9.1 Sequences

1.a. $a_n = \frac{2^{n-1}}{n^2}$

$\left\{ \frac{1}{1}, \frac{2}{4}, \frac{4}{9}, \frac{8}{16}, \frac{16}{25}, \ldots \right\}$

b. $a_n = 9\left(-\frac{2}{3}\right)^n$

Known as a geometric $a_n = ar^n$

$\left\{ 9, -\frac{18}{3}, \frac{36}{9}, -\frac{72}{27}, \frac{144}{81}, \ldots \right\}$

c. $a_n = \frac{n!}{2^n}$

$\left\{ \frac{1}{1}, \frac{1}{2}, \frac{2}{4}, \frac{6}{8}, \frac{24}{16}, \ldots \right\}$

d. $a_n = \frac{2^{n+1}}{3^n}$

could be rewritten as $2\left(\frac{2}{3}\right)^n$ geometric

$a_n = 2\left(\frac{2}{3}\right)^n$

$\left\{ \frac{4}{3}, \frac{8}{9}, \frac{16}{27}, \frac{32}{81}, \frac{64}{243}, \ldots \right\}$

e. $a_n = \frac{(-1)^n}{n(n-1)}$

$\left\{ \frac{1}{2}, -\frac{1}{6}, \frac{1}{12}, -\frac{1}{20}, \frac{1}{30}, \ldots \right\}$

f. $a_n = \frac{x^n}{n!}$

$\left\{ \frac{1}{1}, \frac{x}{1}, \frac{x^2}{2}, \frac{x^3}{6}, \frac{x^4}{24}, \ldots \right\}$

g. $a_n = \dfrac{(-1)^n x^{2n+1}}{(2n+1)!}$

$$\left\{ \frac{x}{1}, -\frac{x^3}{6}, \frac{x^5}{120}, -\frac{x^7}{5040}, \frac{x^9}{362{,}880}, \ldots \right\}$$

h. $a_n = \dfrac{x^{2n}}{(2n)!}$

$$\left\{ \frac{1}{1}, \frac{x^2}{2}, \frac{x^4}{24}, \frac{x^6}{720}, \frac{x^8}{40{,}320}, \ldots \right\}$$

2. a. $\left\{ \frac{1}{3}, \frac{1}{8}, \frac{1}{15}, \frac{1}{24}, \ldots \right\}$ $\quad a_n = \dfrac{1}{n^2-1}, \; n \geq 2$

b. $\left\{ \frac{1}{1}, \frac{1}{3}, \frac{1}{7}, \frac{1}{15}, \frac{1}{31}, \ldots \right\}$ $\quad a_n = \dfrac{1}{2^n - 1}, \; n \geq 1$

c. $\left\{ \frac{1}{1}, \frac{2}{1}, \frac{5}{2}, \frac{10}{6}, \frac{17}{24}, \ldots \right\}$ $\quad a_n = \dfrac{n^2+1}{n!}, \; n \geq 0$

d. $\left\{ -\frac{1}{1}, \frac{2}{1}, -\frac{4}{2}, \frac{8}{6}, -\frac{16}{24}, \ldots \right\}$ $\quad a_n = \dfrac{(-1)^{n+1} 2^n}{n!}, \; n \geq 0$

e. $\left\{ 1, -\frac{3}{5}, \frac{9}{25}, -\frac{27}{125}, \frac{81}{625}, -\frac{243}{3125}, \ldots \right\}$ $\quad a_n = (1)\left(-\frac{3}{5}\right)^n, \; n \geq 0$

geometric

3. a. $\lim_{n\to\infty} \frac{n}{2n+1} = \frac{1}{2}$ converges

L'Hopital's or equal powers of n

b. $\lim_{n\to\infty} \frac{n^2}{n+1} = \infty$ diverges

L'Hopital's or powers of n

c. $\lim_{n\to\infty} \frac{(-1)^n}{(n+1)!} = 0$ b/c $\lim_{n\to\infty} \left|\frac{(-1)^n}{(n+1)!}\right| = 0$ converges

d. $\lim_{n\to\infty} \frac{2^{n+1}}{2^{n-1}+1} = 4$ converges

L'Hopital's or simplify by multiplying by $\frac{1}{2^n}$

e. $\lim_{n\to\infty} \frac{2^n}{(2n)!} = 0$ converges

exponentials vs. factorials

f. $\lim_{n\to\infty} 1 - \frac{2^n}{2^{n+1}} = \frac{1}{2}$ $(1 - 1/2)$ converges

g. $\lim_{n\to\infty} \frac{1+(-1)^n}{n^2} = 0$ converges

non-zero terms $\to 0$

h $\lim_{n\to\infty} \frac{\sin(n\pi/2)}{n(n+1)} = 0$ converges $\lim_{n\to\infty} \left|\frac{\sin\frac{\pi n}{2}}{n(n+1)}\right| = 0$

non-zero terms $\to 0$

4. $a_1 = 3 \quad a_n = 2a_{n-1} + 1$

$\{3, 7, 15, 31, 63, \ldots\}$

$a_1 = 3$

$a_2 = 2a_1 + 1 = 2(3) + 1 = 7$

$a_3 = 2a_2 + 1 = 2(7) + 1 = 15$

etc.

5. $a_n = \frac{1}{n} - \frac{1}{n+1}$ known as telescoping

$\left\{ \left(1 - \frac{1}{2}\right), \left(\frac{1}{2} - \frac{1}{3}\right), \left(\frac{1}{3} - \frac{1}{4}\right), \left(\frac{1}{4} - \frac{1}{5}\right), \left(\frac{1}{5} - \frac{1}{6}\right), \ldots \right\}$

the above does simplify to $\left\{ \frac{1}{2}, \frac{1}{6}, \frac{1}{12}, \frac{1}{20}, \frac{1}{30}, \ldots \right\}$

but actually thats not the point...