More Review Sp. 2015

1. $\int e^{-2x}\,dx$ $\qquad$ $-\frac{1}{2}\int e^{u}\,du$

$u = -2x$ $\qquad$ $-\frac{1}{2}e^{u} + C$

$du = -2dx$

$-\frac{1}{2}du = dx$ $\qquad$ $-\frac{1}{2}e^{-2x} + C$

2. $\int \frac{x^2+4x+1}{x+2}\,dx$

$$\begin{array}{r} x+2 \\ x+2\,\overline{)\,x^2+4x+1} \\ -(x^2+2x) \\ \hline 2x+1 \\ -(2x+4) \\ \hline -3 \end{array}$$

$$\frac{x^2+4x+1}{x+2} = x+2-\frac{3}{x+2}$$

$$\int\left(x+2-\frac{3}{x+2}\right)dx$$

$u = x+2$
$du = dx$

$= \frac{1}{2}x^2 + 2x - 3\ln|x+2| + C$

3. $\displaystyle\int \frac{1}{\sqrt{8-6x-x^2}}\,dx$ $\qquad$ $\displaystyle\int \frac{1}{\sqrt{17-(x+3)^2}}\,dx$

$a=\sqrt{17}$ $\quad u=x+3$

$du=dx$

$-x^2-6x+8$

$-(x^2+6x)+8$

$-(x^2+6x+9-9)+8$

$-(x^2+6x+9)+9+8$

$17-(x+3)^2$

$$\int \frac{1}{\sqrt{a^2-u^2}}\,du$$

$$= \arcsin(u/a)+C$$

$$= \arcsin\left(\frac{x+3}{\sqrt{17}}\right)+C$$

4. $\displaystyle\int \frac{e^x}{e^{2x}+5}\,dx$ $\qquad$ $\displaystyle\int \frac{1}{u^2+a^2}\,du$

$u=e^x$ $\quad a=\sqrt{5}$

$du=e^x\,dx$

$$= \frac{1}{a}\arctan(u/a)+C$$

$$= \frac{1}{\sqrt{5}}\arctan\left(\frac{e^x}{\sqrt{5}}\right)+C$$

#5

$$\int_{-3}^{0} \frac{x}{\sqrt{1-x}}\,dx$$

$u = 1-x \qquad x = 1-u$

$du = -dx$

if change limits:

$x = -3 \rightarrow u = 1-(-3) = 4$

$x = 0 \rightarrow u = 1-0 = 1$

$$-\int_{4}^{1} \frac{1-u}{u^{1/2}}\,du$$

$$-\int_{4}^{1} \left(u^{-1/2} - u^{1/2}\right) du$$

$$-\left[2u^{1/2} - \frac{2}{3}u^{3/2}\right]_{4}^{1}$$

$$= -\left[\left(2(1) - \frac{2}{3}(1)\right) - \left(2(2) - \frac{2}{3}(8)\right)\right]$$

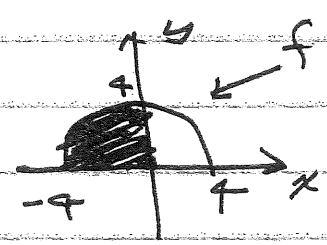
$$= -\left[\frac{4}{3} - \left(-\frac{4}{3}\right)\right] = -\frac{8}{3}$$

#6. $\int \frac{\cos x}{1+\sin x}\,dx$ $\qquad \int \frac{du}{u} = \ln|u| + C$

$u = 1+\sin x$ $\qquad = \ln|1+\sin x| + C$

$du = \cos x\,dx$

#7. $\int_{-4}^{0} \sqrt{16-x^2}\,dx$



$f(x) = \sqrt{16-x^2}$

semi-circle

radius $r=4$

center: $(0,0)$

$= \frac{1}{4}\pi(4)^2 = 4\pi$

~~#8~~ $\int \sec 2y\,dy$ $\qquad \frac{1}{2}\int \sec u\,du$

$u = 2y$

$du = 2dy$ $\qquad = \frac{1}{2}\ln|\sec u + \tan u| + C$

$\frac{1}{2}du = dy$

$= \frac{1}{2}\ln|\sec 2y + \tan 2y| + C$

#9 $\int_0^{\pi/2} (\sin x - \cos x)\,dx = \left[-\cos x - \sin x\right]_0^{\pi/2}$

$= \left(-\cos \pi/2 - \sin \pi/2\right) - \left(-\cos 0 - \sin 0\right)$

$= (0-1) - (-1-0) = 2$

#10 $\int \frac{x}{\sqrt{4-x^2}}\,dx \qquad \int x(4-x^2)^{-1/2}\,dx$

$u = 4 - x^2$

$du = -2x\,dx$

$-\frac{1}{2}\,du = x\,dx$

$= -\frac{1}{2}\int u^{-1/2}\,du$

$= -\frac{1}{2}\left(2u^{1/2}\right) + C$

$= -u^{1/2} + C$

$= -(4-x^2)^{1/2} + C$