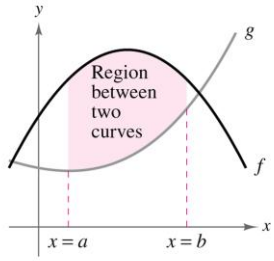
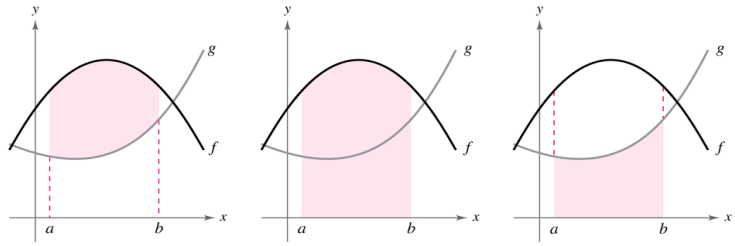
Chapter 7 Applications of Integration

Section 7.1 The Area of a Region between Two Curves



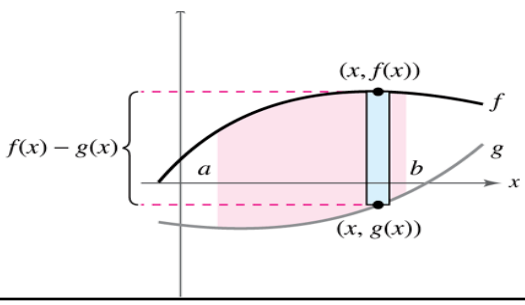


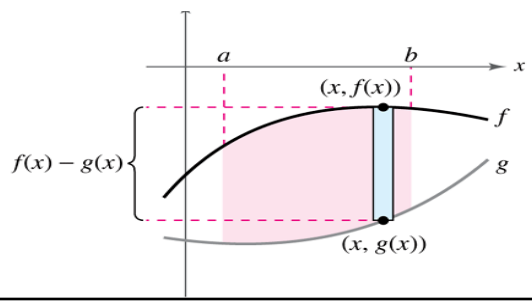
Area of region between f and g	=	Area of region under f	−	Area of region under g
$\int_a^b [f(x) - g(x)]\,dx$	=	$\int_a^b f(x)\,dx$	−	$\int_a^b g(x)\,dx$

Area of a Region Between Two Curves

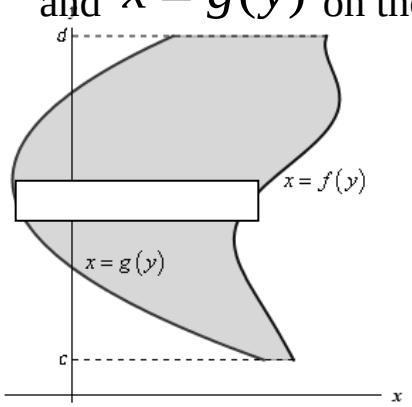
If f and g are continuous on $[a, b]$ and $g(x) \le f(x)$ for all x in $[a, b]$, then the area of the region bounded by the graphs of f and g and the vertical lines $x = a$ and $x = b$ is

$$A = \int_a^b [f(x) - g(x)]\,dx.$$



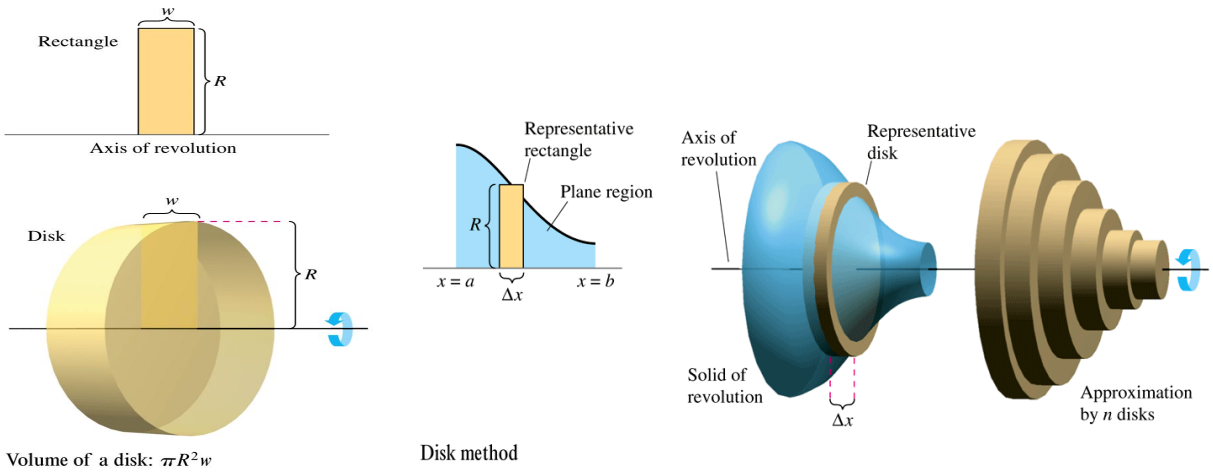


If f and g are continuous on the interval, $[c,d]$, then the area between $x = f(y)$ and $x = g(y)$ on the interval $[c,d]$ with $f(y) \ge g(y)$ is



$$A = \int_c^d \left(f(y) - g(y)\right)dy$$

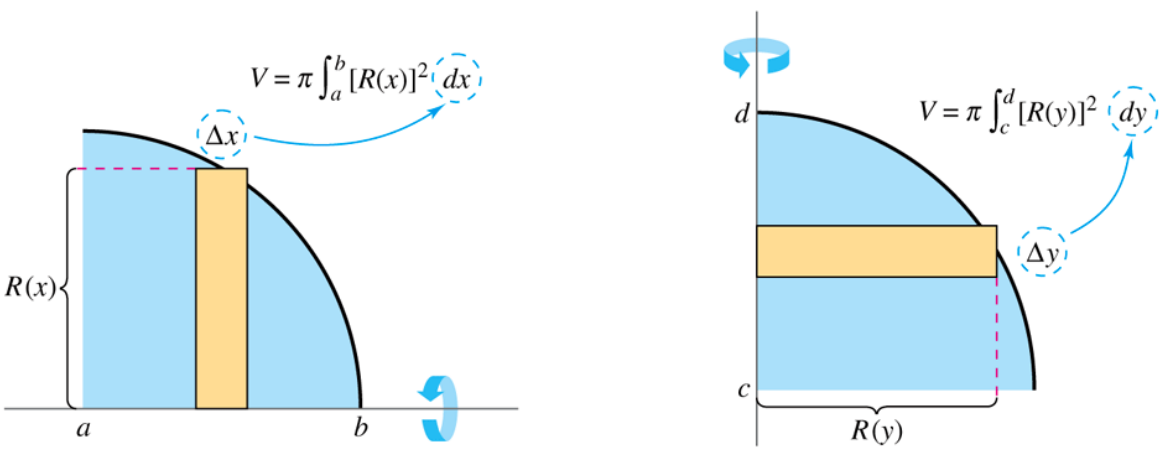
Section 7.2 Volumes of Solids of Revolution: Disk Method



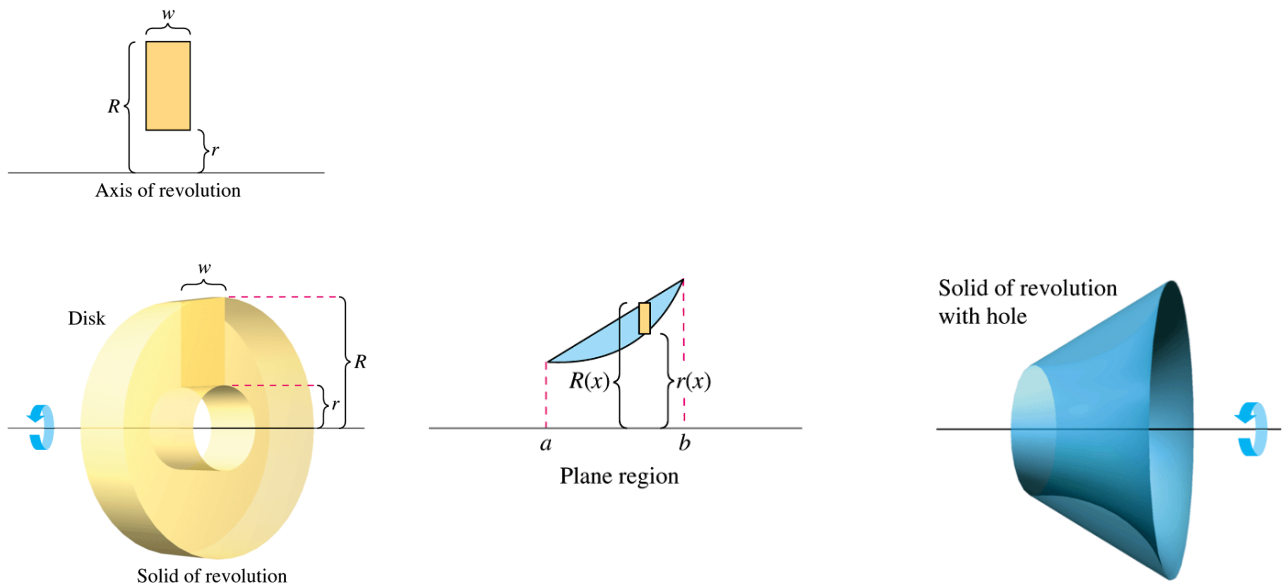
The Disk Method

To find the volume of a solid of revolution with the **disk method,** use one of the following, as shown in Figure 7.15.

Horizontal Axis of Revolution	*Vertical Axis of Revolution*
$\text{Volume} = V = \pi \int_a^b [R(x)]^2\, dx$	$\text{Volume} = V = \pi \int_c^d [R(y)]^2\, dy$



Section 7.2 (continued) Washer Method (Disk Method with a “HOLE”)



Volumes of Solids with Known Cross Sections

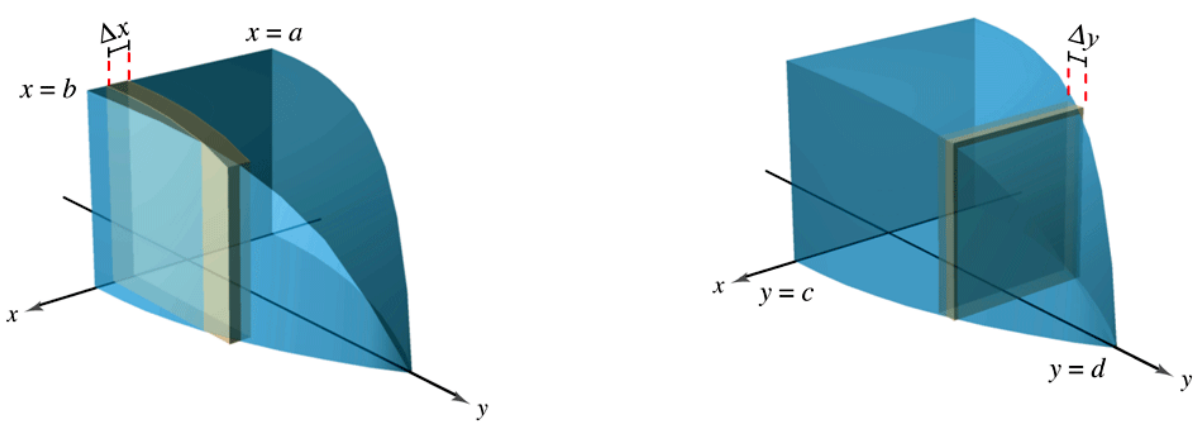
Volumes of Solids with Known Cross Sections

1. For cross sections of area $A(x)$ taken perpendicular to the x-axis,

$$\text{Volume} = \int_a^b A(x)\,dx.$$ See Figure 7.24(a).

2. For cross sections of area $A(y)$ taken perpendicular to the y-axis,

$$\text{Volume} = \int_c^d A(y)\,dy.$$ See Figure 7.24(b).



(a) Cross sections perpendicular to x-axis

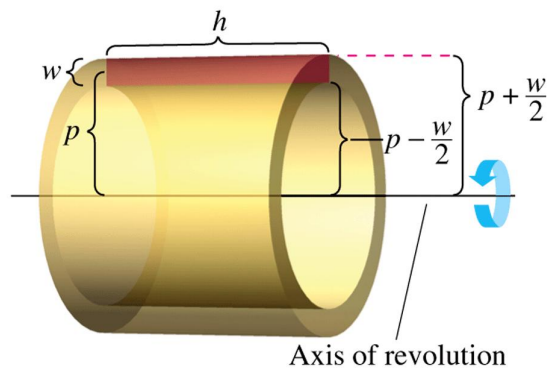
(b) Cross sections perpendicular to y-axis

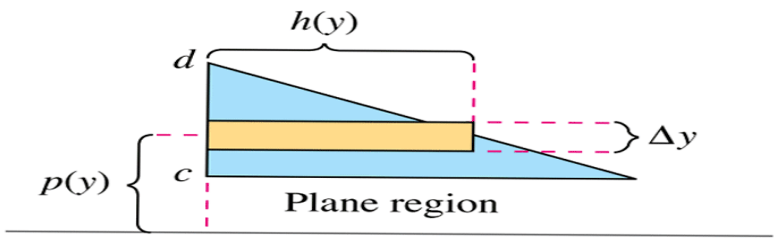
Section 7.3 Shell Method

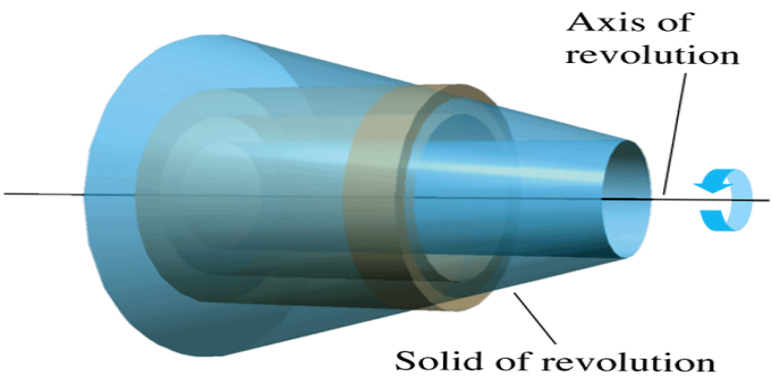
$$width = \Delta x \quad or \quad \Delta y$$

$$p = "average" \text{ radius of a shell}$$

$$h = height \text{ of a shell}$$



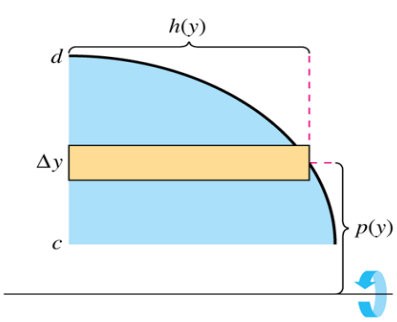




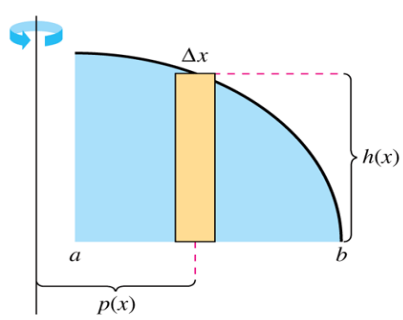
The Shell Method

To find the volume of a solid of revolution with the **shell method,** use one of the following, as shown in Figure 7.29.

Horizontal Axis of Revolution	*Vertical Axis of Revolution*
Volume $= V = 2\pi \int_c^d p(y)h(y)\,dy$	Volume $= V = 2\pi \int_a^b p(x)h(x)\,dx$

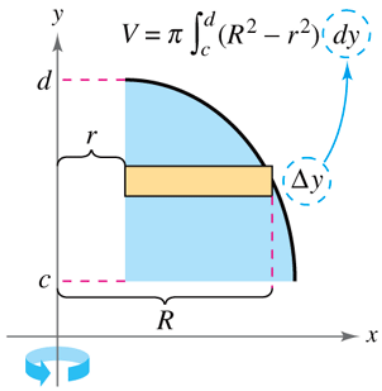


Horizontal axis of revolution

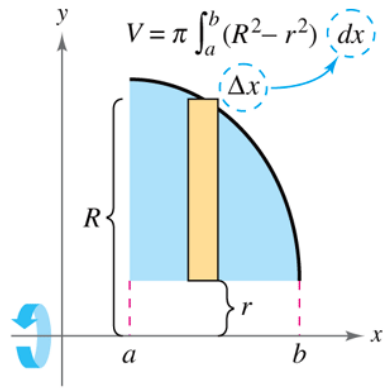


Vertical axis of revolution

Summary of Volumes of Solids of Revolution

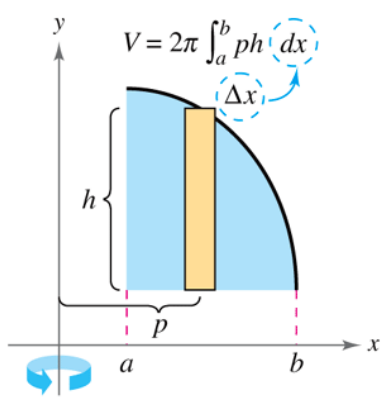


Vertical axis
of revolution

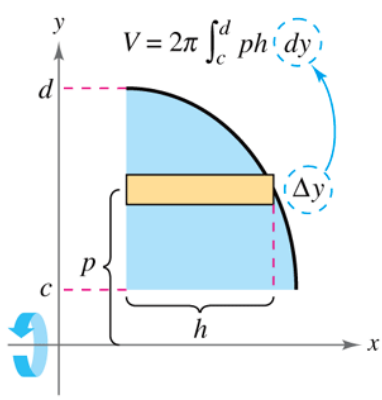


Horizontal axis
of revolution

Disk method: Representative rectangle is perpendicular to the axis of revolution.



Vertical axis
of revolution



Horizontal axis
of revolution

Shell method: Representative rectangle is parallel to the axis of revolution.

Section 7.4 Arc Length and Surface Area of Revolution

Arc Length

Definition of Arc Length

Let the function given by $y = f(x)$ represent a smooth curve on the interval $[a, b]$. The **arc length** of f between a and b is

$$s = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx.$$

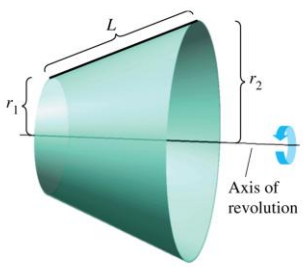
Similarly, for a smooth curve given by $x = g(y)$, the **arc length** of g between c and d is

$$s = \int_c^d \sqrt{1 + [g'(y)]^2}\, dy.$$

Surface Area of Revolution

Definition of Surface of Revolution

If the graph of a continuous function is revolved about a line, the resulting surface is a **surface of revolution.**



Definition of the Area of a Surface of Revolution

Let $y = f(x)$ have a continuous derivative on the interval $[a, b]$. The area S of the surface of revolution formed by revolving the graph of f about a horizontal or vertical axis is

$$S = 2\pi \int_a^b r(x)\sqrt{1 + [f'(x)]^2}\, dx \qquad y \text{ is a function of } x.$$

where $r(x)$ is the distance between the graph of f and the axis of revolution. If $x = g(y)$ on the interval $[c, d]$, then the surface area is

$$S = 2\pi \int_c^d r(y)\sqrt{1 + [g'(y)]^2}\, dy \qquad x \text{ is a function of } y.$$

where $r(y)$ is the distance between the graph of g and the axis of revolution.