Extracting an entanglement signature from only classical mutual information

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Outline

• Shannon and von Neumann Entropy
• Mutual Information – I and J
• Summing J in three bases
• Results
Shannon Entropy

- Shannon entropy is a measure of the uncertainty of a random variable
  - A random variable $A$ with probability distribution $p(a)$
  - $H(A) = - \sum_{a \in A} p(a) \log p(a)$
- Measured in “bits” if log is base 2
- Evenly distributed probabilities gives higher entropy
von Neumann Entropy

- von Neumann entropy is the quantum analog of Shannon entropy
  - A quantum state described by the density matrix $\rho$ has von Neumann entropy
    - $S(\rho) = -\text{Tr}(\rho \log \rho)$
  - Reduces to Shannon entropy upon projective measurements
Mutual Information (1/4)

- Consider two random variables:
  - \( A \) with probability distribution \( p(a) \)
  - \( B \) with probability distribution \( p(b) \)
  - Joint probability: \( p(a,b) \)
- We can define the joint entropy:
  \[
  H(A, B) = - \sum_{a \in A} \sum_{b \in B} p(a, b) \log p(a, b)
  \]
- And also the conditional entropy:
  \[
  H(A|B) = - \sum_{a \in A, b \in B} p(a, b) \log \frac{p(a, b)}{\sum_{a \in A} p(a, b)}
  \]
Mutual Information (2/4)

- Mutual information is a measure of how much information $A$ has in common with $B$
  \[ I(A, B) = H(A) + H(B) - H(A, B) \]
- Pictorially:
Mutual Information (3/4)

- Classically equivalent:
  - \( J(A, B) = H(A) - H(A|B) \)
- For a quantum state \( \rho \):
  - \( I(\rho) = S(\rho^A) + S(\rho^B) - S(\rho) \)
  - \( J \) ... is a little more complicated
    - \( J \) assumes knowledge after a measurement – but in what basis?
    - Assume subsystem \( B \) of \( \rho \) is projectively measured, then we have
      - \( J(\rho)_{\{\Pi^B_b\}} := S(\rho^A) - S(\rho|\{\Pi^B_b\}) \)
Mutual Information (4/4)

- Where

  \[ S(\rho|\{\Pi_b^B\}) = \sum_b p(b)S(\rho_b) \]

  \[ \rho_b = \frac{\Pi_b^B \rho \Pi_b^B}{\text{Tr}[\rho \Pi_b^B]} \]

- \( I \) and \( J \) differ in the quantum framework
- The minimized difference \( I-J \) is known as the *quantum discord*
- \( J \) represents the classical correlations in the system
Summing $J$ in three bases (1/2)

- Example:
  - $J$ is maximal (1) for the singlet state in any basis
  - $J$ is maximal for the maximally correlated mixed state in a singlet basis
- What if we sum $J$ in three mutually unbiased bases? (e.g. HV, AD, RL)
  - $M_J = J(\rho)\{\Pi^B_b\} + J(\rho)\{\Pi^B_{b'}\} + J(\rho)\{\Pi^B_{b''}\}$
  - $M_{JC} = J_C(\rho)\{a,b\} + J_C(\rho)\{a',b'\} + J_C(\rho)\{a'',b''\}$
- These quantities have some interesting properties
Summing $J$ in three bases (2/2)

- They
  - are bounded by 1 for separable states (based upon simulations)
  - reach 3 for maximally entangled states
  - take fewer measurements than a CHSH type test
  - are a measure of how much information two parties can share in multiple bases
Results (1/2)

- Simulation

![Graph showing the relationship between concurrence and $M_{Jc}$ with different states represented by various lines and markers.](image-url)
Results (2/2)

- Experimental Setup
- Singlet state (solid)
- Maximally correlated mixed state (hollow)
Thanks for listening!

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· References