Math 140 Final Exam
Review for Math 141

13. $\int \left(\frac{2x + 5x^{1/2} - 4}{x} \right) dx$

$= \int x^{-1}(2x + 5x^{1/2} - 4)\,dx$ or $\int \left(\frac{2x}{x} + \frac{5x^{1/2}}{x} - \frac{4}{x} \right) dx$

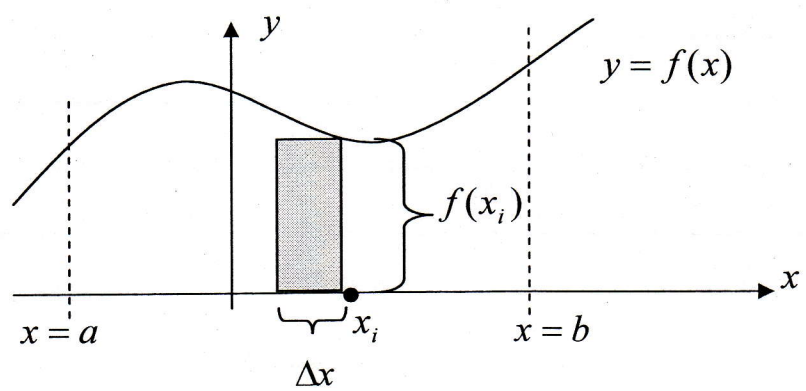
$= \int (2 + 5x^{-1/2} - 4(1/x))\,dx$

$= 2x + 10x^{1/2} - 4\ln|x| + C$

14. Complete the limit definition of the area of a region,

The limit definition of the area of a region bounded by the graphs of $y = f(x)$, $x = a$, $x = b$ and the x-axis is given by

$$A = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

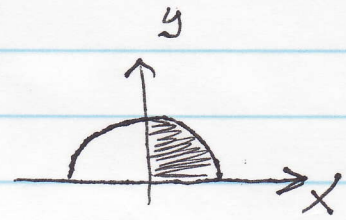


where

Δx is the width of each rectangle, and
x_i are the right endpoints of each rectangle, and
$f(x_i)$ are the heights of each rectangle.

15. $\int_0^1 \sqrt{1-x^2}\,dx = \frac{1}{4}\pi(1)^2 = \pi/4$

$f(x) = \sqrt{1-x^2}$, $x=0$, $x=1$, x-axis



16. $\int \frac{x}{\sqrt{25-x^2}}\,dx = \int x(25-x^2)^{-1/2}\,dx$

$u = 25-x^2$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

substitute: $-\frac{1}{2}\int u^{-1/2}\,du$

$= -\frac{1}{2}(2u^{1/2}) + c$

$= -u^{1/2} + c$

$= -(25-x^2)^{1/2} + c$

17. $\int (1+\tan y)\,dy = y - \ln|\cos y| + c$

18. $\int \frac{1}{\sqrt{6-4x-x^2}}\,dx$

complete the square

$-x^2-4x+6$

$-(x^2+4x+4-4)+6$

$-(x+2)^2+10$

$10-(x+2)^2$

$\int \frac{1}{\sqrt{10-(x+2)^2}}\,dx$

$a = \sqrt{10}$

$u = x+2$

$du = dx$

$= \arcsin\left(\frac{x+2}{\sqrt{10}}\right) + c$

19. $\displaystyle\int \frac{3x^3+15x+1}{x^2+5}\,dx$ long division

$$\begin{array}{r|l} & 3x \\ \hline x^2+5 & 3x^3+15x+1 \\ & -(3x^3+15x) \\ \hline & 1 \end{array}$$

$$\int 3x + \frac{1}{x^2+5}\,dx$$

$u = x \quad a = \sqrt{5}$

$du = dx$

$$= \frac{3}{2}x^2 + \frac{1}{\sqrt{5}}\arctan\frac{x}{\sqrt{5}} + C$$

20. $\displaystyle\int xe^{x^2}\,dx$

$u = x^2$

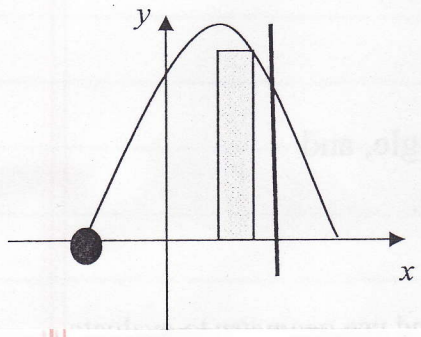
$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

substitute: $\displaystyle\frac{1}{2}\int e^u\,du = \frac{1}{2}e^u + C$

$$= \frac{1}{2}e^{x^2} + C$$

21. Find the area of the region bounded by the graphs of the equations, $f(x) = 4 + 3x - x^2$, $x = 2$ and the x-axis, as shown, in the figure.



x-intercepts: $4 + 3x - x^2 = 0$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$x = 4,\ x = -1$

$$A = \int_{-1}^{2} (4 + 3x - x^2)\,dx = \left[4x + \frac{3}{2}x^2 - \frac{1}{3}x^3\right]_{-1}^{2}$$

$$= \left(4(2) + \frac{3}{2}(2)^2 - \frac{1}{3}(2)^3\right) - \left(4(-1) + \frac{3}{2}(-1)^2 - \frac{1}{3}(-1)^3\right)$$

$$= \frac{27}{2}$$

Bonus:

$y = y' \cos^2 x$

$\frac{1}{\cos^2 x}\,dx = \frac{1}{y}\,dy$

$\sec^2 x\,dx = \frac{1}{y}\,dy$

$\int \sec^2 x\,dx = \int \frac{1}{y}\,dy$

$\tan x + C_1 = \ln|y|$

$y = e^{\tan x + C_1}$

$y = Ce^{\tan x}\ (C = e^{C_1})$