5}
\]  

(3)

Proof. This equation is equivalent to,

\[
\frac{2x+14}{x+5} = \frac{4}{x+5}
\]  

(4)

Then, by numerator, we have \(2x+14 = 4\), which means that \(x+5 = 0\), thus we have \(x = -5\).

But we also have \(x+5\) as denominator, thus this equation does not have any solution.

2 Quadratic Equations

2.1
Solve this quadratic equation,

\[
4x^2 + 13x + 16 = 0
\]  

(5)

Proof. As for this problem, \(a = 4\), \(b = 13\) and \(c = 16\). Thus by formula we have quadratic equation,

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-13 \pm \sqrt{169 - 4 \times 4 \times 16}}{8} = \frac{-13 \pm \sqrt{-87}}{8} = \frac{-13 \pm i\sqrt{87}}{8}
\]  

(6)
2.2
Solve this quadratic equation,
\[4x^2 + 13x - 6 = 0\] \hspace{1cm} (7)

Proof. As for this problem, \(a = 4\), \(b = 13\) and \(c = -6\). Thus by formula we have quadratic equation,
\[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-13 \pm \sqrt{169 - 4 \times 4 \times (-6)}}{8} = \frac{-13 \pm \sqrt{265}}{8}\] \hspace{1cm} (8)

2.3
Using the discriminant to decide the number of the solutions of the following quadratic equations.
\[x^2 + 6x + 9 = 0\] \hspace{1cm} (9)
\[x^2 + x + 1 = 0\] \hspace{1cm} (10)

Proof. The discriminant of the first equation is following,
\[\Delta = 6^2 - 4 \times 1 \times 9 = 36 - 36 = 0\] \hspace{1cm} (11)
Thus, by the property we got in section 1.4, we know this equation has **only one real solution**.
For the second equation, its discriminant is following,
\[\Delta = 1^2 - 4 \times 1 \times 1 = 1 - 4 = -3\] \hspace{1cm} (12)
Thus, by the property we got in section 1.4, we know this equation has **two no-real solutions**.

3 Other Types Equations
3.1
Solving the following equation,
\[x^3 + 6x^2 + 3x + 18 = 0\] \hspace{1cm} (13)

Proof. The original equation is equivalent to,
\[x^2(x + 6) + 3(x + 6) = 0\] \hspace{1cm} (14)
which is also equivalent to,
\[(x^2 + 3)(x + 6) = 0\] \hspace{1cm} (15)
Then, let \(x + 6 = 0\), we get \(x = -6\). Let \(x^2 + 3 = 0\), we will get \(x = i\sqrt{3}\) or \(x = -i\sqrt{3}\).
3.2

Solving the following equation,

\[ x^{-2} + 2x^{-1} - 15 = 0 \]  (16)

Proof. Let \( u = \frac{1}{x} \). Then, we will have,

\[ u^2 + 2u - 15 = 0 \]  (17)

By formula, we know \( a = 1, b = 2, c = -15 \). Then,

\[
    u = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{2^2 - 4 \times 1 \times (-15)}}{2} = \frac{-2 \pm \sqrt{64}}{2} = \frac{-2 \pm 8}{2} = -5 \text{or} 3
\]  (18)

As \( x = \frac{1}{u} \), thus, we know \( x = -\frac{1}{5} \) or \( x = \frac{1}{3} \).

3.3

Solving the following equation,

\[ \sqrt{x} + 15 = x - 5 \]  (19)

Proof. By square the equation, we have,

\[ x + 15 = (x - 5)^2 \]  (20)

which is equivalent to,

\[ x + 15 = x^2 - 10x + 25 \]  (21)

\[ x^2 - 11x + 10 = 0 \]  (22)

By formula, we know \( a = 1, b = -11, c = 10 \). Then,

\[
    x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{11 \pm \sqrt{(-11)^2 - 4 \times 1 \times 10}}{2} = \frac{11 \pm \sqrt{81}}{2} = \frac{11 \pm 9}{2} = 10 \text{or} 1
\]  (23)

Then, let \( x = 10 \) in the original equation, we will have,

\[ \sqrt{10 + 15} = 10 - 5 \]  (24)

which is right.

let \( x = 1 \) in the original equation, we will have,

\[ \sqrt{1 + 15} \neq 1 - 5 \]  (25)

which is wrong.

Thus, for original equation, we only have solution, \( x = 10 \).