

CHAPTER 13

INTRODUCTION TO MULTIPLE CORRELATION

Chapter 12 introduced you to the concept of partialling and how partialling could assist you in better interpreting the relationship between two primary variables. Another correlational technique that utilizes partialling in its derivation is called **MULTIPLE CORRELATION**. The main purpose of multiple correlation, and also **MULTIPLE REGRESSION**, is to be able to predict some criterion variable better. Thus, while the focus in partial and semi-partial correlation was to better understand the relationship between variables, the focus of multiple correlation and regression is to be able to better predict criterion variables. The data set below represents a fairly simple and common situation in which multiple correlation is used.

STUDENT	SATV	SATM	GPA
1	570	755	3.4
2	648	611	3.3
3	571	650	1.9
4	578	584	2.1
5	550	680	2.5
6	669	701	2.9
7	630	605	2.6
8	502	528	2.3
9	641	764	2.5
10	623	509	2.6

These data relate to the situation where we have information on the Scholastic Aptitude Test, both Verbal and Math sections, and want to use that information to predict college grades (GPA). Generally speaking, we will have SATV and SATM data prior to when students are admitted to college, and then will collect the criterion or GPA data later. For information purposes, the descriptive statistics and correlations are given below.

DESCRIPTIVE STATISTICS

	N	MEAN	STDEV
SATV	10	598.2	52.1
SATM	10	638.7	87.5
GPA	10	2.610	0.479

CORRELATIONS

	SATV	SATM
SATM	0.250	
GPA	0.409	0.340

What if we are interested in the best possible prediction of GPA? If we use only one predictor variable, either SATV or SATM, the predictor variable that correlates most highly with GPA is SATV with a correlation of .409. Thus, if you had to select one predictor variable, it would be SATV since it correlates more highly with GPA than does SATM. As a measure of how predictable the GPA values are from SATV, we could simply use the correlation coefficient or we could use the coefficient of determination, which is simply r squared. Remember that r squared represents the proportion of the criterion variance that is predictable. That value or coefficient of determination is as follows.

$$r^2_{\text{(SATV)(GPA)}} = (.409)^2 = 0.167281$$

Approximately 16.7 percent of the GPA criterion variance is predictable based on using the information available to us by having students' SATV scores. Actually, we might have done the regression analysis using SATV to predict GPA and these results would have been as follows. This is taken from Minitab output; see page 108 for another example.

The regression equation is: PREDICTED GPA = 0.36 + 0.00376 SATV

Predictor	Coef	Stdev	t-ratio	p
Constant	0.359	1.780	0.20	0.845
SATV	0.003763	0.002966	1.27	0.240

$$s = 0.4640 \quad R\text{-sq} = 16.7\% \quad R\text{-sq(adj)} = 6.3\%$$

Notice that in the output from the regression analysis includes an r squared value (listed as R-sq) and that value is 16.7 percent. In this regression model, based on a Pearson correlation, we find that about 17% of the criterion variance is predictable. But, can we do better? Not with one of the two predictors. However, we see that the best single predictor of GPA in this case is SATV accounting for approximately 16.7 percent of the criterion variance. The correlation we find between SATV and GPA is .409. But, it is obvious that we have a second predictor (SATM) that, while not correlating as high with GPA as SATV, does still have a positive correlation of .34 with GPA. In selecting SATV as the best single predictor, it is not because the SATM has no predictability but rather that SATV is somewhat better. Perhaps it would make sense to explore the possibility of combining the two predictors together in some way to take into account the fact that both do correlate with the criterion of GPA. Is it possible that by combining SATV and SATM together in some way that we can improve on the prediction that is made when only selecting SATV in this case? By using both SATV and SATM added together in some

fashion as a new variable, can we find a correlation with the criterion that is larger than .409 (the larger of the two separate r values) or account for more criterion variance than 16.7%?

Remember I mentioned when using Minitab to do the regression analysis, that the basic format of the regression command is to indicate first which variable is the criterion, then indicate how many predictor variables there are, and then finally indicate which columns represent the predictor variables. For a simple regression problem, there would be one criterion and one predictor variable. However, the regression command allows you to have more than one predictor. For example, if we wanted to use both SATV and SATM as the combined predictors to estimate GPA, we would need to modify the regression command as follows: c3 is the column for the GPA or criterion variable, there will be 2 predictors, and the predictors will be c1 and c2 (where the data are located) for the SATV and SATM scores. Also, you can again store the predicted and error values. See below.

```
MTB > regr c3 2 c1 c2 c10(c4);          <---- Minitab command line  
SUBC> resi c5.                        <---- Minitab subcommand line
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The regression equation is: $GPA = -0.18 + 0.00318 SATV + 0.00139 SATM$

$s = 0.4778$ $R\text{-sq} = 22.8\%$ $R\text{-sq(adj)} = 0.7\%$

As usual, the first thing that appears from the regression command is the regression equation. In the simple regression case, there will be an intercept value and a slope value that are attached to the predictor variable. However, in this multiple regression case, the regression equation needs to have the second predictor variable included and there will be a "weight" or "slope-like" value also attached to the second predictor variable, SATM in this instance. To make specific predictions using the equation, we would need to substitute both the SATV and SATM scores into the equation and then come up with the predicted GPA value. For example, for the first student who obtained 570 on SATV and 755 on SATM, the predicted GPA value would be as follows.

$$\begin{aligned} \text{PREDICTED GPA}' &= -.18 + .00318 (570) + .00139 (755) \\ &= 2.68 \end{aligned}$$

If the predicted GPA is 2.68 and the real GPA is 3.4, that means that this prediction is $2.68 - 3.4 = -.72$ in error; that is, the predicted value in this case underestimates the true value by .72 of a GPA unit. Since the predicted GPA values and errors have been stored in columns, we can look at all the data. As is normally the case, some of the predictions are overestimates (case 3 for example) and some are underestimates (case 8 for example). In fact, if you added up the errors, you would find (within round-off error), that the sum would be 0 since the overestimates and underestimates will balance out over the full set of data. Look at the data and statistics below.

STUDENT	SATV	SATM	GPA	PredGPA	ErrGPA
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1	570	755	3.4	2.68174	0.718265
2	648	611	3.3	2.72990	0.570098
3	571	650	1.9	2.53920	-0.639197
4	578	584	2.1	2.46986	-0.369860
5	550	680	2.5	2.51406	-0.014059
6	669	701	2.9	2.92158	-0.021575
7	630	605	2.6	2.66434	-0.064343
8	502	528	2.3	2.15049	0.149506
9	641	764	2.5	2.91998	-0.419977
10	623	509	2.6	2.50886	0.091142

CORRELATIONS

	SATV	SATM	GPA	PredGPA
SATM	<u>0.250</u>			
GPA	<u>0.409</u>	<u>0.340</u>		
PredGPA	0.858	0.712	0.477	
ErrGPA	0.000	0.000	0.879	0.000

The first 3 correlations at the top of the matrix are the intercorrelations amongst the 3 primary variables: the two predictors and the criterion (these are underlined). Notice that the correlation of each predictor with the residuals is 0 (these are slanted); recall from the discussion of partial correlation the fact was mentioned that the predictor will be uncorrelated with the residual or error values. The most important correlation in the matrix is between GPA, or the actual criterion, and the predicted GPA: this correlation is .477 (this has been bolded). When we combine the two predictors in some optimal way by using the weights in the regression equation (.00318 for SATV and .00139 for SATM), we find that this combination produces a correlation of .477 with the criterion. In the literature, **THE CORRELATION BETWEEN THE ACTUAL CRITERION VARIABLE AND THE PREDICTED CRITERION VARIABLE** (based on a weighted combination of two or more predictors) **IS CALLED THE MULTIPLE CORRELATION**. The symbolism is as follows.

$$R_{Y.12} = r_{(GPA)(GPA)} = \text{multiple correlation}$$

In simple regression, it is commonplace to use a "small" r to indicate correlation but to differentiate it from the multiple predictor case, where we use capital R for multiple correlation. The subscripts Y.12 simply mean (in this case) that Y is the criterion variable that is being predicted by a best weighted combination of predictors 1 and 2. Again, note that this multiple correlation value is .477.

Recall that when we used the single predictor that most highly correlated with GPA, we selected SATV since it had the higher of the two predictor-criterion correlations: .409 versus .34 for SATM. Also recall that SATV accounted for approximately 16.7 percent (.409 SQUARED = 16.7%) of the criterion variance. Thus, did we do better than a

correlation of .409 or 16.7% predictable variance? In this case we did! Compared to the simple correlation of SATV with GPA which was .409, the multiple correlation combining both predictors together is .477, a difference of .068 in the higher direction. As for the proportion of predictable variance or r squared value, .477 squared = 22.8 percent or a gain of 22.8 - 16.7 = 6.1 percent in predictable variance. Whether you consider the increase in the correlation or the increase in the r squared value (r squared versus R squared), there is some improvement in the prediction of the criterion. Here is an instance where the combination of two predictors does a better job of estimating the criterion than only using one predictor variable. While there is some improvement in the predictability in this case, that does not always happen. The factors that determine whether there is improvement or not will be discussed later.

Multiple R Formula

In the section on partial correlation, a shortcut formula for finding the partial r value was presented that was based on the intercorrelations of all three variables. There is a comparable shortcut formula for the multiple correlation that works in the case where there are two predictors and one criterion. See the formula below.

$$R_{Y.12} = \sqrt{\frac{r_{1Y}^2 + r_{2Y}^2 - 2r_{1Y}r_{2Y}r_{12}}{1 - r_{12}^2}}$$

The relevant correlations for R are shown below.

$$r_{12} = .25 \quad r_{1Y} = .409 \quad r_{2Y} = .34$$

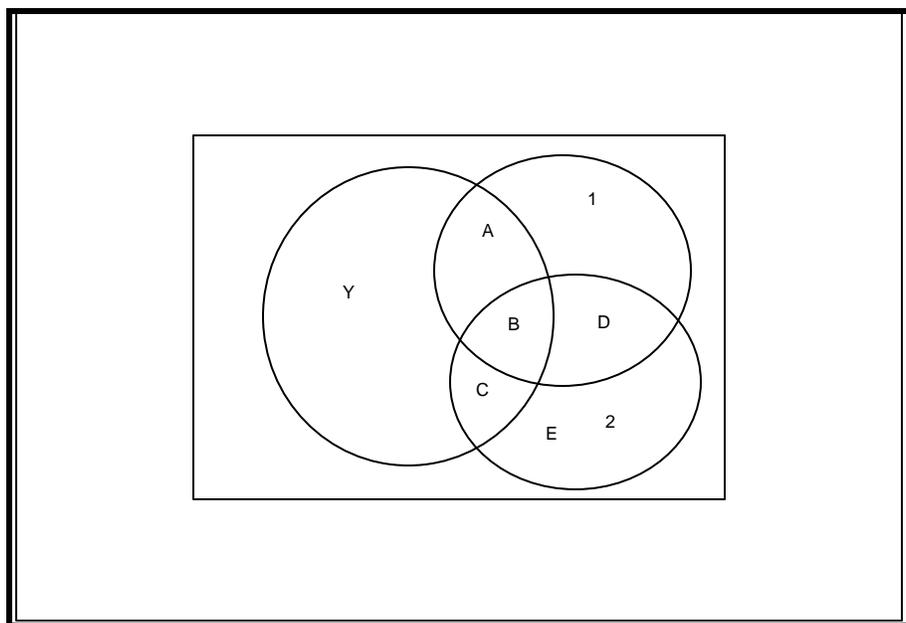
$$R_{Y.12} = \sqrt{\frac{.409^2 + .34^2 - 2 * .409 * .34 * .25}{1 - .25^2}} = .477$$

In the formula, make sure you take the square root! Of course, if you square the multiple R value of .477, you obtain the proportion of criterion variance that is predictable based on using the combination of the two predictors to estimate the criterion. This proportion is 22.8 percent which is one part of the regression output when doing the multiple regression equation. It is very commonplace in the multiple correlation literature to report R squared as the relationship strength indicator.

As was indicated earlier, there is a connection between the partialling process that

was discussed in Chapter 12, and the process of obtaining and interpreting the multiple correlation. The connection is based on the notion that if we could obtain the amount of variance that is predictable on the criterion based on using both predictors, then that would be R squared. And, if one knows R squared, it is a simple matter to take the square root to obtain the multiple correlation value or R. Let's see how the partialling process can help us find the multiple correlation.

Recall from the example above, that there were positive intercorrelations among all 3 variables: each predictor correlated with the criterion (.409 and .34), and the two predictors correlated with each other (.25). We could depict this situation by using the following Venn diagram. Overlapping circles are used to indicate when two variables correlate with each other: the more they overlap, the more they correlate.



In this case, since all variables correlate with one another, all 3 circles will overlap to some extent. The Y circle is meant to represent the variance on the criterion, and the 1 and 2 circles are meant to represent the variances on the two predictor variables. The small lettered areas A, B, C and D represent where each of the variables overlaps with each other. Not only does predictor variable 1 overlap and correlate with Y (areas A and B), does predictor variable 2 overlap and correlate with Y (areas B and C), note that the two predictor variables overlap and correlate with one another (areas B and D).

If you want to explore how well the combination of predictor variables 1 and 2 estimates or predicts the criterion variable Y, you need to see the extent to which the predictors overlap with Y. In this case, whereas 1 overlaps with Y in areas A and B, and 2 overlaps with Y in areas B and C, both 1 and 2 overlap with Y in areas A plus B plus C. Therefore, the combination of predictors 1 and 2 overlaps more (in total) with the criterion variable Y than does either predictor variable separately. Since the overlap of 1 and 2 is

larger than 1 only or 2 only, the overall overlap (which represents the R squared amount or proportion of predictable variance computation) will be greater using both predictor variables in this case. If we could calculate the area A + B + C relative to the Y area, that would give us the R squared or proportion of Y variance that is needed. With this R squared amount, we can then easily take the square root and that will give us the multiple correlation value or R. Let's see how this is accomplished.

The first thing we do is to select one of the two predictor variables and find the overlapping area between that predictor and the criterion variable Y. Let's start with SATV which is the first predictor variable. The area where it overlaps with Y is area A + B and is found by the coefficient of determination which is simply the square of the correlation between 1 and Y. From the correlation matrix above, this correlation is .409; therefore, r square = .409 squared or 16.7 percent or .167 as a proportion. This is the square of a zero order correlation between predictor variable 1 or SATV and the criterion variable Y or GPA. See below.

$$r_{1Y}^2 = .409^2 = 16.7 \%$$

The second step is to consider the overlapping area between predictor variable 2 and the criterion Y variable. This is represented by the areas B + C. Again, we could find that proportionate area by using the square of the correlation between 2 and Y and that would be: .34 squared or 11.56 percent.

$$r_{2Y}^2 = .34^2 = 11.56 \%$$

However, if we look at the overlap of 1 with Y or 16.7 percent and the overlap of 2 with Y or 11.56 percent. this totals to 16.7 + 11.56 or about 28.26 percent of the total area of Y. Since we have already done the multiple regression analysis and have found the multiple correlation of .477, that means the the "correct" amount of overlapping variance is .477 squared or about 22.8 percent. Therefore, the amount of overlapping that we have by taking the square of each predictor-criterion correlation (16.7 + 11.56 = 28.26) yields an amount that is too large. Why?

If you look at the diagram, you can see that 16.7 percent is the combination of areas A + B, and that the 11.56 percent is the combination of the areas B + C. So, if we use both of these values, we have included the B area twice. We need some way to look at the total of the overlapping areas of 1 with Y and 2 with Y but only include the B area once. Here is how that can be done.

Notice that predictor variable 1 (SATV) overlaps or correlates with predictor variable 2 (SATM). Recall that if you use some variable as a predictor of another variable,

then that predictor will correlate 0 with the residual or error scores. So, what about the possibility of using SATV (variable 1) as the predictor of SATM (variable 2)? The correlation between them is .25 and the overlapping areas of B + D would represent the r squared amount or .25 squared. However, the areas on SATM (variable 2) also include C and E and represent the non-overlapping part of the relationship between SATV (1) and SATM (2). Non-overlapping areas mean uncorrelated segments and this area would represent the error score part of the SATM variance. Therefore, if we first use SATV or variable 1 to predict SATM or variable 2 so as to partial SATV from SATM, the residual or error scores on SATM will represent the areas of C + E. Then, if we look at the correlation between the error scores on SATM having removed the influence of SATV, we could examine the semi-partial correlation of SATM with the criterion Y or the GPA variable. The reason this is a semi-partial correlation is because we remove the influence of SATV from SATM only and do not also remove the influence of SATV from GPA or Y. [Note: if you used the partial correlation formula, you would be removing the area of B again.] Thus, we need the first order semi-partial correlation of SATM with GPA having removed the influence of SATV from SATM only.

One way to obtain this semi-partial correlation would be to do the regression analysis using SATV to predict SATM and then obtain error or residual scores or ErSATM. Then we could correlate this ErSATM variable with the GPA variable and that would be our semi-partial correlation. Another way would be to use the shortcut formula for the semi-partial correlation. Remember (from Chapter 12) that it looks almost identical to the partial correlation formula but only has one radical term in the denominator. In this case, the primary relationship is between SATM and GPA, and the regression analysis would apply to using SATV to predict SATM. Therefore, the correlation between SATV and SATM is the only one that would appear in the denominator. The semi-partial formula in this case would be as follows.

$$r_{Y(2.1)} = \frac{r_{Y2} - r_{Y1} r_{12}}{\sqrt{1 - r_{12}^2}}$$

$$= \frac{.34 - (.409 * .25)}{\sqrt{1 - .25^2}}$$

$$= .246$$

The semi-partial correlation of .246 represents the relationship between SATM and

GPA with the influence of SATV removed from SATM only. But, it is not actually the semi-partial correlation we want here to represent the C slice of the overlap between SATM and GPA. What we actually need is the coefficient of determination which is the square of the correlation of .246 which equals .061 or about 6.1 percent. Therefore, what we now have are the following parts.

$$r_{Y1}^2 = .409^2 = 16.7 \text{ percent}$$

$$r_{Y(2.1)}^2 = .246^2 = 6.1 \text{ percent}$$

))))))))))

22.8 percent

Since the 22.8 percent value represents the overlapping variance area, which is the coefficient of determination, then this represents the R squared value. Thus, if we want the multiple correlation value or R, all we have to do is to take the square root of the .228 value = .477, and recall that was the multiple correlation value found above.

Multiple Regression Equation Formulas

Before leaving this example, I want to show you the computational formulas for finding the multiple regression equation: in this case, there are two predictors and one criterion variable. You may want to review the computational formulas for finding the slope and intercept for the simple regression equation that are presented in Chapter 10. The information that is used to find the slope in the simple regression formula is the correlation between the two variables (predictor and criterion) and the two standard deviations. For finding the intercept, we would need the slope that has been previously calculated and also the mean values (for the predictor and criterion). To find the multiple regression equation, we use similar descriptive statistics and correlational output. The relevant data are on pages 142 and 143.

For finding the "weight" that goes with predictor 1:

$$b_1 = \frac{r_{Y1} - r_{Y2} r_{12} (S_Y / S_1)}{(1 - r_{12}^2) (S_1)}$$

$$= \frac{.409 - .34 * .25 * .479}{(1 - .25^2) * 52.1} = .007$$

For finding the "weight" that goes with predictor 2:

$$b_2 = \frac{r_{Y2} - r_{Y1} r_{12} (S_Y / S_1)}{(1 - r_{12}^2) (S_2)}$$

$$.34 - .409 * .25 * .479 / ((1 - .25^2) * 87.5) = .003$$

For finding the "intercept" value or a:

$$a = \bar{Y} - b_1 (\bar{X}_1) - b_2 (\bar{X}_2)$$

$$= 2.61 - (.007 * 598.2) - (.003 * 638.7) = -3.493$$

Thus, by using the formulas and working by hand, the multiple regression is as follows.

$$GPA' = -3.493 + .007 SATV + .003 SATM$$

Unfortunately, in some cases when working these equations by hand, there can be considerable round off error. Look at the values obtained by hand above and the corresponding values from Minitab's calculations: the regression equation is **GPA' = - 0.18 + 0.00318 SATV + 0.00139 SATM**. You will see that there is considerable round off error. Not only does that impact on the actual b1, b2 and a values, it would also impact on the predicted GPA values if you substitute into the regression equation. One reason for the round off error in this case is that the b1 and b2 values are multiplied by relatively large means: 598.2 and 638.7. Even a small error in b1 and/or b2, when multiplied by a large mean, will lead to considerable error. The best word of advice on this subject is to let something like Minitab do your computational work. Since Minitab and other statistical packages use many, many decimal places in the calculations, this will avoid serious round off problems.

Final Notes on Multiple Correlation

It is important to understand that the example shown above represents the simplest case for multiple correlation and multiple regression where there are two predictor variables. While this is not uncommon, you must realize that many multiple regression analyses involve more than two predictor variables. Even the example where SATV and SATM were combined to predict COLGPA is generally one where high school grade point average would also be one of the predictors. Thus, in that case, there would normally be 3 predictor variables. In only showing you an example of two predictor variables, I have deliberately keep the work to a minimum and the formulas presented for finding either the multiple correlation itself or the components of the multiple regression equation are in their simplest forms. While a statistical package can find the multiple regression equation with more than two predictors easily, the computational formulas for finding these same values are not that simple.

Note 2

When you find the multiple regression equation, there will be what are commonly called "weights" attached to each of the independent or predictor variables. For example, in the equation using SATV and SATM to predict GPA, the "weight" attached to the SATV variable was about .003 (from the Minitab output) and the "weight" attached to the SATM variable was about .001. One could easily attach an "importance" meaning to these weights and interpret these as meaning that the variable with the largest weight is the more important one to the prediction. While this may be true in some cases, it is not necessarily true since the formulas for finding these weights involve not only correlations but also standard deviations. Thus, to some extent, the characteristics of each predictor variable (i.e., their descriptive statistics) will determine whether there is a relatively small or relatively large number for the weight. For example, if we had an equation where one predictor variable was high school grade point average and the other predictor was SATV scores, and we were combining them to predict college grade point average, the weight for the SATV predictor must necessarily be very small. This is because the equation will be multiplying the weight times an SATV value which is like 450, or 500 or 620. Since the criterion variable is a grade which is on a 4 point scale, one cannot multiply an SATV number by a large weight or else the predicted college GPA value would be something like 354, which does not fit on the grade scale. Thus, it is very important to resist trying to equate the "weights" in the multiple regression equations to meaning "importance".

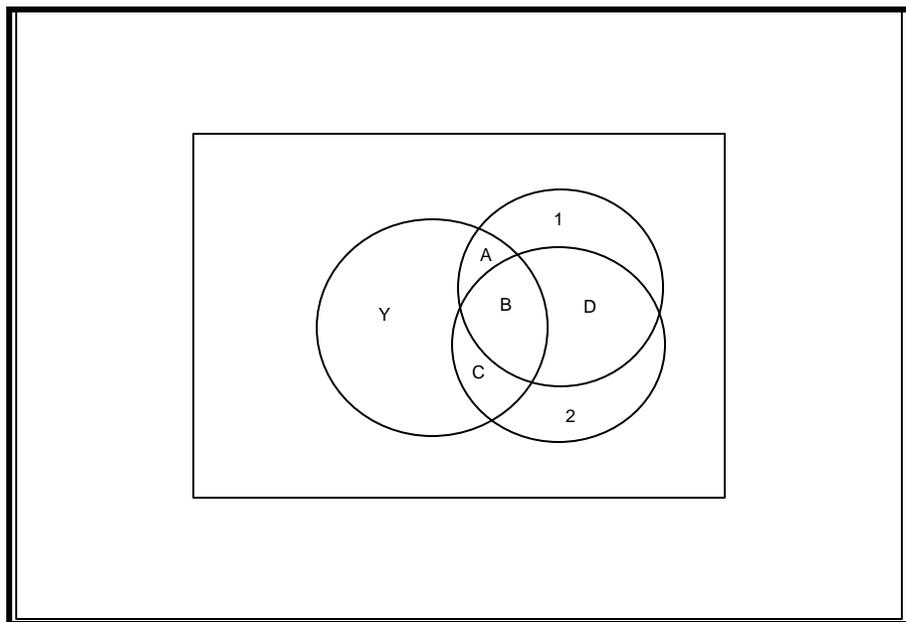
Note 3

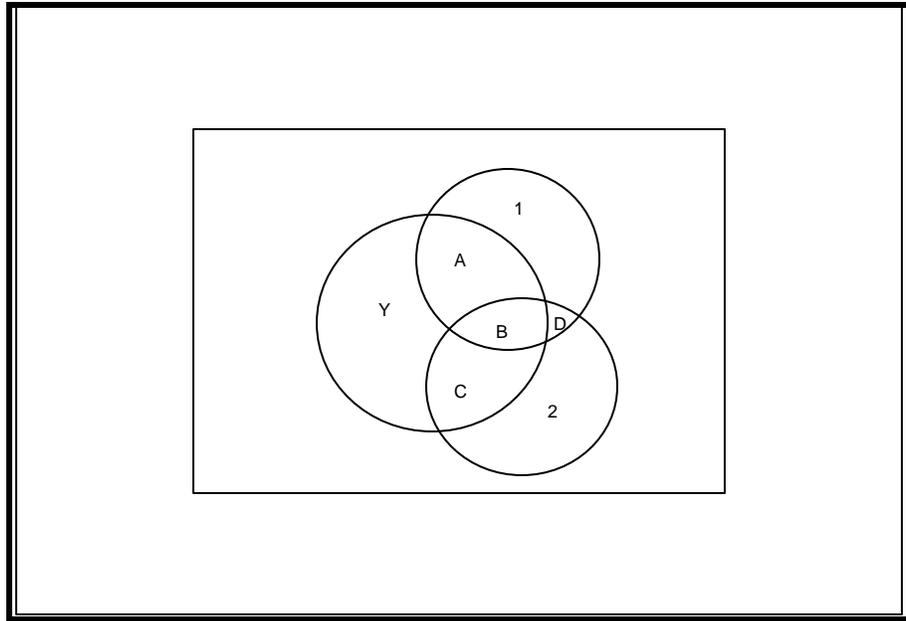
This final note deals with the context in which combining two or more predictor variables will enhance or increase the predictability of some criterion variable. In the first case, we should recognize that in order for even one predictor to be used to estimate some criterion, there must be some reasonable correlation (either + or -) with the criterion. In terms of our Venn diagram circles above, this simply means that the predictor variables must overlap to some extent with the criterion. This also is true if there are more than two predictors. Predictors will not be useful in predicting the criterion unless they overlap to some extent with the Y variable. This fact is a given. It also follows, generally, that the more

the predictors overlap with the criterion, the better the criterion can be predicted; ie, the more total part of the Y circle will be overlapped by the predictors.

However, what is not precisely clear is how the relationships among the predictors will impact on whether combining predictor variables together will improve the predictability of the criterion. Note that in the example above, where the combination of the two predictors added about 6 percent predictable variance over and beyond what only one predictor would do, we see that the correlation between the two predictors was relatively low, a value of .25 from the intercorrelation table. But, what would happen if the two predictors correlated much more highly with each other? To explore that question, examine two more Venn diagrams, shown separately, on the next page.

Recall from the discussion above, the key to seeing how well the predictors estimate the criterion variable is the extent to which the overlap of the predictors accounts for as much of the criterion or Y circle as possible. In the first figure, we have a case where both predictors overlap with the criterion but the predictors also overlap or correlate with each other to a high degree. If we want to see what the total overlapping area is between the two predictors with the criterion, we need to total up areas A, B and C. But, since areas A and B represent the overlap of predictor variable 1 with the criterion, notice that the additional area in C is so small that it's addition to areas A + B would be only minimal. Because the predictor variables 1 and 2 overlap so much with each other (areas B and D), the area of overlap of 1 with 2, which includes area B, has already been accounted for in the relationship between predictor variable 1 and Y.





However, look at the lower graph. In this case, I have tried to keep the $A + B$ and the $B + C$ parts relatively the same but, what is different in this figure? Well, note that I have made much less overlap between the predictor variables. In this case, I have made the diagram reflect the fact that the predictors correlate less with each other, than in the top diagram. The net impact of that is that the $A + B$ area between predictor 1 and Y, compared to the $B + C$ area between predictor 2 and Y, have a much smaller common B segment. This is because the common segment between predictors 1 and 2, the $B + D$ area, is much, much smaller. By including the relationship between predictor 2 in our equation, we are now taking over the C part, which is a large slice not accounted for by the relationship of predictor 1 with Y. By having two predictors that correlate less with each other (assuming of course the same general relationships between each predictor and the criterion), we now account for a larger total segment of the Y criterion variance. What do you think would happen if the predictors correlated 0 with each other; that is, the 1 and 2 circles would rotate away from each other so that there was no $B + D$ area of overlap between them?

Thus, the general situation can be stated as follows. Combining two or more measures together to predict some criterion variable will be best or maximally beneficial when each of the predictors correlates with the criterion to a reasonable degree, but where the **PREDICTORS CORRELATE AS LITTLE WITH EACH OTHER AS POSSIBLE**. While the principle seems easy enough to understand, the practical implementation of this principle is not as easy to accomplish as it may appear. If there are several variables that you are considering for the prediction of some criterion, the main reason why you may select these "potential" predictors is the fact that you know that each of them (perhaps from some literature) correlates with the criterion. This means that some of these variables may be very similar to one another. If that is true, then it may be the case that the predictors will correlate with each other at a moderate level. Thus, the trick is to select predictors that

correlate reasonably well with the criterion but do not correlate very highly with each other. If I had a simple rule to make this task easier for you, I would "state it", but I don't. The best suggestion I have is that a careful examination of the relevant literature will allow you to be in a better position to make good decisions about which set of predictors may be formed to best predict some criterion. "Knowing the field" is the best solution to the above dilemma.

Practice Problems

Junior level math exam scores (1) and intelligence scores (2) were used to predict physical science final examination scores (Y). Here are the data.

Math Scores	IQ	Phys Sci Exam
62	118	91
63	109	89
58	110	79
57	112	78
59	120	78
52	110	72
51	104	70
47	122	69
49	102	67
50	98	64

- A. Find the descriptive statistics and intercorrelations of the variables. Which predictor is best for predicting the criterion? How much do the predictors correlate with each other?

- B. Find the multiple correlation using Math and IQ to predict Phys Sci scores. What would the predicted Phys Sci score be for someone who obtained 57 on Math and 112 on IQ?
- C. Compare the multiple correlation to the best single correlation (from B above). Is there any difference?
- D. Compare the coefficient of determination from B and D. How much gain, if any, is there in the percentage of criterion variance that is predictable using both predictors rather than the best single one?