Math 141 Section 9.2 Series Worksheet

1.a. $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$

$$\sum_{n=0}^{\infty} (1)\left(\frac{1}{2}\right)^n$$

$S_0 = 1$

$S_1 = 1 + \frac{1}{2} = \frac{3}{2}$

$S_2 = 1 + \frac{1}{2} + \frac{1}{4} = \frac{7}{4}$

$S_3 = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \frac{15}{8}$

$S_4 = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} = \frac{31}{16}$

b. $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} + \ldots$

$$\sum_{n=0}^{\infty} (1)\left(-\frac{1}{2}\right)^n$$

$S_0 = 1$

$S_1 = 1 - \frac{1}{2} = \frac{1}{2}$

$S_2 = 1 - \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$

$S_3 = 1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} = \frac{5}{8}$

$S_4 = 1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} = \frac{11}{16}$

c. $\sum 1 + \frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \ldots$

$$\sum_{n=0}^{\infty} (1)(2/3)^n$$

$S_0 = 1$

$S_1 = 1 + \frac{2}{3} = 5/3$

$S_2 = 1 + \frac{2}{3} + \frac{4}{9} = \frac{19}{9}$

$S_3 = 1 + \frac{2}{3} + \frac{4}{9} + \frac{8}{27} = \frac{65}{27}$

$S_4 = 1 + \frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \frac{16}{81} = \frac{211}{81}$

d. $\sum_{n=0}^{\infty} 10(2/5)^n$

$S_0 = 10$

$S_1 = 10 + 4 = 14$

$S_2 = 10 + 4 + \frac{8}{5} = \frac{78}{5}$

$S_3 = 10 + 4 + \frac{8}{5} + \frac{16}{25} = \frac{406}{25}$

$S_4 = 10 + 4 + 8/5 + \frac{16}{25} + \frac{32}{125} = \frac{2062}{125}$

1 e. $$\sum_{n=0}^{\infty} \frac{4^{n+1}}{5^n} = \sum_{n=0}^{\infty} 4\left(\frac{4}{5}\right)^n$$

$$S_0 = 4$$

$$S_1 = 4 + \frac{16}{5} = \frac{36}{5}$$

$$S_2 = 4 + \frac{16}{5} + \frac{64}{25} = \frac{244}{25}$$

$$S_3 = 4 + \frac{16}{5} + \frac{64}{25} + \frac{256}{125} = \frac{1476}{125}$$

$$S_4 = 4 + \frac{16}{5} + \frac{64}{25} + \frac{256}{125} + \frac{1024}{625} = \frac{8404}{625}$$

f. $$\sum_{n=0}^{\infty} \frac{5^{n-1}}{2^n} = \sum_{n=0}^{\infty} \frac{1}{5}\left(\frac{5}{2}\right)^n$$

$$S_0 = \frac{1}{5}$$

$$S_1 = \frac{1}{5} + \frac{1}{2} = \frac{7}{10}$$

$$S_2 = \frac{1}{5} + \frac{1}{2} + \frac{5}{4} = \frac{39}{20}$$

$$S_3 = \frac{1}{5} + \frac{1}{2} + \frac{5}{4} + \frac{25}{8} = \frac{203}{40}$$

$$S_4 = \frac{1}{5} + \frac{1}{2} + \frac{5}{4} + \frac{25}{8} + \frac{125}{16} = \frac{1031}{80}$$

1. a. $\sum_{n=0}^{\infty} (1)(1/2)^n$

$r = 1/2$, $|1/2| < 1$ convergent geometric

$$S = \frac{1}{1-1/2} = 2$$

b. $\sum_{n=0}^{\infty} (1)(-1/2)^n$

$r = -1/2$, $|-1/2| < 1$, conv. geometric

$$S = \frac{1}{1-(-1/2)} = \frac{1}{3/2} = \frac{2}{3}$$

c. $\sum_{n=0}^{\infty} (1)(2/3)$

$r = 2/3$, $|2/3| < 1$, conv. geom.

$$S = \frac{1}{1-2/3} = 3$$

d. $\sum_{n=0}^{\infty} 10(2/5)^n$

$r = 2/5$, $|2/5| < 1$, conv. geom.

$$S = \frac{10}{1-2/5} = \frac{10}{3/5} = 50/3$$

e. $\sum_{n=0}^{\infty} 4(4/5)^n$

$r = 4/5$, $|4/5| < 1$, conv. geom.

$S = \frac{4}{1-4/5} = \frac{4}{1/5} = 20$

f. $\sum_{n=0}^{\infty} \frac{1}{5}\left(\frac{5}{2}\right)^n$

$r = 5/2$, $|5/2| \geq 1$ divergent geom. <u>no sum!</u>

2. $\sum_{n=0}^{\infty} \left(\frac{1}{n+1} - \frac{1}{n+3}\right) = (1 - 1/3) + (1/2 - 1/4) + (1/3 - 1/5) + \ldots$

$S_4 = (1 - \cancel{1/3}) + (1/2 - \cancel{1/4}) + (\cancel{1/3} - 1/5) + (\cancel{1/4} - 1/6)$

$= 1 + \frac{1}{2} - \frac{1}{5} - \frac{1}{6}$

$S_n = 1 + \frac{1}{2} - \frac{1}{n+1} - \frac{1}{n+2}$

$S = \lim_{n\to\infty} S_n = \lim_{n\to\infty}\left(1 + 1/2 - \frac{1}{n+1} - \frac{1}{n+2}\right)$

$= 1 + 1/2 - 0 - 0 = 3/2$ converges

telescoping series

3. $\sum_{n=1}^{\infty} \frac{(n+1)!}{n!}$

n^{th} term test

$\lim_{n\to\infty} \frac{(n+1)!}{n!} = \lim_{n\to\infty} (n+1) = \infty \neq 0$ series diverges

4. $\sum_{n=0}^{\infty} \frac{2n^2+1}{n^2+1}$

n^{th} term test

$\lim_{n\to\infty} \frac{2n^2+1}{n^2+1} = \frac{1}{2} \neq 0$ series diverges

(use L'Hopitals)

5 $\sum_{n=1}^{\infty} \frac{n}{2n^2+1}$

n^{th} term test

$\lim_{n\to\infty} \frac{n}{2n^2+1} = 0$ test fails

(use L'Hopital's)

6. $\sum_{n=1}^{\infty} \frac{(-1)^n n}{2n+1}$

n^{th} term test

$\lim_{n\to\infty} \frac{(-1)^n n}{2n+1} = DNE \neq 0$ series diverges

(odd terms $\to -\frac{1}{2}$
even terms $\to 1/2$)

7 $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n^2+1}$

nth term test

$\displaystyle\lim_{n\to\infty} \frac{2^n}{n^2+1} = \infty \neq 0$ series diverges

(Use L'Hopital's)

8. $\displaystyle\sum_{n=0}^{\infty} 8\left(-\frac{4}{5}\right)^n$

geometric series

$r = -4/5$, $|-4/5| < 1$, converges $S = \dfrac{8}{1-(-4/5)}$

$S = \dfrac{8}{9/5}$

$S = 40/9$

9. $\displaystyle\sum_{n=0}^{\infty} \frac{4^{n+1}}{5^n} = \sum_{n=0}^{\infty} 4(4/5)^n$

geometric series

$r = 4/5$, $|4/5| < 1$, converges $S = \dfrac{4}{1-4/5} = 20$

10. $\displaystyle\sum_{n=0}^{\infty} \frac{2^n}{2^n+1}$

nth term test

$\displaystyle\lim_{n\to\infty} \frac{2^n}{2^n+1} = 1 \neq 0$ series diverges

(use L'Hopital's)

$$\sum_{n=1}^{\infty} \frac{2}{n^2+2n} = \sum_{n=1}^{\infty} \frac{2}{n(n+2)}$$ telescoping

use partial fractions

$$\frac{A}{n} + \frac{B}{n+2} = \frac{2}{n(n+2)}$$

$A(n+2) + Bn = 2$

$n = -2 \qquad -2B = 2$

$B = -1$

$n = 0 \qquad 2A = 2$

$A = 1$

$$\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+2} \right)$$

$$S_4 = \left(1 - \frac{1}{3}\right) + \left(\frac{1}{2} - \frac{1}{4}\right) + \left(\frac{1}{3} - \frac{1}{5}\right) + \left(\frac{1}{4} - \frac{1}{6}\right)$$

$$S_4 = 1 + \frac{1}{2} - \frac{1}{5} - \frac{1}{6}$$

$$S_n = 1 + \frac{1}{2} - \frac{1}{n+1} - \frac{1}{n+2}$$

$$S = \lim_{n \to \infty} S_n = \lim_{n \to \infty} \left(3/2 - \frac{1}{n+1} - \frac{1}{n+2} \right)$$

$$= 3/2 - 0 - 0 = 3/2$$

converges by definition of convergent series