1. (3 points) If \( F(x) = \frac{1}{x - 3} \), find the following limits. If it does not exist, write DNE.

(a) \( \lim_{x \to 4} F(x) = \lim_{x \to 4} \frac{1}{x - 3} = \frac{1}{4 - 3} = 1 \) (by theorem on limits of rational functions)

(b) \( \lim_{x \to 3^+} F(x) = \lim_{x \to 3^+} \frac{1}{x - 3} \) stays at 1 small positive

(c) \( \lim_{x \to 3} F(x) \)

DNE (\( \lim_{x \to 3^+} \frac{1}{x - 3} = \infty \), \( \lim_{x \to 3^-} \frac{1}{x - 3} = -\infty \))

2. (2 points) If \( H(x) = \begin{cases} x + 1, & x < 0 \\ 2, & 0 \leq x < 1 \\ 3 - x, & x \geq 1 \end{cases} \), find the following limits. If it does not exist, write DNE.

(a) \( \lim_{x \to 0^-} H(x) = \lim_{x \to 0^-} x + 1 = 1 \)

DNE since \( \lim_{x \to 0^-} H(x) = \lim_{x \to 0^-} x + 1 = 1 \) \( \neq \) \( \lim_{x \to 0^+} H(x) = \lim_{x \to 0^+} 2 = 2 \)

(b) \( \lim_{x \to 1^-} H(x) = \lim_{x \to 1^-} 2 = 2 \)

\( \lim_{x \to 1^-} H(x) = \lim_{x \to 1^-} 2 = 2 \) \( \Rightarrow \lim_{x \to 1^-} H(x) = 2 \)

\( \lim_{x \to 1^+} H(x) = \lim_{x \to 1^+} 3 - x = 3 - 1 = 2 \)
3. (5 points) Recall that \( f(x) \) is continuous at \( x = a \) if and only if all three of the following things happen:

\[
\begin{align*}
(1) \quad & f(a) \text{ exists,} \\
(2) \quad & \lim_{x \to a} f(x) \text{ exists, and} \\
(3) \quad & \lim_{x \to a} f(x) = f(a).
\end{align*}
\]

With this in mind, answer the following questions.

(a) The function

\[
f(x) = \begin{cases} 
  \frac{x^2 - 1}{x - 1}, & x \neq 1, \\
  4, & x = 1 
\end{cases}
\]

is not continuous at \( x = 1 \). Why not?

(3) fails

(b) The function

\[
f(x) = \begin{cases} 
  \frac{1}{3}x + 4, & x < 3 \\
  2x - 1, & x > 3 
\end{cases}
\]

is not continuous at \( x = 3 \). Why not?

(1) and (3) fail

(c) The function

\[
f(x) = \begin{cases} 
  \frac{1}{3}x + 4, & x \leq 3 \\
  2x - 5, & x > 3 
\end{cases}
\]

is not continuous at \( x = 3 \). Why not?

(2) and (3) fail:

\[
\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} \frac{1}{3}x + 4 = \frac{1}{3}(3) + 4 = 5, \\
\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} 2x - 5 = 2(3) - 5 = 1
\]