1. (3 points) Maximize $Q = xy$, where $x$ and $y$ are positive numbers such that $\frac{4}{3}x^2 + y = 16$.

If $\frac{4}{3}x^2 + y = 16$, then $y = 16 - \frac{4}{3}x^2$.

So $Q = xy = x \left( 16 - \frac{4}{3}x^2 \right) = 16x - \frac{4}{3}x^3$.

I.e., $Q(x) = 16x - \frac{4}{3}x^3$.

We want to maximize $Q(x)$ on $(0, \infty)$.

$Q'(x) = 16 - 4x^2 \Rightarrow$ critical values are: $x = 2$ or $x = -2$

Since $Q''(x) = -8x$, $Q''(2) = -16 < 0$,

so $x = 2$ is the location of an absolute max. Therefore, the absolute max value of $Q$ is $Q(2) = 16 - \frac{4}{3}(2)^2$.
2. (7 points)

(a) Find $y'$ if $y^2 + x^2 = 5 + xy$.

$2yy' + 2x = xy' + y$

$2yy' - xy' = y - 2x$

$y' \left(2y - x\right) = y - 2x$

$y' = \frac{y - 2x}{2y - x}$

(b) Find the slope of the tangent line to the curve $x^3 + 2y^3 = 6$ at the point $(2, -1)$.

$3x^2 + 6y^2 y' = 0$

$6y^2 y' = -3x^2$

$y' = \frac{-3x^2}{6y^2} = \frac{-x^2}{2y^2}$

So $y' \bigg|_{(2, -1)} = \ldots$