Series 9.2-9.4

a. $\sum_{n=0}^{\infty} \frac{2^{n-1}}{3^n} = \sum_{n=0}^{\infty} \frac{1}{2}\left(\frac{2}{3}\right)^n$ geometric

$r = 2/3 \quad 0 < |2/3| < 1$ conv. $S = \frac{\frac{1}{2}}{1-2/3} = \frac{3}{2}$

b. $\sum_{n=1}^{\infty} \frac{2^n}{2^{n+1}+1}$ nth term test

$$\lim_{n\to\infty} \frac{2^n}{2^{n+1}+1} = \lim_{n\to\infty} \frac{\cancel{\ln 2}\, 2^n}{\cancel{\ln 2}\, 2^{n+1}} = \frac{1}{2} \neq 0$$

series div.

c. $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$ LCT to div. p-series $\sum_1^{\infty} \frac{1}{n}$

$$\lim_{n\to\infty} \left(\frac{\frac{n}{n^2+1}}{\frac{1}{n}}\right) = \lim_{n\to\infty}\left(\frac{n}{n^2+1}\cdot\frac{n}{1}\right) = \lim_{n\to\infty} \frac{n^2}{n^2+1} = 1$$

finite and pos.

both series div.

d. $\sum_{n=0}^{\infty} \frac{1}{n^2+3n+2}$

$$\frac{1}{n^2+3n+2} = \frac{A}{n+1} + \frac{B}{n+2}$$

$1 = A(n+2) + B(n+1)$

$n=-2$ $\quad 1 = -B \rightarrow B = -1$

$n=-1$ $\quad 1 = A$

$$\sum_{n=0}^{\infty} \frac{1}{n+1} - \frac{1}{n+2} = \underbrace{\left(1 - \frac{1}{2}\right)}_{n=0} + \underbrace{\left(\frac{1}{2} - \frac{1}{3}\right)}_{n=1} + \underbrace{\left(\frac{1}{3} - \frac{1}{4}\right)}_{n=2} + \ldots$$

$S_2 = 1 - \frac{1}{4}$ so $S_n = 1 - \frac{1}{n+2}$

$S = \lim_{n\to\infty} S_n = S = \lim_{n\to\infty} \left(1 - \frac{1}{n+2}\right) = 1$ conv.

e. $\sum_{n=1}^{\infty} \frac{1}{n\sqrt[3]{n}} = \sum_{n=1}^{\infty} \frac{1}{n^{4/3}}$ p-series $p = 4/3 > 1$ conv.

f. $\sum_{n=0}^{\infty} \left(\pi/2\right)^n$ geometric $r = \pi/2$

$|\pi/2| \geq 1$ div.

g. $\sum_{n=1}^{\infty} \frac{n^2}{n^4+1}$ use LCT to conv. p-series $\sum_{1}^{\infty} \frac{1}{n^2}$

$$\lim_{n\to\infty} \left(\frac{\frac{n^2}{n^4+1}}{\frac{1}{n^2}} \right) = \lim_{n\to\infty} \left(\frac{n^2}{n^4+1} \cdot \frac{n^2}{1} \right) = \lim_{n\to\infty} \left(\frac{n^4}{n^4+1} \right)$$

$= 1$ pos. finite

both series converge

h. $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \cdots = \sum_{n=0}^{\infty} \frac{n+1}{n+2}$

or

$$\sum_{n=1}^{\infty} \frac{n}{n+1}$$

n^{th} term test

$\lim_{n\to\infty} \left(\frac{n+1}{n+2} \right) = 1 \neq 0$ series diverges

i. $\sum_{n=1}^{\infty} \frac{2^n}{n^2+1}$

n^{th} term test

$\lim_{n\to\infty} \frac{2^n}{n^2+1} = \frac{\infty}{\infty}$

$\lim_{n\to\infty} \frac{\ln 2 \cdot 2^n}{2n} = \frac{\infty}{\infty}$

$\lim_{n\to\infty} \frac{(\ln 2)^2\, 2^n}{2} = \infty \neq 0$ series diverges

j. $\sum_{n=0}^{\infty} \frac{(-1)^n 5}{2^n} = \sum_{n=0}^{\infty} 5\left(-\frac{1}{2}\right)^n$

geometric $r = -1/2$ $0 < |-1/2| < 1$ converges

$S = \frac{5}{1+1/2}$

$= \frac{5}{3/2}$

$= \frac{10}{3}$