EE 350 Signals and Systems Midterm Exam 1
6:30-7:45 p.m., Mon. Oct. 10, 2005
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Instructor: G. Kesidis

room ?

INSTRUCTIONS:

- Answer all questions.
- **Show** your arguments and **justify** the steps.
- You do **not** need to numerically simplify your answers.
- Circle your answers where appropriate.
- The total marks of the exam is 50.
- The duration is 75 minutes.

Ques. No.	Mark
1	/20
2	/15
3	/15
Total	/40

Name: KESIDIS

Student ID: SOLUTIONS

Problem 1:
For $t \in$Reals, define $g(t) = 3t[u(t) - u(t-4)]$, where u is the unit step, and

$$x(t) = \sum_{k=-\infty}^{\infty} 4g(t-5k)$$

(a) Plot x and g.

(b) Is x periodic? If so, state its period.

For a certain linear, time-invariant system, suppose v is the zero state response (ZSR) to u and w is the ZSR to $tu(t)$.

(c) Find the ZSR to g in terms of v and w.

(d) Find the ZSR to x in terms of v and w.

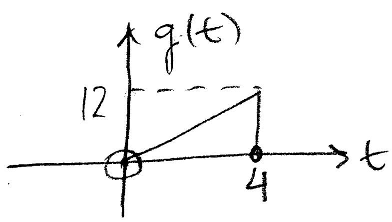
(e) Is g a power signal or energy signal or neither? If a power (respectively, energy) signal, find its power (respectively, energy).

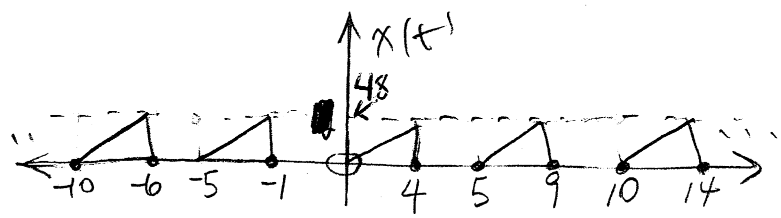
mean

(f) Is x a power signal or energy signal or neither? If a power (respectively, energy) signal, find its power (respectively, energy).

Mean

(a)





(b) Yes, x has period $\boxed{5}$

(c) $g(t) = 3t\,u(t) - 3t\,u(t-4)$

$= 3t\,u(t) - 3(t-4)u(t-4) - 12\,u(t-4)$

$\therefore$ by LTI, ZSR to g is $\boxed{y(t) \triangleq 3w(t) - 3w(t-4) - 12v(t-4)}$

(d) Also, by LTI, ZSR to x is $\boxed{\sum_{k=-\infty}^{\infty} 4y(t-5k)}$ where y is given in (c)

(e) g is clearly an energy signal with energy $\int_0^4 (3t)^2 dt = 9 \cdot \frac{1}{3} t^3 \Big|_0^4$
$= \boxed{192}$

(f) x is clearly a power signal with (mean) power $\frac{1}{5}\int_0^4 (3t)^2 dt = \boxed{\frac{192}{5}}$

Problem 2:
For the system

$$y(t) = 5f(t) + \int_{-\infty}^{t} f(\tau)\mathrm{d}\tau$$

with input f and output y:

(a) Determine whether the system linear.

(b) Determine whether the system time-invariant.

(c) Determine whether the system causal.

(d) Find the impulse response, h.

(e) By graphical convolution means, find the zero state response, $y(t)$ for $t \geq 0$, to the pulse

$$f(t) = u(t) - u(t-2).$$

[redacted]

Let y_k be the response to f_k & let α_1, α_2 and T be arbitrary scalars

(a) Response to $\alpha_1 f_1 + \alpha_2 f_2$ is

$$5(\alpha_1 f_1(t) + \alpha_2 f_2(t)) + \int_{-\infty}^{t} (\alpha_1 f_1(\tau) + \alpha_2 f_2(\tau))\, d\tau$$

Since integral is linear

$$= \alpha_1\left(5f_1(t) + \int_{-\infty}^{t} f_1(\tau)\,d\tau\right) + \alpha_2\left(5f_2(t) + \int_{-\infty}^{t} f_2(\tau)\,d\tau\right)$$

$$= \alpha_1 y_1(t) + \alpha_2 y_2(t) \quad \forall t$$ so system is linear

(b) Response to $f_1(t-T)$ is $\qquad s \triangleq \tau - T$

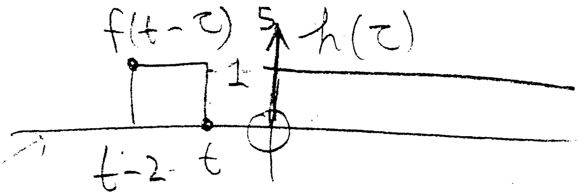
$$5f_1(t-T) + \int_{-\infty}^{t} f_1(\tau - T)\,d\tau \overset{\downarrow}{=} 5f_1(t-T) + \int_{-\infty}^{t-T} f_1(s)\,ds$$

$$= y_1(t-T) \quad \forall t$$, so system is time-invariant

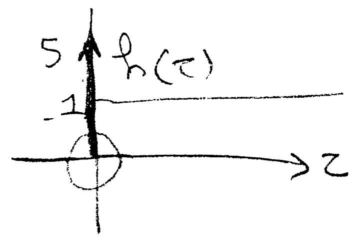
(c) if $f(t) = 0$ for all $t < 0$, then for any $t < 0$, $y(t) = 5 \cdot 0 + \int_{-\infty}^{t} 0 \cdot d\tau = 0$
so system is causal

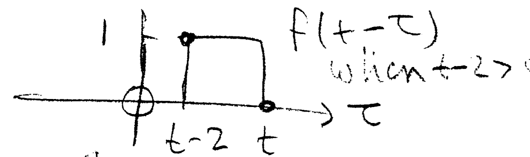
(d) $h(t) = 5\delta(t) + \int_{-\infty}^{t} \delta(\tau)\,d\tau = \boxed{5\delta(t) + u(t)}$ where u is the unit step

(e) turn over



(e) By graphical means





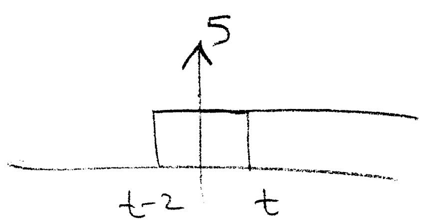
Case 1: $t \leq 0 \Rightarrow h(\tau)f(t-\tau) = 0 \ \forall \tau$

$\Rightarrow y(t) = \int_{-\infty}^{\infty} h(\tau)f(t-\tau)\,d\tau = 0$

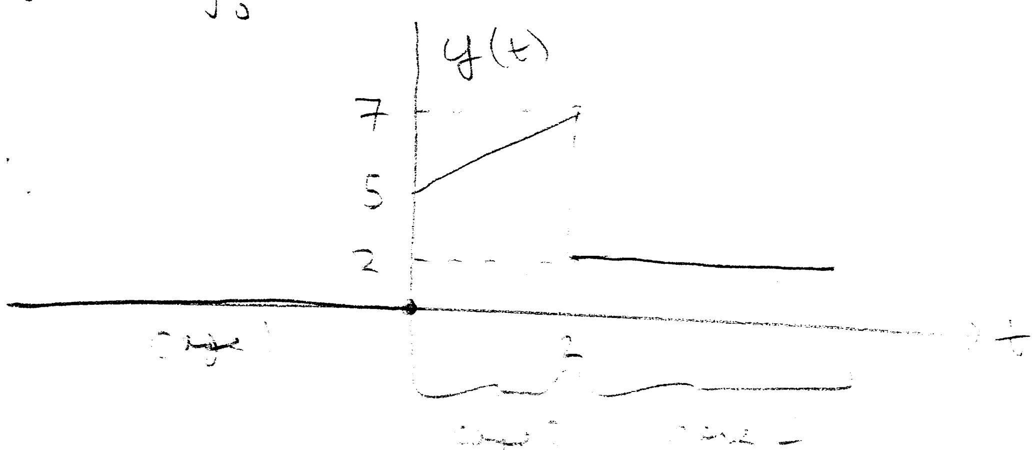
Case 2: $t-2 > 0 \Rightarrow h(\tau)f(t-\tau) = f(t-\tau) \ \forall \tau$

$\Rightarrow y(t) = 2$ (area under f)

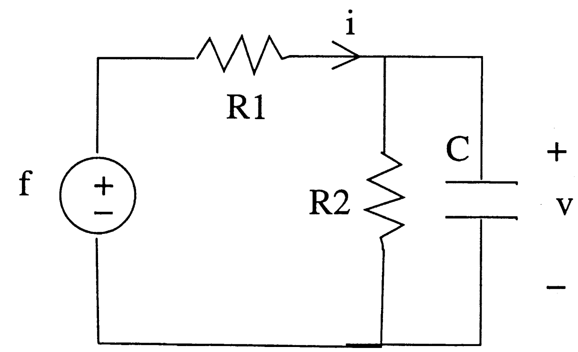
Case 3: $t > 0$ and $t-2 \leq 0$ i.e. $0 < t \leq 2$



$$y(t) = \int_0^t (5\delta(\tau) + 1)\,d\tau = 5 + t$$



Problem 3:



For this linear, time-invariant circuit with input f and output v,

(a) Find the differential equation describing the input-output relationship by first writing one nodal (KCL) equation.

Now assume the initial current $i(0) = 3A$, $R_1 = R_2 = 1\Omega$ and $C = 0.25\text{F}$.

(b) Find the total response $v(t)$, $t \geq 0$, to the input $f(t) = 3\sin(2t)$ by finding:

(i) The zero input response, v_{ZI}.

(ii) The zero state response, v_{ZS}. Hint: first find the ZSR to complex exponential signals $e^{j\omega t}$ and then write the ZSR in terms of (real valued) sines and cosines.

(a) $$\frac{f-v}{R_1} - \frac{v}{R_2} - C\frac{\overbrace{dv}^{Dv}}{dt} = 0 \Rightarrow \boxed{Dv + \frac{1}{C}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)v = \frac{1}{CR_1}f}$$

(b) $Dv + 8v = 4f$

(i) characteristic value λ satisfies $\lambda + 8 = 0 \Rightarrow \lambda = -8$

$\Rightarrow$ system mode is e^{-8t} $\Rightarrow \boxed{v_{ZI} = Ce^{-8t}, \; t \geq 0}$

to find initial condition $v(0)$, note

$f(0) - R_1 i(0) = v(0)$ by KVL

Since $f(0) = 0$, $v(0) = v_{ZI}(0) = -1 \cdot 3 = -3$

$\Rightarrow \boxed{C = -3}$

Problem 3 (Cont.):

(ii) ZSR to $f(t) = 3\sin 2t = \frac{3}{2j} e^{2jt} - \frac{3}{2j} e^{-2jt}$

is

$$v_{zs}(t) = \overbrace{A e^{-8t}}^{\text{mode}} + \frac{3}{2j} H(2j) e^{2jt} - \frac{3}{2j} H(-2j) e^{-2jt}$$

where the transfer function $H(s) = \frac{P(s)}{Q(s)} = \frac{4}{s+8}$

and A is such that $v_{zs}(0) = 0$

i.e., $A = -\frac{3}{2j} H(2j) + \frac{3}{2j} H(-2j)$